INDIAN INSTITUTE OF TECHNOLOGY
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NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)

Discrete Mathematics

Module-03
Mathematical Induction
Lecture-02
Mathematical Induction (2)

With
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Today we will discuss set operations and laws of set operations.

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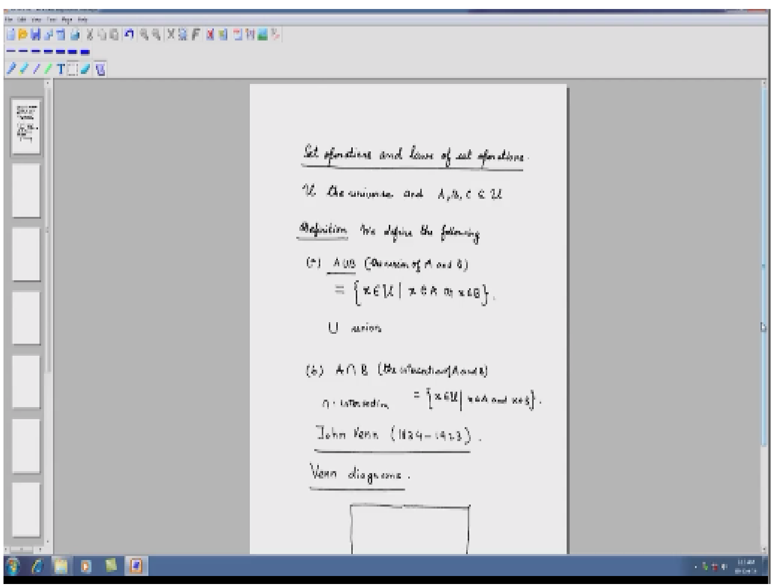

Now our starting point is U the universe and A,B,C etc which are subsets of U now we define several operations involving these sets A,B,C we define the following a) A U B the union of set A and set B is all x strictly speaking all x in u such that x belonging to A or x ∈ B now this is intuitively very clear this only means that given any two sets A and B I can consider a set which contains all the elements of A and all the elements of B needless to say that it will contain the elements which are both in A and B.

And this bigger set is denoted by A U B the symbol this symbol is the symbol of union of two sets so this is an operation on sets the next operation that we define is called the intersection of two sets we write intersection as this and B so this symbol is called intersection and this is the set of all x belonging to u such that x ∈ A and x ∈ B at this point I would like to mention a diagrammatic representation of sets proposed by John Venn who lived between 1834 to 1923.

The diagrammatic representation that he proposed is famous as Venn diagrams the idea behind Venn diagram is very simple it just says that when we are considering the universe u will write a rectangle or draw a rectangle.

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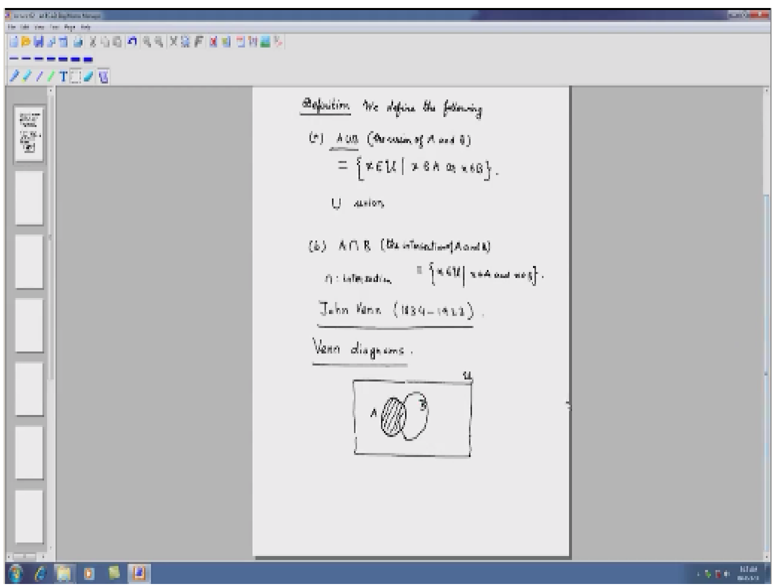

And call that u our Universal set and the subsets or the sets of the universal sets that we discuss that we discussed within this framework which are of course subsets of u so these sets that is ABC and all the others will be drawn as circles inside u we are the interior of the circles may be shaded when I write this means that A consists of the elements that are within this circle now suppose we take another set B in general it will have some common area shared with the circle corresponding to A. Let us call it B let us go to the next page and draw the situation one more time.

