Mathematical methods and its applications
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Lecture – 09
Cauchy - Euler equation

Hello friends. Welcome to my lecture on Cauchy-Euler Equations.

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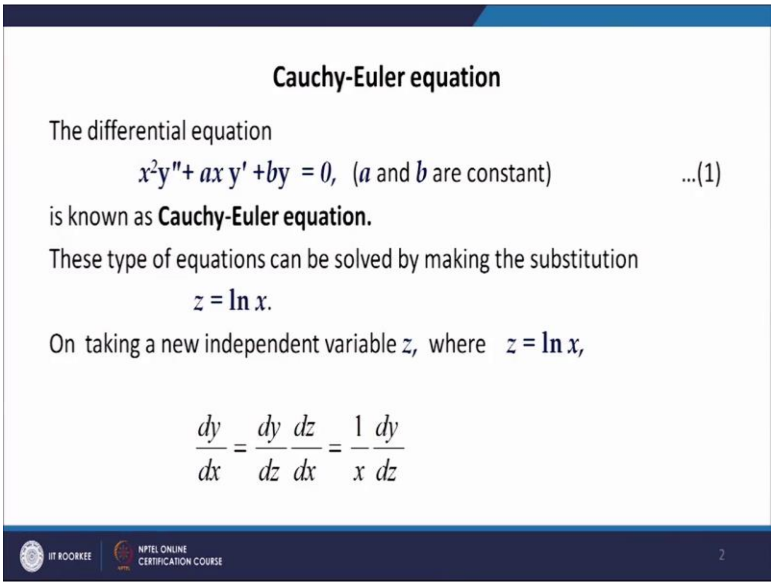

The differential equation of second-order x square y double dash plus ax y dash plus by equal to 0, where a and b are constants is known as a Cauchy-Euler equation. Now, you can see that this is a second-order linear differential equation with variable coefficients, these coefficients of y double dash is the coefficient of y double dash is x square the coefficient of y dash is a into x. So, the coefficients of the derivates are functions of x.

Now, in general an ordinary second-order linear equation with variable coefficients is not easy to solve, but some special clause of second-order linear differential equations with variable coefficients can be solved. So, this Cauchy-Euler equation belongs to one such clause of differential equation, where you can see that the power of the monomials say x square the power of x square is 2, and the derivative y double dash is of second-order. So, similarly in the second term, the power of x is 1 and derivatives of first order and then we have v into y equal to 0. So, in every term, the power of x and the order of the derivative in that term are same. So, such kind of differential equations are called as Cauchy-Euler equations.

So, we are taking a first homogeneous case of actually, this is a homogeneous case of Cauchy-Euler equation. Now, such case such type of equations can be reduced to linear differential equation with constant coefficients. And we know how to solve a linear differential equation with constant coefficients. So, by making the substitution, say z equal to ln x by making the substitution z equal to ln x, we shall convert this equation to Cauchy-Euler equation to ordinary linear differential equation with constant coefficients and solved it there and then replace the wherever z y x. So, let us put z equal to ln x in this, let us consider z equal to ln x now.

So, we are changing the independent variable from x to z by considering z equal to ln x or x equal to e to the power z.

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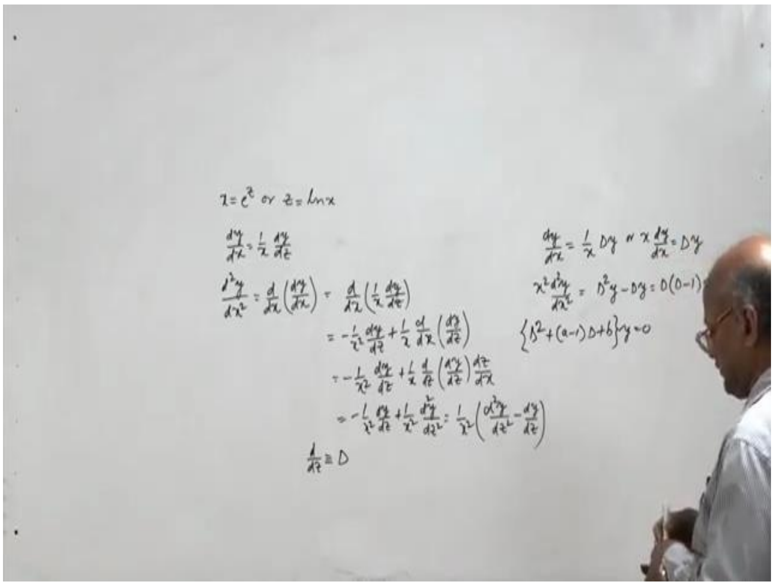

So, when you take z equal to x equal to e to the power z or z equal to ln x let us find the values of the derivatives of y with respect to x. So, when you find dy by dx we can write dy by dz into dz by dx, and dz by dx you can see from here z equal to ln x that it is 1 by x. So, we have dy by dx equal to 1 over x dy by dz now. So, dy by dx we get as 1 over x dy over dz. Now, let us find d square by y dx square. So, d square by y dx square is nothing but the first derivative of dy by dx dy by dx, we can put from here. So, d over dx of 1 over x dy by dz and this is equal to now this is a product of two functions of x. So, we can find the derivative of this product.

So, derivative of 1 over x is minus 1 over x square dy by dz and then 1 over x times d over dx of dy by dz. And dy by dz can be put as d over dx of dy by dx, dy by dx is equal to 1 over x 1 over dy by dz. So, we write as dy by dz. So, d over dx this is minus 1 over x square dy by dx and dy by dz and then 1 over x dy dy by dz sorry, not this is dy by dx ok. So, this is minus 1 over x square dy by dz and then here I can write 1 over x d over dz dy by dz into dz by dx right that is.

So, 1 over x square dy by dz; and dz by dx, we know dz by dx is 1 over x. So, 1 over x square dy y d square y by dz square. So, we get 1 over x square times d square y by dz square minus dy by dz. Or we can say x square d square y by dx square x square d square y by dx square equal to d square y by dz square minus dy by dz. Now, let us denote the differential operator d over dz by D.

