Mathematical methods and its applications
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Lecture – 36
Application of Laplace Transforms –I

Welcome to the series of lecture series on mathematical methods and its applications. So, in the lectures, we have seen about Laplace transforms their property, inverse Laplace transforms various functions, how to find the Laplace inverse and Laplace transform, etcetera.

So, now, we will see applications of Laplace transforms. What are the various applications of Laplace transforms? So, Laplace transform can be used in solving various ordinary differential equations which we faced in various engineering problems like in engineering; like electrical engineering problems or in mechanical engineering problems we face some ordinary differential equations or partial differential equations which can be solved using Laplace transforms some integral equations, some involving discontinuous functions which can be solved using Laplace transforms.

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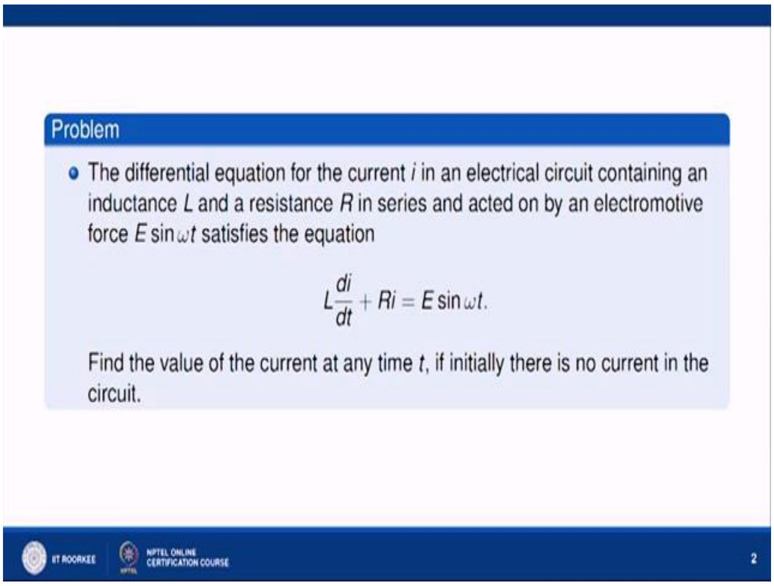

So, let us see the application Laplace transforms: part 1. So, the first problem is the differential equation for a current i in an electrical circuit containing an inductance L and a resistance R in series and acted on by an electromotive force given by E sin omega t satisfies the differential equation this.

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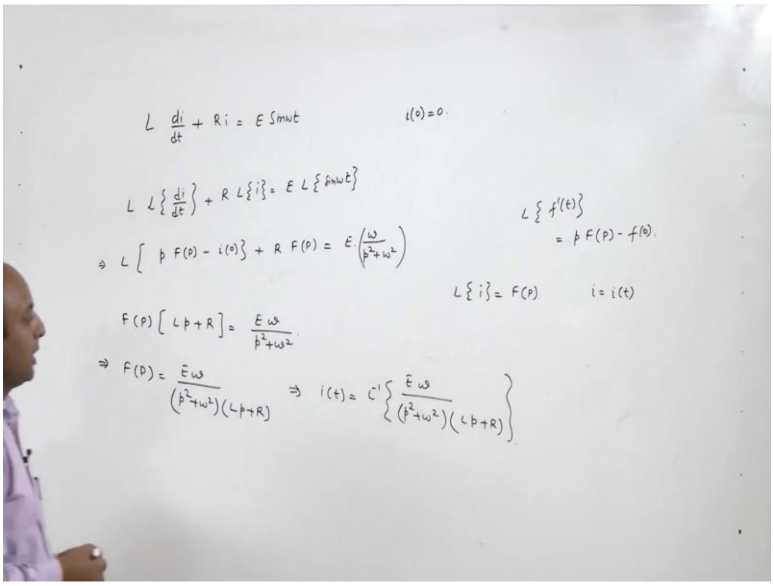

So, what is the differential equation? It is the d L d i by d t plus R i is equal to E sin omega t E sin omega t is the EMF; electro motive force. So, we have to find out the value of the current at any time t if initially there is no current in the circuit that is it is given to us at i at 0 is 0 that is i at t equal to 0 is 0. It is given to us. So, how to solve? So, we can solve this problem using our differential equations techniques also, but we can solve the same problem using Laplace transforms also. So, how can we solve this using Laplace transform? Let us see.