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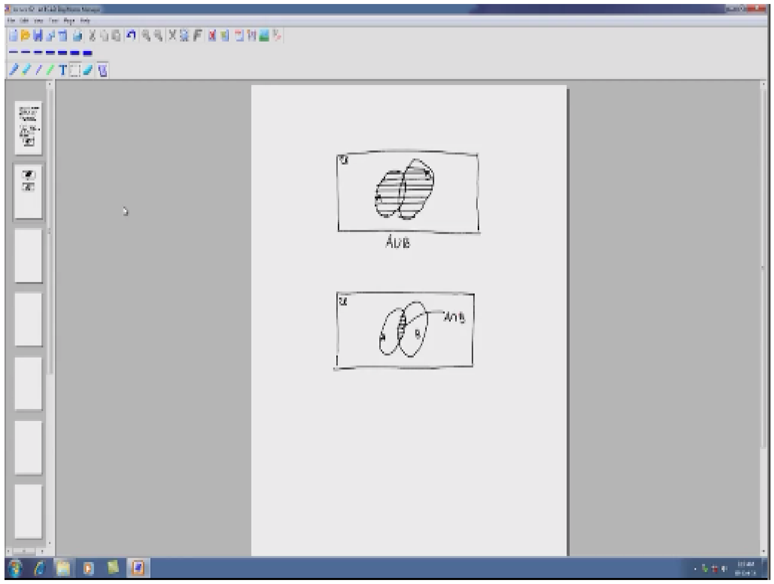

So here we have the universal set u or simply the universe I am representing a set A by the inner portion of a circle and B the inner portion of another circle if we want to know the set A U B this will consist of the shaded area which consists of the region that is covered by A and the region covered by B, so this whole shaded area is A U B again we take A and B if we consider only the region that is within the interior of the circle corresponding to A and the interior of the circle corresponding to B.

Then we get the set A ∩ B now this pictorially gives us the sets A U B and A ∩ B and these diagrams are very useful in getting a getting an intuitive feeling of into the feeling of operations on sets the next operation that we discuss is called symmetric difference now let us check the definition of symmetric difference this is the third the third subpart of the first definition that we have started.

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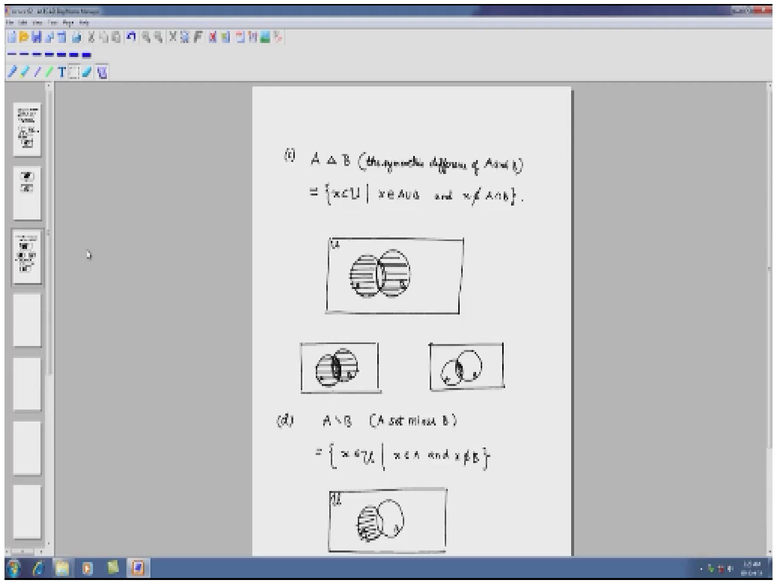

Symmetric difference is denoted by Δ so again we have two sets and I am writing A Δ B this is the symmetric difference of the set A and the set B and this consists of all the elements in u such that x ∈ A U B and x does not ∈ A ∩ B now let us take the help of the Venn diagram to understand this operation suppose these are my sets A and B so I am looking for the region which is inside A U B and which is outside A ∩ B let us recall what is A U B and what is A ∩ B A U B is the shaded region.

That covers the whole of A and B and A ∩ B is this part so if I superimpose these two diagrams and we will find that this part is A ∩ B and therefore I have to cut this portion out I do not want elements there and all the other elements will be in the symmetry difference therefore the symmetric difference is a set which can be represented by the Venn diagram as the shaded region within A U B.

So this is hole of A U B leaving aside the portion A ∩ B there are two more operations that I would like to discuss over here one is A set - B A set - B this is the set of all x such that x ∈ A and x does not ∈ B if you look at the Venn diagram corresponding to this operation then we see something like this again we have the universal set and we are taking two sets A and B within that a universal set and I want the elements which are in A but not in B. Therefore I shade the portion of A which is outside A ∩ B this shaded region corresponds to A set - B or simply A – B.