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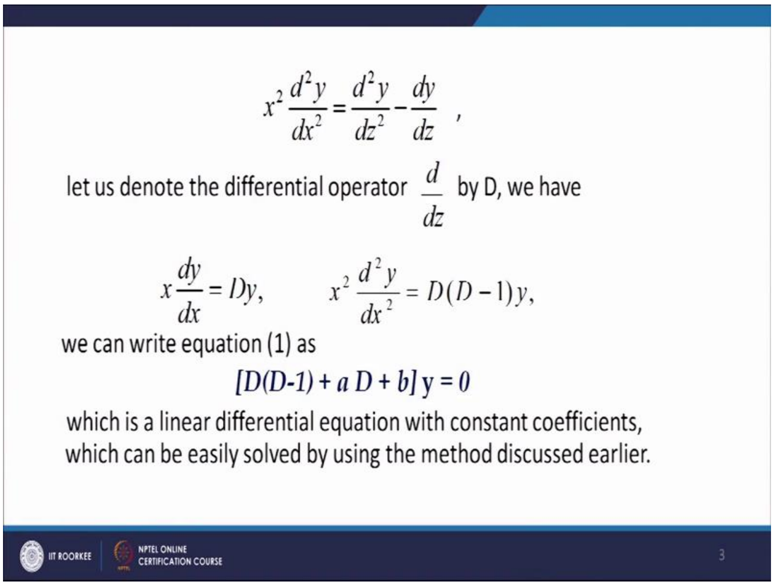

If we denote the differential operator d over dz by D then dy by dx will be 1 over x Dy or we can say x dy by dx will be Dy and d square y by x square d square y by dx square will be this will be equal to d square y by dz square; so D square y minus Dy which is same as D into D minus 1 y now. So, then the equation 1 can be written as the equation 1 you can see this is equation 1. So, x square y double dash is D into D minus 1 by a times x y dash is dy and then we have plus b y equal to 0. So, we get the operator D over D minus 1 plus a D plus b operating on y equal to 0.

Now, this is nothing but D square minus D plus a, so plus a minus 1 into D plus b operating on y equal to 0 which is a linear differential equation with constant coefficients where we independent variable has been changed from x to z. And it can be solved by the method of linear differential equation with constant coefficient which we have already discussed.

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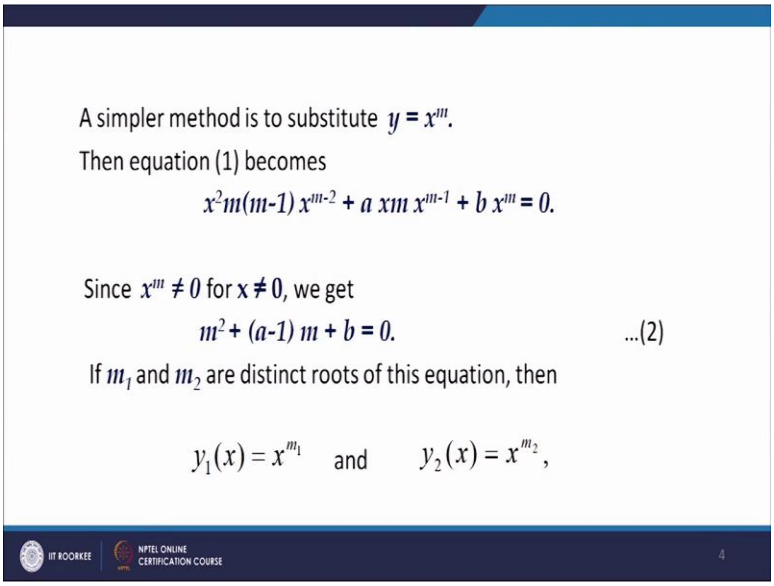

So, we can find the solution of this equation, and then replace z by l n x there. Now, there is another method which is rather simpler to find the solution of the differential equation x square y double dash plus a x y dash plus dy equal to 0. In this method, we put y equal to x to the power m in that equation. So, we will get x square y double dash when you differentiate twice x to the power m you get m into m minus 1 x to the power m minus 2. So, you get m into m minus 1 x to the power m, after you multiply by x square. And similarly a times x y dash will be m into a x to the power m plus b x to the power m equal to 0, so that is what we have written here x square. Now, m into m minus 1 x to the power m minus 2 plus a x m x to the power m minus 1 plus a x m equal to 0.

Now, since x to the power m is not equal to 0 when x is not equal to 0 we shall get from this equation m into m minus 1 plus a x a into m plus b equal to 0, which is nothing but m square plus a minus 1 into m plus b equal to 0. Now, this is a second-order, this is a coordinated equation in m. And if we find the roots of this if they are say m 1 and m 2 if they are distinct roots then you get y 1 solution of the given differential equation as y 1 x equal to x to the power m 1. And the other solution is y 2 x equal to x to the power m 2 now x m 1 and m 2 are distinct. So, y 1 over y 2 will not be a constant and therefore, x to the power m 1 and x to the power m 2 will be linearly independent to each other. So, we can write the form as a fundamental system of solutions of the given equation for all x for which that defined.

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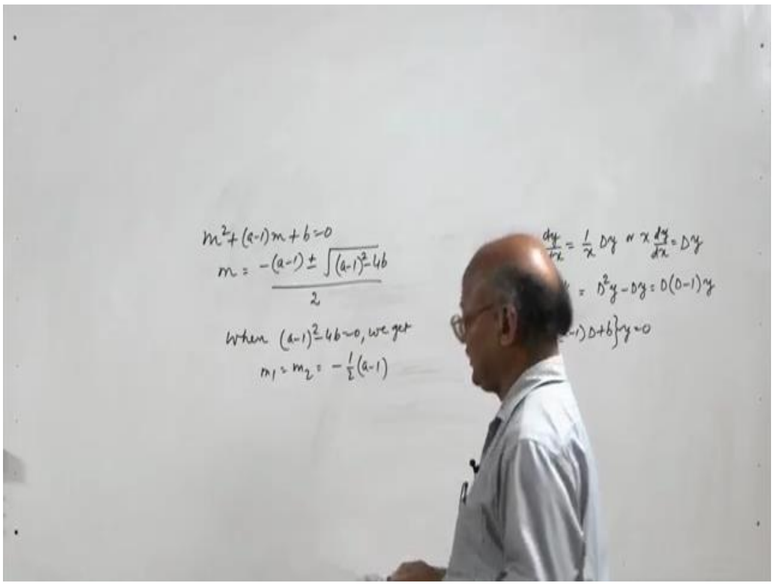

So, general solution of the equation in this case can be written as y x equal to c 1 x to the power m 1 plus c 2 x to the power m 2. Now, the other possibilities that the 2 roots m 1 and m 2 are equal now we had the equation like this. The equation is m square plus a minus 1 into m plus b equal to 0. When we solve it for the values of m, we get minus a minus 1 plus minus plus under root, so when the two roots are equal means when the discriminate is equal to 0. So, when a minus 1 whole square minus 4 v equal to 0, we get the case of double root, we get m 1 equal to m 2 equal to minus half a minus 1.