So, first you take Laplace both the sides. So, this L is inductance. So, lap L into Laplace transform of d i by d t plus R is constant Laplace transform of i is equals to E is constant Laplace transform of sin omega t. So, it is L Laplace transform of d i by d t. We already know that Laplace transform of f dash t is what? F dash t is nothing but p F p minus f 0, where F p is Laplace transform of f t. Now here f t is; here it is i dash t; d i by d t it is i dash derivative of i respect to t. So, what the Laplace transform of this using that expression is; will be it is p F p minus F 0, F 0 means i 0 plus R into suppose Laplace transform of i is F p which is equals to E and sin omega t is omega upon P square plus omega square.

So, and assuming Laplace transform of i as F p. You can call it some other function also F p. So, it is P into Laplace transform of i which are call; which I am calling as F p minus i 0 or f 0 plus R into Laplace transform of i which I am calling as F p equal to E into Laplace transform of sin omega t it is sin omega upon P square plus omega square. Now i 0 is 0 is given to us you can collect F p. So, F p will be nothing but L p plus R, it is 0 and it is E omega upon P square plus omega square.

So, from here F p is nothing but E omega upon P square plus omega square into L p plus R. So, what will be this is F p and F p is nothing but Laplace transform of i. So, what will be i? Basically i is nothing but function of t i is equal to i of t i is nothing but function of t here. So, this implies i t will be nothing but Laplace inverse of E omega upon P square omega square into L p plus R. So, now, find out the solution of this differential equation i t we have to find out the Laplace inverse of this expression, this is the only thing left now. So, how can you find out Laplace inverse of this? So, let us focus on the Laplace inverse now Laplace inverse of this expression.

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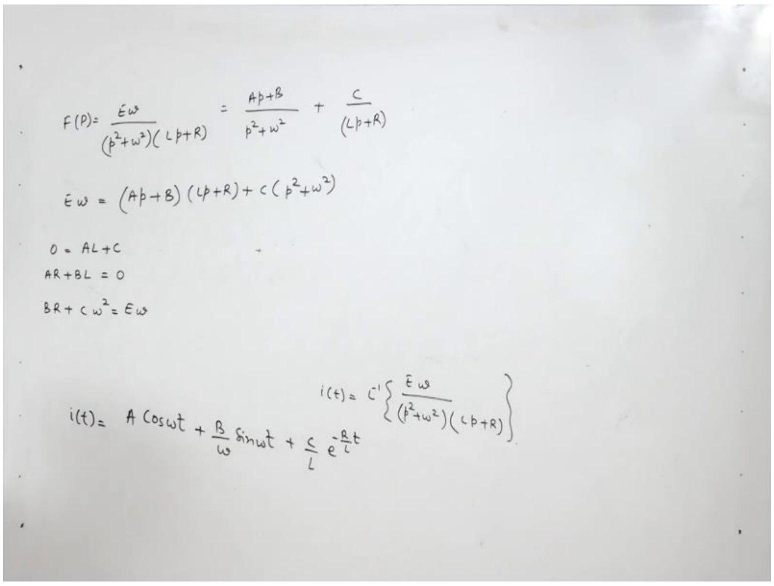

So, this F p is what is E omega, E omega is constant free from P, it is P square omega square into L p plus R. So, one way is you can make partial fractions or you can use compellation theorem. So, we can make partial fraction as well. So, it is A p plus B upon P square plus omega square and plus C upon L p plus R.

So, E omega will be nothing but A p plus B, we can find out A, B and C. Now compare the coefficients both the sides P coefficients of P square here is 0 and here. It is AL plus C coefficient of P square. Now coefficients of P suppose coefficient of P will be nothing but it is AR plus BL and here it is nothing. So, it is 0 and constant quantity is B R plus C omega square which is E omega. So, using these 3 expressions, we can easily find out the value of A, B, C, sub where you substitute these values over here and take the Laplace inverse both the sides. So, we can find out what will be i t, i t can we find out. So, what will be i t? I t will be nothing but Laplace inverse of this expression means this expression. So, this Laplace inverse this will be nothing but A into cos omega t plus B upon omega sin omega t plus C upon L. We can take out common which is nothing but E to the power minus R by L times t. So, these A, B, C can be find out using these 3 expressions using these 3 equations. So, we can substitute the values of A, B, C, from these 3 equations over here which will give us the value of i t function i t.