So, in this case one solution we can write that is x to the power m 1 or x to the power minus half a minus 1, how to obtain the other solution. So, what we will do is we will apply the method of variation of parameters. So, one solution is this is 1 by 2 into 1 minus a, m 1 is 1 by 2 into 1 minus a. So, one solution is y 1 2 equal to x to the power half 1 minus a. Now, let us assume other solution y 2 to be some u times y 1, where u is the unknown function which is to be determined in a such a way that y 2 is a solution of the given differential equation second solution. So, u is a some function of x.

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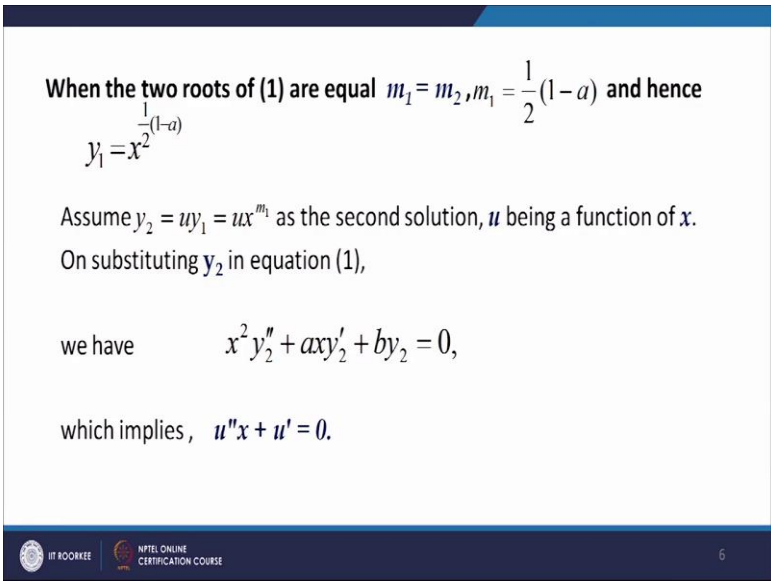

Now, let us substitute y 2 in equation one. So, when we substitute y 2 in equation one since y 2 is a solution of that equation we will get x square y 2 double dash plus a x y 2 dash plus b y 2 equal to 0. Now, let us see how we get u double dash x plus u dash equal to 0. Once we arrive here, we shall be able to do determine the value of u very easily. So, let see how we arrive here. So, let us put our differential equation is this one. So, our differential equation is x square y. So, we have x square y 2 double dash plus a x y 2 dash b plus b y equal to 0, and we have assumed y 2 equal to u times y 1 y 2 equal to u times y 1.

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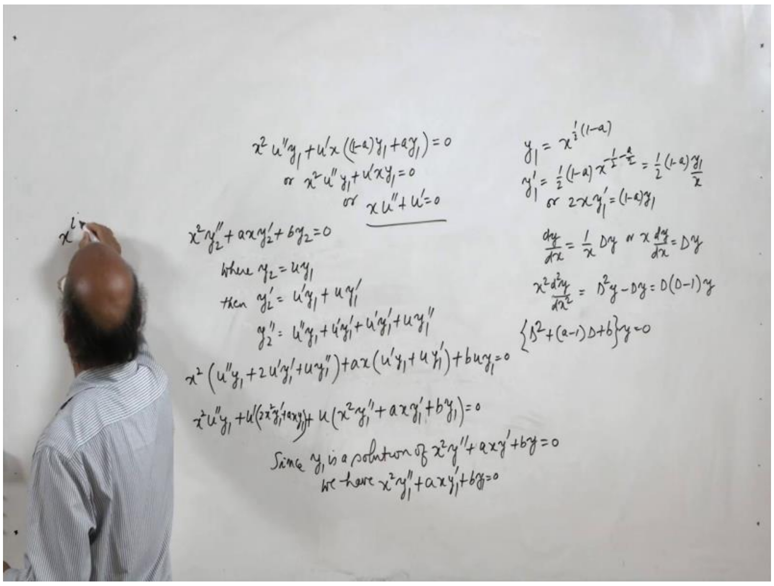

So, let us find its first and second derivatives. So, then y 2 dash equal to u dash y 1 plus u y 1 dash, and y 2 double dash will be equal to u double dash y 1 plus u dash y 1 dash plus u dash y 1 dash plus u y 1 double dash. Let us substitute these values of y 2 y 2 dash and y 2 double dash in this equation we get x square y 2 double dash means u double dash y 1 plus 2 u dash y 1 dash plus u y 1 double dash plus a x y 2 dash will be u dash y 1 plus u y 1 dash plus b u y 1 equal to 0. Now, we have x square u double dash y 1 this is term this one term which contains u double dash and the term containing u dash is what, so we get, let us first write y 1 double dash terms. So, u times I think I will write u times x square y 1 double dash plus u times a x y 1 dash plus b y 1.

What is left out is here we will write this as. So, we have taken this term, we have taken this term, we have taken this term, we have taken that. So, a x u dash y 1 and we have this one 2 x square u 1 dash y 1 dash. So, u dash times 2 x square y 1 dash and then a x y 1 equal to 0. Now, let us see since y 1 is solution of the given differential equation x square y double dash y 1 double dash plus a x y 1 dash plus b y 1 equal to 0. So, since y 1 is the solution of this equation, we have x square y 1 double dash plus a x y 1 dash plus b y 1 equal to 0 [FL]. So, this term vanishes.

And further more let us see further more y 1 equal to x to the power 1 by 2 into 1 minus a. So, let us differentiate this. So, y 1 dash equal to 1 by 2 into 1 minus a x to the power 1 by 2 into 1 minus a minus 1. So, we get minus half minus a by 2. And this can be written as so 1 by 2 1 minus a into y 1 divided by x. So, we get x y 1 dash equal to 2 x y 1 dash equal to 1 minus a into y 1. So, or 2 x y 1 dash equal to 1 minus a into y 1.

So, let us see what we get now. So, this equation reduces to x square u double dash y 1 and then u dash let us write x outside. So, u dash x and then 2 x y 1 dash 2 x y 1 dash is 1 minus a into y 1. And here we have a y 1 equal to 0. So, we will get x square u double dash y 1 and then this say y 1 and y 1 will cancel, we get u dash x y 1 equal to 0, or we can say now y 1 is x to the power half one minus a. Let us remove this y 1 and x so or we will get x u double dash plus u dash equal to 0. So, we get x u double dash plus u dash equal to 0. Now, we can easily solve this when you integrate, you can write u double dash y u dash equal to minus 1 over x.

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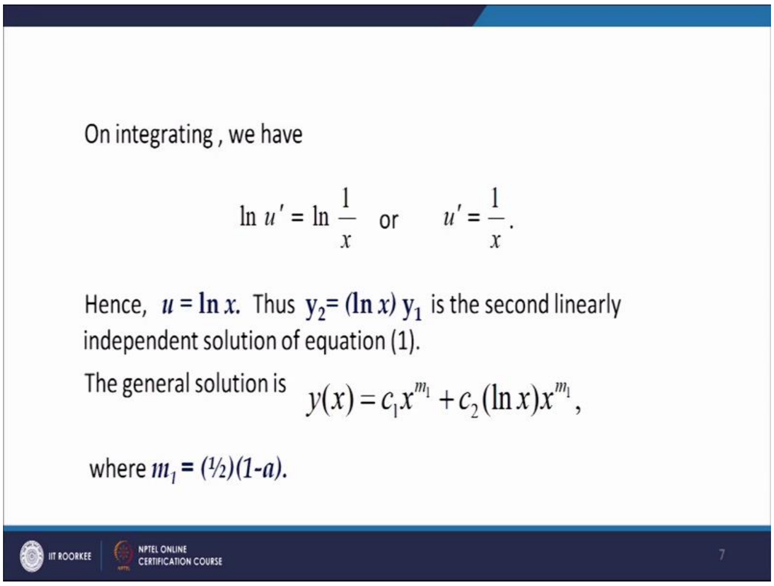

So, integrating we will get l n u dash equal to l n 1 over x and which will give you u dash equal to one by x. So, we get u equal to l n x, and thus y 2 equal to u y 1 will be equal to l n x into y 1. Now, y 2 over y 1 is equal to l n x. So, y 2 over y 1 is not a constant value therefore, y 2 is second linearly independent solution of equation 1. So, we have got two linearly independent solutions of the equation variable with variable coefficients Cauchy-Euler equation. So, the general solution will be y x equal to c 1 y 1; y 1 is x to the power m 1, m 1 is call by 1 by 1 by 2 minus a and then c 2 times l n x into x to the power m 1. So, this is the case of auxiliary equation have been two equal roots.

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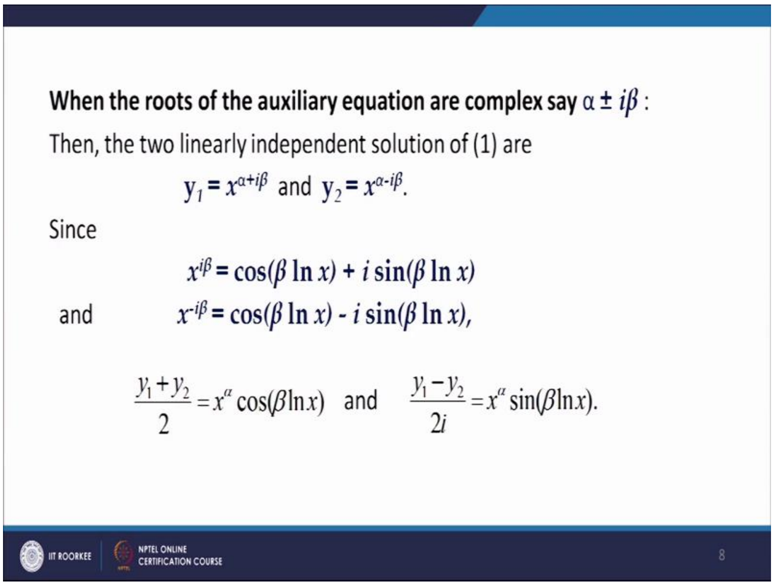

Now, let us go to the case where the roots of the auxiliary equation or complex c alpha plus minus i beta. Now, such alpha i plus i beta alpha i minus i beta are both distinct roots. So, we can write the two linearly independent solutions of the equation 1 as y 1 equal to x to the power alpha plus i beta, and y 2 equal to x to the power alpha minus i beta, but we are always looking for real solutions. So, what we will do is we will try to find two is linearly independent real solutions of the Cauchy-Euler equation.

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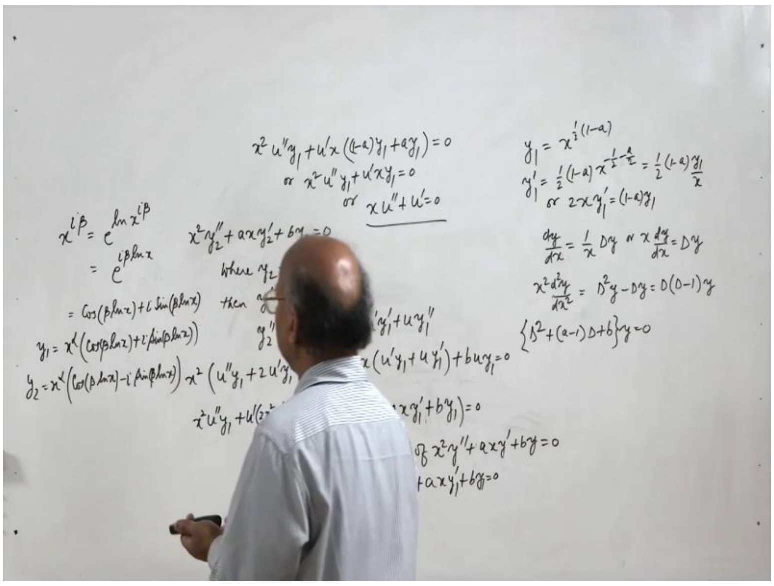

So, let us note that we can write sin x to the power i beta as x to the power i beta as e to the power l n x to the power i beta which is equal to e to the power i beta l n x. Now, let us apply Euler's formula, this is e to the power i theta is cos theta plus i sin theta. So, cos beta l n x plus i sin beta l n x we shall have. So, e x to the power i beta will be cos beta l n x plus i sin beta l n x likewise x to the power minus i beta, there is a x to the power minus i beta is equal to cos beta l n x minus i sin beta l n x. And now so y 1 will be equal to y 1 plus y 2 y 2 y 1 is equal to x to the power alpha cos beta l n x plus i sin beta l n x y 2 is x to the power alpha cos beta l n x minus i sin beta l n x.