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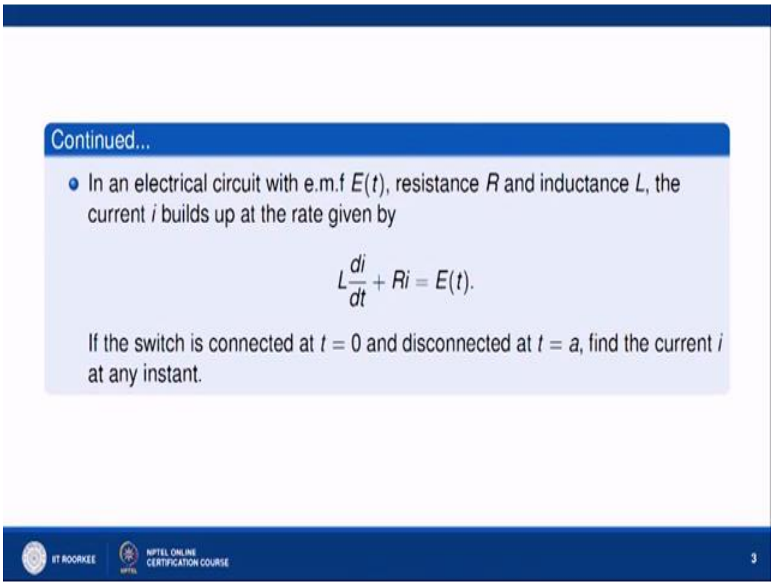

Now, see the next problem again it is a electrical circuit problem in a electrical circuit with e m f E t resistance R and inductance L the current i builds up at the rate given by this equation. Now here if the switches is connected at t equal to 0 and disconnected t equal to a find the current i at any time t. So, how can we proceed for this problem?

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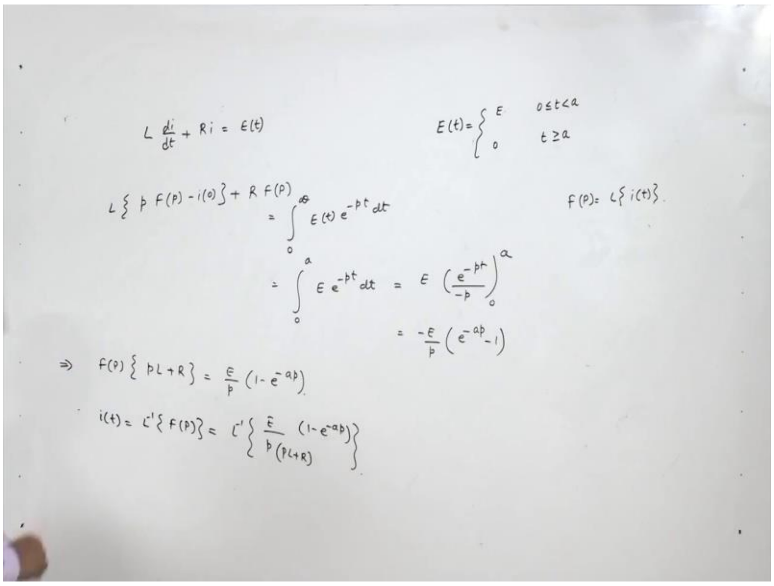

So, what is the governing equation here i d i by d t plus R i equals to E t, the same equation it is LCR circuit equation I mean e m f resistance and inductance, this is the equation given by differential equation given by this expression. Now it is given that in a switches is connected is equal to 0 and disconnected is equal to A, what was it mean that electromotive force which is E t, it is some constant quantity say E when t varying from 0 to a and disconnected at t equal to a means it is 0 when t is greater than or equal to. So, this is how electromotive force works for this particular problem.

Now, to solve to find i at n instance t, we will use Laplace transforms. So, it is take Laplace transform both the side, it is L it is again Laplace transform of d i by d t which is given by P into F p minus i 0 plus R into F p is equals to is equal to E t. E t nothing but this expression I mean given by this expression. So, it is 0 to infinity because you have to find out Laplace transform of E t. So, Laplace transform of E t is nothing but 0 to infinity E t into E power minus P t d t and it is nothing but 0 to a E; e [FL] power minus P t d t otherwise it is 0. So, which is equal to E; e to the power minus P t upon minus P 0 to a and which is equals to minus E by P E to the power minus A p minus 1. So, these are this is the right hand side and what is F p? F p is nothing but Laplace transform of i t this I am assuming here. Now you assumed that a t equal to 0 initially there is no current in the circuit. So, i 0 is 0. So, i 0 is 0. So, this implies basically F p P L plus R is equals to E upon P 1 minus E [FL] power minus A p because i 0 is 0 initially I am assuming that there is no conductance circuit, it is 0 F p is L p plus R and right hand side is E by P 1 minus E [FL] power minus A p.