So, adding y 1 and y 2, we get 2 x to the power alpha cos beta l n x, or we can say y 1 plus y 2, y 2 is x to the power alpha cos beta l n x, and y 1 minus y 2 divided by 2 i is equal to x to the power alpha sin beta l n x. Now, our equation is a homogeneous equation. So, if y 1 and y 2 are two solutions of that then y 1 plus y 2 by 2 and y 1 minus y 2 by 2 or also solutions of that. So, x to the power alpha cos beta l n x and x to the power alpha sin beta l n x they are also solutions of the Cauchy-Euler equation homogeneous Cauchy-Euler equation.

Now, further we note that x to the power alpha cos beta l n x and x to the power sin beta alpha l n x they are independent of each other, because when you divide one by the other it does not come out to be a constant. So, they are both linearly independent. And therefore, the general solution is can be written as y equal to x to the power alpha c 1 cos beta l n x plus c 2 sin beta l n x to the power alpha cos beta l n x x to the power alpha sin beta l n x can be can be set to form a fundamental set. So, this is the general solution in this case.

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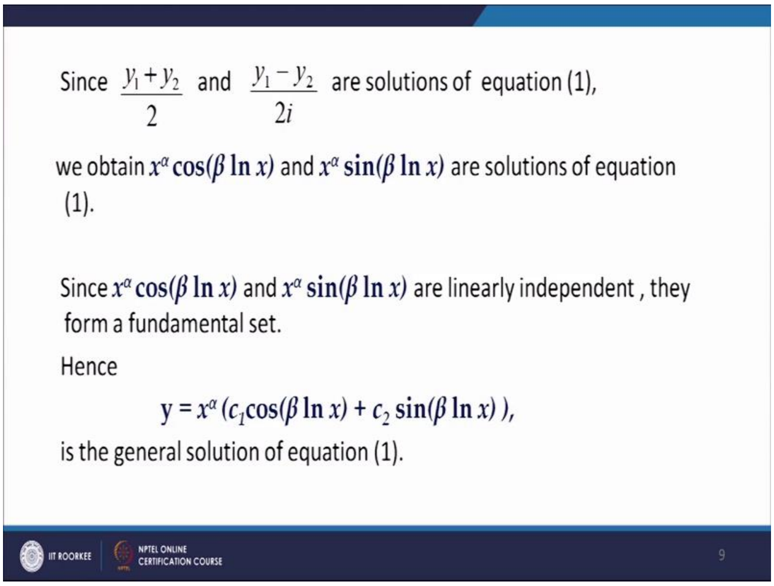

Now, let us take an example x square y double dash plus 2 x y dash minus 2 y equal to 0.

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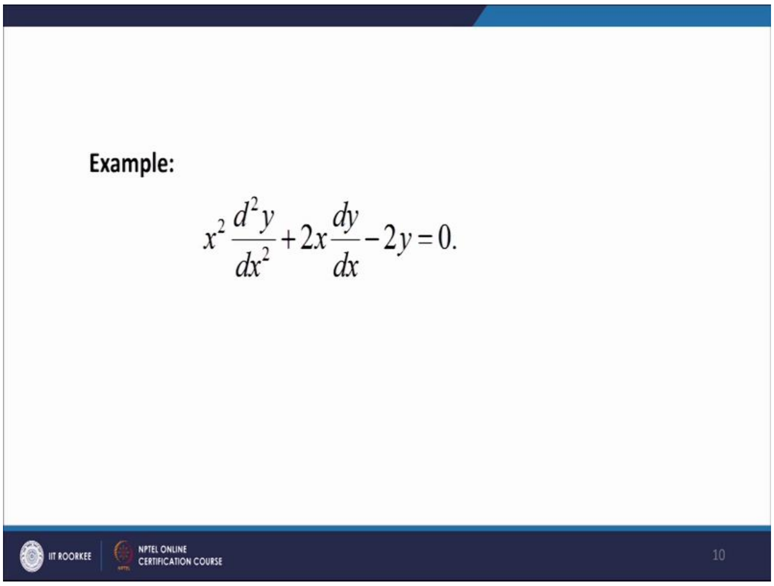

And let us see how we solve it.

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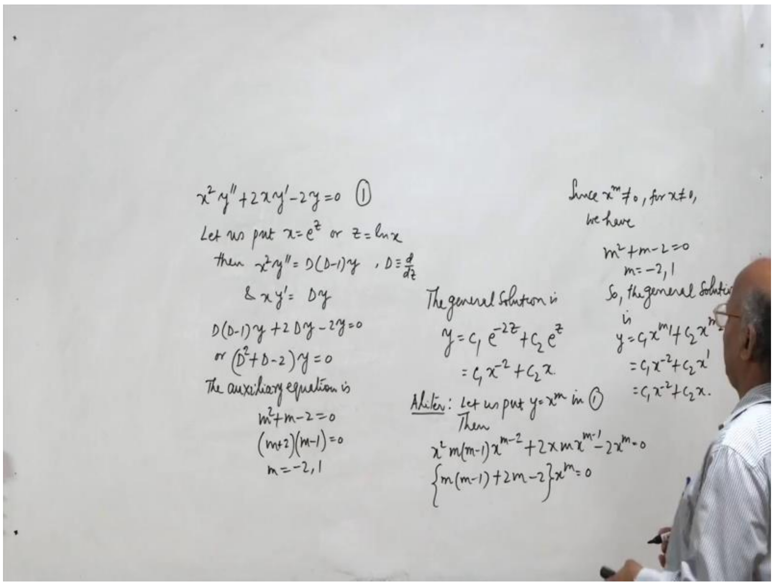

So, x square y double dash minus plus 2 x y dash minus 2 y equal to 0; so let us put x equal to e to power z or z equal to l n x. Then we have seen that x square y double dash is equal to D into d minus 1 y and where D is a d over dz; and x y dash is equal to D y. So, let us put these values. So, D D minus 1 y plus 2 D y minus 2 y equal to 0 or we can say D square plus 2 Dy minus D y is D y plus D minus 2 operating on y equal to 0. Now, this is a second-order linear differential equation with constant coefficients the dependent variable is by the independent variable is z we have d square y over dz square plus dy by dz minus 2 y equal to 0.

So, to find the solution of this, we write the auxiliary equation m square plus m minus 2 equal to 0, and the factors of this equation or m minus 2 m plus 2 into m minus 1 equal to 0. So, m is equal to minus 2 and 1, they are the two distinct roots of this auxiliary equation and therefore, the general solution is y equal to c 1 e to the power minus 2 z plus c 2 e to the power z. Now, let us now from the variable z, we have to convert to the variable x e to the power z is x. So, x square is e to the power 2 z. So, this is c 1 x to the power minus 2 and then we have c 2 x. So, we get the general solution as y equal to c 1 x to the power minus 2 plus c 2 x.

Now, we can find the solution of this by that other method. So, let us find the solution by the other method let us put y equal to x to the power m in the given equation, let me call it as equation 1. Then what do we have m into x square times m into m minus 1 x to the power m minus 2 plus 2 x y dash is m into x to the power m minus 1 minus 2 x to the power m equal to 0. Or we can say m into m minus 1 plus 2 m minus 2 x to the power m equal to 0. Now, again x to the power m is never 0 when x is not equal to 0. So, we get since we have m square m into m minus 1 plus 2 m minus 2 equal to 0, which gives you m square plus m minus 2 equal to 0, the roots are m equal to minus 2 and 1

So, the general solution is y equal to c 1 x to the power m 1 plus c 2 x to the power m 2, this is a case of two distinct roots. And m 1 is minus 2, so c 1 x to the power minus 2 plus c 2 x to the power m 2 is 1. So, we get c 1 x to the power minus 2 plus c 2 x same as in the previous case. So, this is how we will solve the Cauchy-Euler equation in the homogeneous form. Now, let us consider the Cauchy-Euler equation can be generalized to such type of equations also, where we see that the power of alpha x plus beta is same as the order of the derivative occurring in that term.

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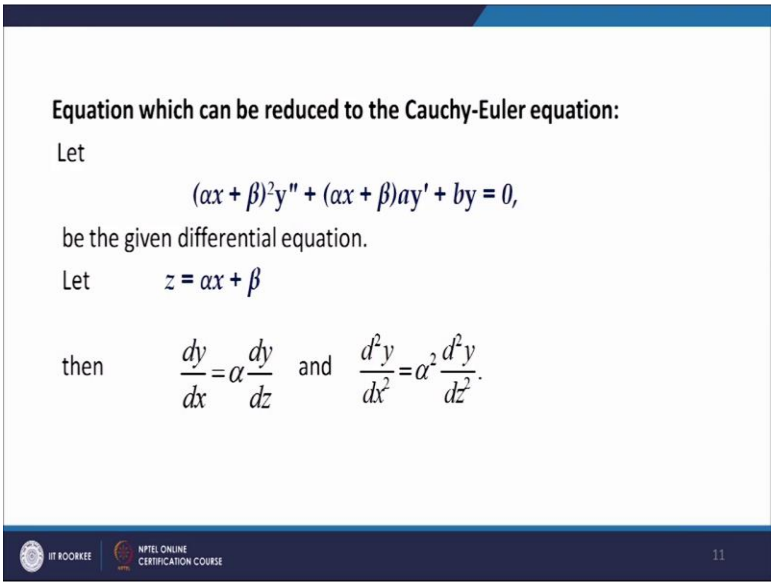

So, here alpha x plus beta power is 2, and we have the derivative of second-order. Here alpha x plus beta power of alpha beta is 1, and we have the derivative of first order. So, in such a case, what we do is we shall convert this to Cauchy-Euler equation if we can define z equal to alpha x plus beta. So, when we take z equal to alpha x plus beta, we can find dy by dx and d square y by dx square. So, dy by dx here will be dy by dz into dz by dx, and dz by dx is alpha. So, we write alpha times dy by dz. And from here d square y by dx square will be alpha times d over dx of dy by d z.

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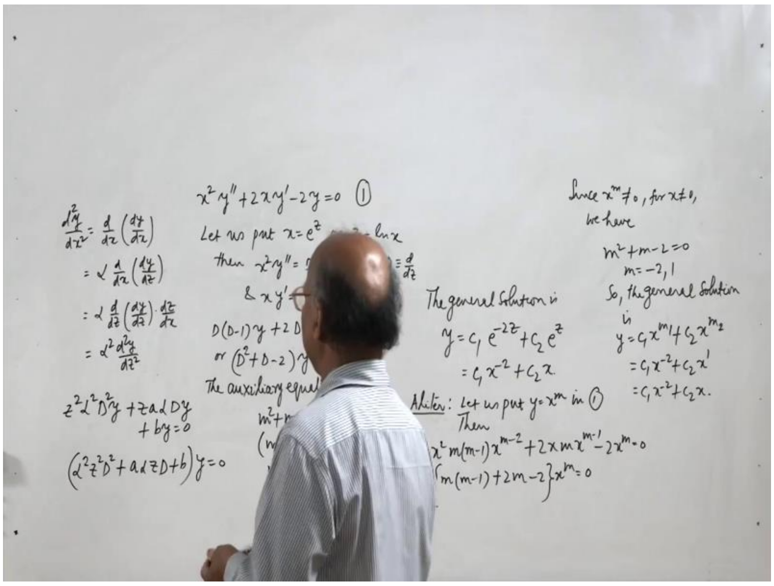

So, d square y over dx square this will be equal to d over dx of dy by dx. Now, dy by dx is alpha times dy by d z. So, alpha times, alpha is a constant, we can write it outside. Now, this is alpha times d over dz of. So, this is dz by dx is alpha. So, we have alpha square d square y by dz square. So, by defining alpha x plus beta as z, we can write this equation in the Cauchy-Euler equation form. So, then we shall have alpha square let me write alpha square. So, alpha x plus beta whole square y double dash that alpha x that will be z square y double dash is alpha square d square y, where d now denoting by D over dz plus z alpha y dash is no, it is a, sorry it is a.

So, z into a then alpha then Dy and then plus b y equal to 0, or we can say this is nothing but alpha x square z square D square plus a alpha z D plus b operating on y equal to 0. Now, this is a Cauchy-Euler equation in the independent variable z. Now, this can be divided by alpha x square, and we can bring it to the standard Cauchy-Euler form where the coefficient of z square d square y by dz square is unity. So, this can then we solved by the methods which we have just now discussed. And then after finding the solution in terms of z, we can put z equal to alpha x plus beta. So, we will get the solution.