Now, the only thing is find i t which is nothing but Laplace inverse of F p and F p from here is nothing but Laplace inverse of E by A p into P L plus R 1 minus E [FL] power minus A p. So, now, we have to find out the Laplace inverse of this expression this is the only problem left now. So, how to find Laplace inverse of this?

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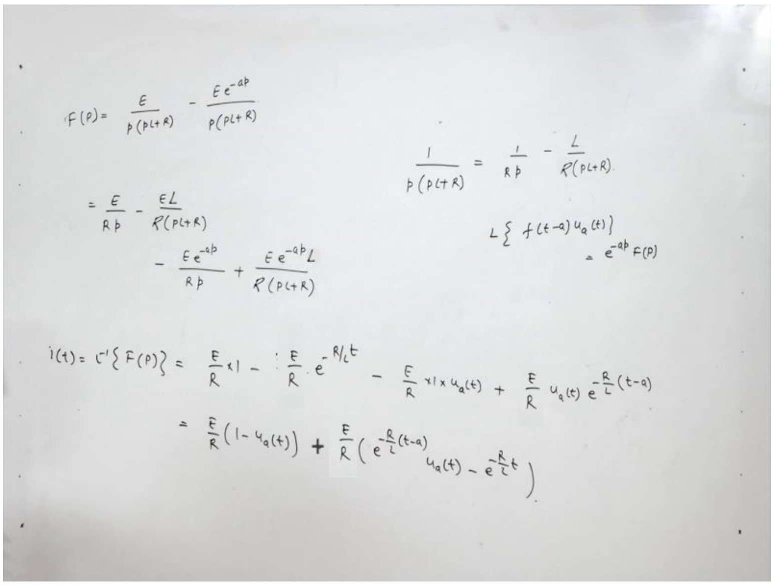

Now, again what is F P? F p is E upon P into P L plus R minus E into E [FL] power minus A p upon P into P L plus R. This is F p and we have to find out Laplace inverse of this F p. So, first let us find the partial fraction of this expression. So, this is partial fraction can be find out when you put is equal to 0. So, this is R P plus when you put equal to this. So, it is minus L by R into P L plus r. So, you can check when you take the L C M you can get the right hand side. So, this is the partial fraction of this expression. So, what will be F P? Now it is E upon R P minus E into E to the power minus A p upon R P minus plus E into E to the power minus A p now the Laplace inverse that is i t will be Laplace inverse of F p. So, this will be the Laplace transform of this expression. So, we can further divided into 2 when t is more than a. So, it is 1. So, it is 1, it is 1. So, that will give the i t when t is greater than a and when t is less than a and greater than 0 it is 0 it is 0. So, that we give gives the value of i t when t is lying between 0 and a 0 and a. So, this will be the i t for this expression. So, this is how we can solve some problems using Laplace transforms.

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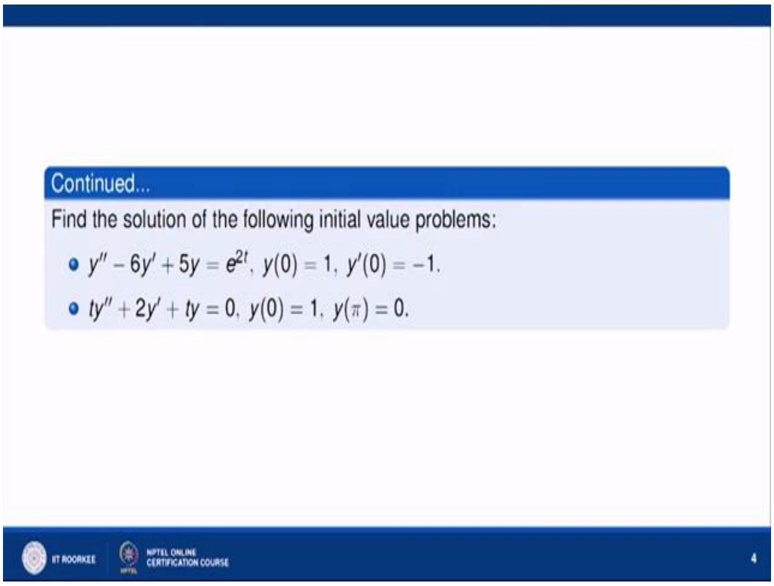

Now, we will see some more problems which can be solved using Laplace transforms these are simple problems just to illustrate how can we use Laplace transform to solve ordinary differential equations.