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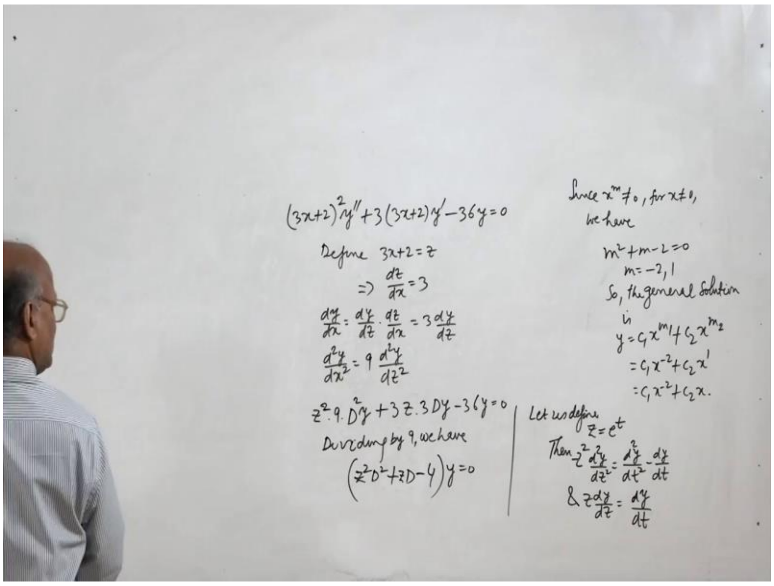

Now, let us say for example, let us look at this differential equation. So, for example, we have the differential equation three x plus 2 by three x plus 2 whole square y double dash and then we have three times 3 x plus 2 y dash plus minus 36 y equal to 0. So, define 3 x plus 2 equal to z then dz by dx equal to 3. So, dy by d x equal to dy by dz into dz by d x. So, we get 3 times dy by dz and d square y by dx square is equal to alpha x square. So, this will be 3 square. So, 9 times d square y by dz square, similarly we get. So, we shall have 3 x plus 2 whole square. So, z square y double dash, y double dash is 9 times D square y, then we have 3 times z, 3 times z y dash is 3 times Dy and then we have minus thirty six y equal to 0. Now, we can divide this equation by 9. So, dividing by 9, we have z square D square plus z D minus 4 z dy equal to 0. Now this is a Cauchy-Euler equation with the coefficients here 1 and here we have minus 4.

So, it can be then solved by defining z equal to; so now the independently variable z. Let us define z equal to e to the power t. Then we shall have z square D square y will be equal to than z square D square y over dz square will be equal to D square y over dt square minus dy by dt and z dy by dz will be equal to dy by dt.

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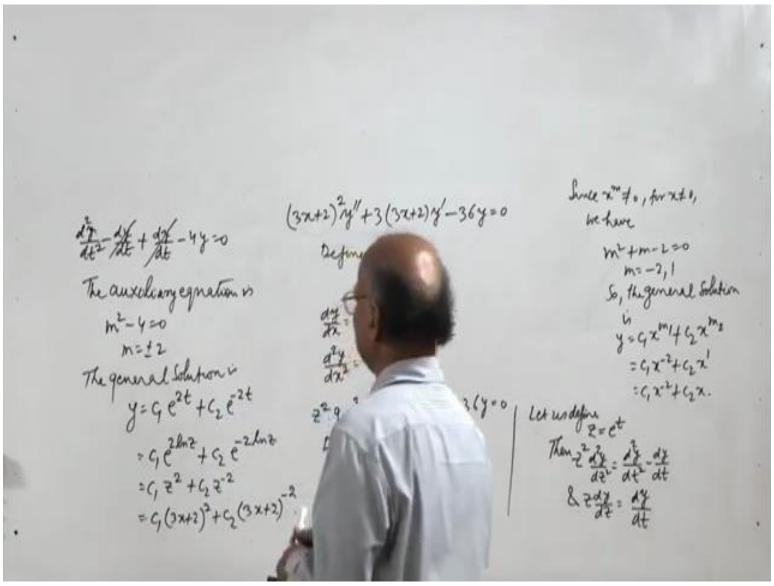

So, we shall have. So, we will get d square y by dt square minus dy by dt from z square d square y plus z dy z dy, we will get dy by dt and then minus 4 y equal to 0. So, this will cancel. We have d square y over dt square minus 4 y equal to 0 which is a linear differential equation of second-order with constant coefficient.

So, the auxiliary equation is m square minus 4 equal to 0. So, m equal to plus minus 2. So, we get the general solution as y equal to c 1 e to the power 2 t plus c 2 e to the power minus 2 t. Now, let us put t as l n z. So, c 1 e to the power 2 l n z plus c 2 e to the power minus 2 l n z, and this is equal to c 1 z to the power 2 c 2 z to the power minus 2, but z is equal to 3 x plus 2. So, we get c 1 3 x plus 2 square, and c 2 3 x plus 2 minus 2, this is the general solution in this case. Now, let us take a non-homogeneous Cauchy-Euler equation we have seen how to solve Cauchy-Euler equation in the homogeneous form.

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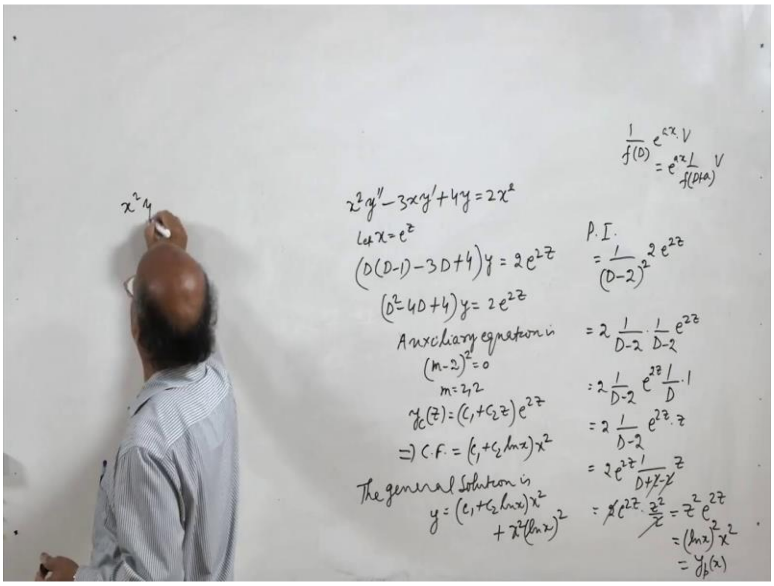

So, let us just find the particular integral. So, what we will do is, let us consider x square y double dash minus 3 x y dash plus 4 y equal to. Now, we have to remember one thing here when we solve Cauchy-Euler equation in the non-homogeneous form the method where we put x equal to e to the power z that is applicable here in the case of non-homogeneous equation. But the other method where we have put y equal to x to the power m that is not applicable here, because y equal to x to the power m does not reduce the Cauchy-Euler equation into a linear differential equation with constant coefficients. So, y equal to x to the power m method can be used only in the case where the Cauchy-Euler equation is in the homogeneous form, but x equal to e to the power z can be reduces the Cauchy-Euler equation to linear differential equation with constant coefficients, so that will be applied to solve non-homogeneous Cauchy-Euler equations.