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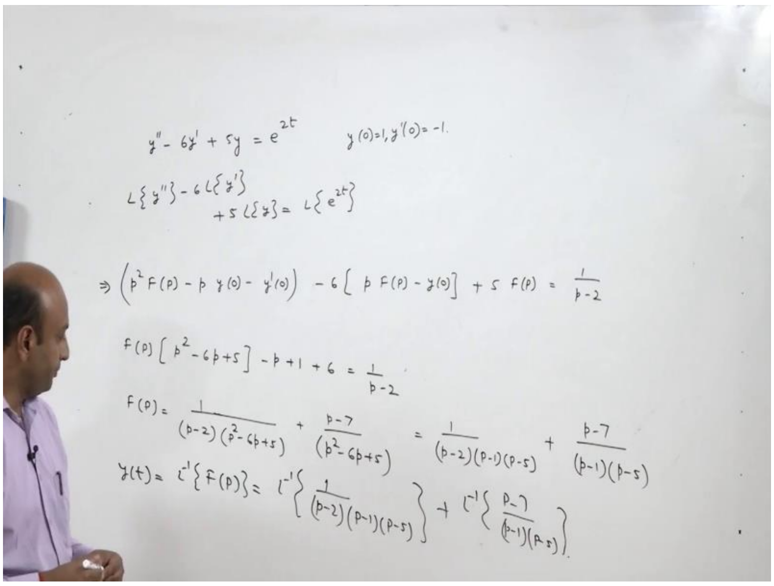

So, the problem is simple, it is y double dash minus 6 y dash plus 5 y is equal to E [FL] power 2 t and y 0 is 1 and y dash 0 is also 1. It is given; it is 1 minus 1. So, it is 1 y 0 is 1 and y dash 0 is minus 1. So, now how to solve this problem is a Laplace transforms. So, first take Laplace transform both the sides. So, it will be Laplace transform of y double dash minus 6 times Laplace transform of y dash plus 5 times Laplace transform of y is equals to Laplace transform of E [FL] power 2 t. Now from the Laplace transform derivatives, we know that the Laplace transform of y double dash is nothing but P square F p minus P f 0 minus f dash t. Now you call it f 0 or you can call it y 0 there is no problem now because here instead of f we are having y.

So, minus 6 and Laplace of y dash will be nothing but p F p minus y 0 plus 5 times Laplace of Laplace of y is F p and it is nothing but 1 upon P minus 2 Laplace of y I am calling as F p. So, y 0 is 1 and y dash 0 is minus 1 it is given. So, F p will be that is P square minus 6 P plus 5 and y 0 is 1 that is minus P minus minus plus 1 and it is minus minus plus 6 into 1 is 6 is equals to 1 upon P minus 2. It is 1 is minus 1. So, it is P it is plus 1 minus minus plus 6 into 1 is 1. So, F p will be nothing but 1 upon t minus 2 into t square minus 6 P plus 5 and it is this will go to that side. So, it is plus P minus 5 of 7 6 plus 1 is 7. So, it is P minus 7 upon P square minus 6 P plus 5. So, it is nothing but 1 upon P minus 2 into it is nothing but 5 and 1. So, that is P minus 1 into P minus 5 plus it is P minus 7 upon P minus 1 into P minus 5. Now what will be y t? Y t will be nothing but Laplace inverse of F p. So, Laplace inverse of this can be find out using partial fractions. So, you can use partial fractions to find out the Laplace of this. So, that will give the final answer of final expression for y t.

Now, the next now we will see the next problem involving t also in the y y double dash or y let us solve this problem. So, this problem can be completed you use you take the partial fraction make the partial fractions and then take the Laplace inverse of these expressions.