So, let us put x equal to e to the power z. So, then we will have D into D minus 1 y minus 3 D plus 4. So, x is equal to e power z. So, we get two times e to the power 2 z. Let us change the right hand side also 2 z. So, we have D square minus 4 D plus four y equal to 2 e to the power 2 z. Now, auxiliary equation is m minus 2 whole square equal to 0. So, m equal to 2 2, so we get case of complex roots. So, y c x, in this y c z because independent variable is z, so y c z will be equal to c 1 plus c 2 z e to the power 2 z.

Now, let us separate z equal to l n x. So, we get the complementary function c of s c 1 plus c 2 log x into x square. So, this is the complementary function, let us find the particular integral. So, particular integral is 1 upon d minus 2, whole square operating on two times e to the power 2 z. Now, 2 is a constant, we cannot write outside the operator, e to the power 2 z, when you replace D by 2 here it becomes 0. So, what we do is 2 times 1 over d minus 2 1 over D minus 2 operating on e to the power 2 z.

Now, let us apply the formula 1 over f D e to the power a x into v that formula apply let us apply. So, e to the power a x comes out we have when v is a function of x. So, in case f D becomes 0 under placing d by a, we apply this formula. So, this is equal to 2 times 1 over d minus 2, this will be e to the power 2 z 1 over d plus 2 minus 2, so D operating on 1. This mean 1 over d means integral of 1. So, we get z; so e to the power 2 z into z.

Now, again e to the power 2 z into b this is b. So, we get two times e to the power 2 z 1 over d plus 2 minus 2 operating on z, 1 over D when operates on z we get integral of z. So, 2 times, so z square e to the power 2 z writing z in terms of x we write z equal to ln x. So, this is ln x whole square and e to the power 2 z in x square. So, this is particular integral y p x, this is y p x. So, general solution is y equal to c 1 plus c 2 ln x into x square plus x square ln x whole square, this is how we solved this equation.

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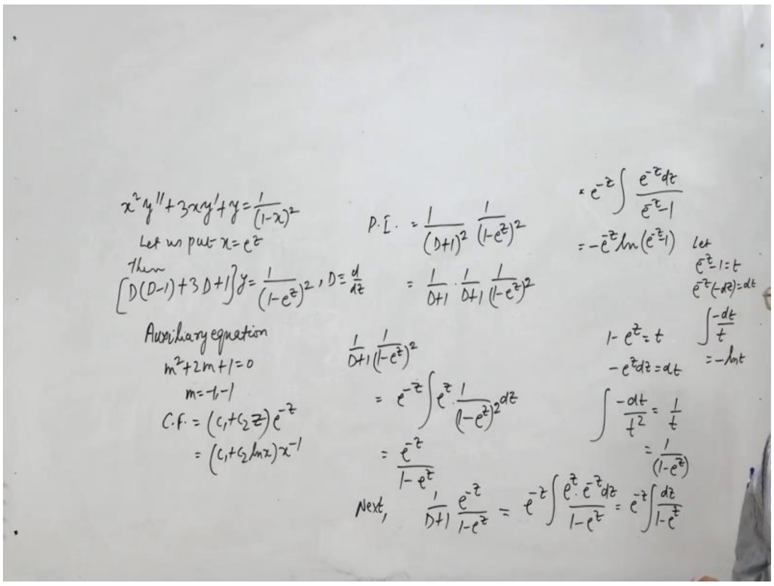

And the one more differential equation we have taken let me just give a hint here, because no formula will be applicable here we have to apply the general method to find the particular integral. So, x square y double dash plus 3 x y dash plus equal to 1 over 1 minus x square whole square. So, again let us put z x equal to e to the power z. So, then

D into D minus 1 plus 3 D plus 1 y equal to 1 over 1 minus e to the power z whole square. Now, this is D square minus D, so we have D square plus 2 D plus 1. So, auxiliary equation is m square plus 2 m plus 1 equal to 0, which is m equal to minus 1 minus 1. So, complimentary function will be c 1 plus c 2 ln z c 1 plus c 2 z and then e to the power minus z converting to x we have c 1 plus c 2 lnx x to the power minus 1.

And for particular integral what we do is. So, particular integral is equal to 1 over this is D square plus 2 D plus 1. So, D plus 1 whole square operating on 1 over 1 minus e to the power z whole square. Now, this is 1 over D plus 1 into 1 over D plus 1 operating on 1 over 1 minus e to the power z whole square. So, let us first find the effect of 1 over D plus operating on 1 over 1 minus e to the power z whole square. So, 1 over D plus 1 1 operates on 1 over one minus e to the power z whole square, what we get is e to the power minus z integral e to the power z in to 1 upon 1 minus e to the power z whole square dz.

Now, here we apply the applied the formula of the general method. If you do remember the general method we had said that 1 over D minus alpha one operates on Q, Q is a function of x and D is d over dx. So, then we had the formula e to the power alpha x integral e to the power minus alpha x Q dx, where D was d over dx. Now, here remember D is d over dz. So, we will have e to the power alpha is minus 1. So, e to the power minus z e power z upon this. Now, to integrate this let us put 1 minus e to the power z equal to t. So, we get minus e to the power dz, where dz equal to dt. So, we will get integral minus dt upon t square. So, this will be 1 by t, and 1 by t means 1 over 1 minus e to the power z. So, this will be e to the power minus z divided y 1 minus e to the power z. So, when 1 over D plus 1 operates on this, we will get this again let us operate 1 over D plus 1 on this. Next, 1 over D plus 1 operating on e to the power minus z divided by one minus e to the power z.

Let us find out the result of this. So, again we apply this formula alpha is minus 1. So, we get e to the power minus z integral e to the power z into e to the power minus z dz divided by 1 minus e to the power z. And this will give you how much e to the power minus z integral dz divided by 1 minus e to the power z. Let us see how we find the integral here. So, e to the power minus z, we multiply here e to by e to the power minus z in the numerator and denominator, and get e to the power minus z dz divided by e to the power minus z minus 1. Now, let us put e to the power minus z minus 1 equal to t. So, then e to the power minus z into minus dz will be dt. And therefore, we shall have e to the power minus z dz will be minus dt. So, minus dt divided by t, which will be minus ln t. So, this will be equal to minus e to the power minus z ln t means e to the power minus z minus 1.

Now, this will be equal to e to the power z is x. So, minus 1 over x ln 1 over x minus 1, so this is what we get. And hence the general solution is y equal to c 1 plus c 2 lnx into x to the power minus 1 minus 1 over x ln 1 minus x by x. So, this is how we will solve this equation. With this, I will conclude my lecture.

Thanks.