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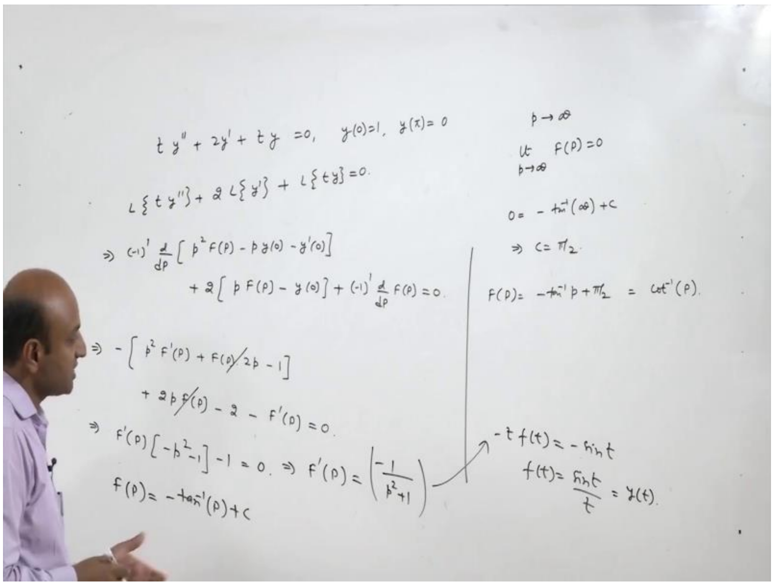

Now next problem t y double dash plus 2 y dash plus t y equal to 0 and it is give to you that t y at 0 is 1 and y at pi is some value 0 this is given to you. Now how to solve this problem again, you will use Laplace transforms you take Laplace transform both the sides. So, Laplace transform of t y double dash plus 2 times Laplace transform of y dash plus Laplace transform of t y dash t y equal to 0.

Now what is Laplace transform y double dash? I mean f double dash f double dash t is nothing but P square F p minus P f 0 minus f dash 0 and Laplace transform of t f double dash would be what will be nothing but minus 1 [FL] power 1 d by d P of Laplace transform f double dash t by the property of multiplication with t. So, now, Laplace transform of t into y double dash would be what will be minus 1 [FL] power 1 d by d P of Laplace transform of y double dash. So, Laplace transform of y double dash is P square F p minus P into y 0 minus y dash 0 plus 2 into Laplace transform of y dash Laplace transform of y dash is nothing but p F p minus y 0 plus Laplace transform of t into y dash again is nothing but minus 1 [FL] power 1 d by d P of Laplace transform of y which is F p it is equal to 0 because I am assuming the Laplace transform of y t as F p.

Now, let us simplify this it is minus derivative with respect to p. So, it is P square into f dash P plus F p into 2 P minus derivative of with respect to P is y 0, y 0 is 1 and derivative this is 0 plus 2 p F p and y 0 is 1 th minus 2 minus f dash P is equal to 0 it is 2 F p 2 F p y 0 is 1. So, 2 into 1 is 2. Now it is nothing but this implies f dash P into minus

P square minus 1 with F p, what will be having? Now it is minus this and plus this. So, both will cancel out it is minus minus plus 1 minus 2. So, it is minus 1 equal to 0. So, what will be f dash P? F dash P will be nothing but 1 upon negative P square plus 1. So, what will be F P? You integrate both sides F p will be nothing but minus tan inverse P plus C. Now you find Laplace inverse both the sides to find out y t. So, y t will be nothing but this implies, it is f dash P minus P square minus 1. So, it is it is minus plus 1 it is minus minus plus 1 plus 1 minus 2 is minus 1 and it is 1 upon P square plus 1 and P square plus 1 is tan inverse P plus C.

Now, to calculate this C we will use corollary of existence theorem for Laplace transforms we know that as P tend to infinity F p must tend to 0. So, take P tend to infinity both the sides. So, at P tend to infinity limit P tend to infinity F p must be 0. So, this is 0 equal to minus tan inverse pi by 2 tan must infinity plus c. So, this implies c. So, implies C is pi by 2. So, F p is nothing but minus tan inverse P plus pi by 2 and pi by 2 minus tan inverse P is nothing but cot inverse p.

So, now we have to find out Laplace inverse of this F p. So, take the derivative both the sides or we can take Laplace inverse here itself if we take Laplace inverse here. So, it is nothing but minus t f t and it is nothing but minus of sin t. So, f t will be nothing but sin t by t if we take the Laplace inverse here itself we can we can proceed from here also from here to take if you want to find out the Laplace inverse of F p again we have to differentiate it. So, we will go back to a same expression then the Laplace inverse of this will be nothing but sin t upon t. So, this is nothing but y t now y at pi must be 0 which is satisfied y at pi is 0. So, which is satisfied, hence this is the expression their solution for this problem.

Now, similarly this problem can be solved using Laplace transforms whenever we involve with (Refer Time: 26:37) delta function right hand side. So, such problems can also be solved using Laplace transforms you take Laplace transform both the sides and then take Laplace inverse that will give you the final answer of the same problems.

Thank you.