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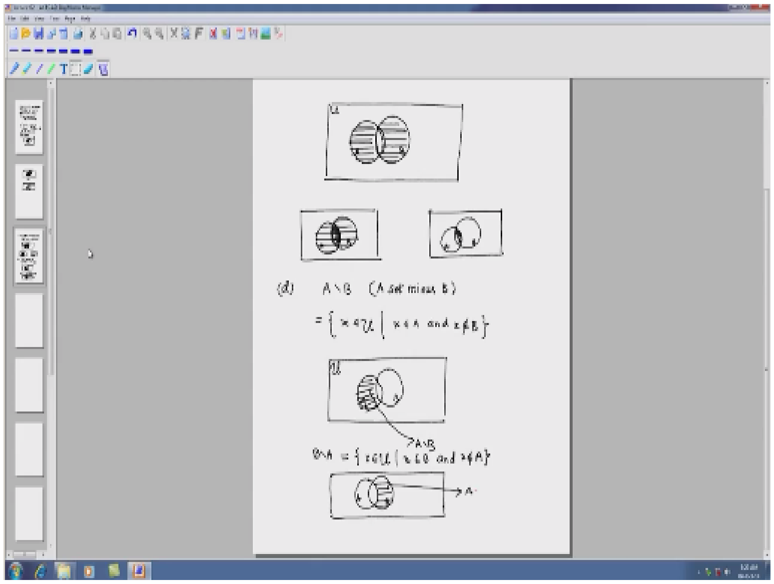

Now if we inverted this situation and if we had taken B - A we would have gotten x ∈ u and X ∈ B and X does not ∈ A now in Venn diagram that means the region which is inside B but not in A ∩ B so if you have A here and B here this is the region corresponding to B - A this is B I am sorry this is B - A gasps we can combine these two things and get a different description of symmetric difference of A and B.

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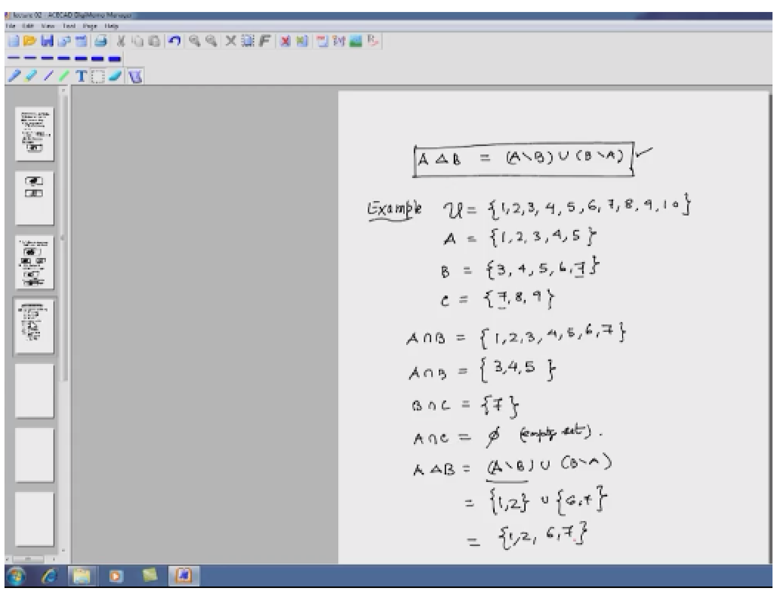

The symmetric difference of A and B can also be thought of as A set - B U B set - A we have to remember this now let us look at some examples suppose our universal set is integers from 1 to 10 and let us consider the sets ABC has then A U B is 1, 2, 3, 4, 5, 6 and 7 so that is the integers which are both in A and B if you want to find out A ∩ B that will take only the integers which are common to both A and B and we see that 3 is common to both A and B 4 is both common to A and B and same is 5 therefore it will be 3, 4 and 5.

Now if we take B ∩ C B ∩ C is only 7 because we see that the seven is the only integer which is common to both B and C and then if we consider A ∩ C we see that a is one 1, 2, 3, 4, 5 and C is 7, 8 , 9 therefore A ∩ C does not have any element so it is the empty set 5 we have defined empty set in the last lecture and here we see an example of the empty set now let us consider the symmetric difference between A and B.

So in order to do that I have to find out elements which are in A but not in B and the elements which are in B and not in A we can as well use the relation that we have obtained over here that is A - B U B - A if I do that and first let us consider A - B so let us compare A and B 1 is in A but not in B 2 is in A but not in B but 3, 4, 5 all are in A are all are in B therefore from A - B is going to be only 1 and 2 Union now we see that 3, 4 and 5 are common to both A and B and in B we have two more terms or two more elements 6 and 7.

Which are not in A therefore those elements will appear in B - A and thus we have the set 1, 2 unions X set 6, 7 and taking the Union we have 1, 2, 6, 7 this is a symmetric difference between A and B one more operation that we need to consider is said to be the complement so we have a set A which is of course a subset of the universal set.

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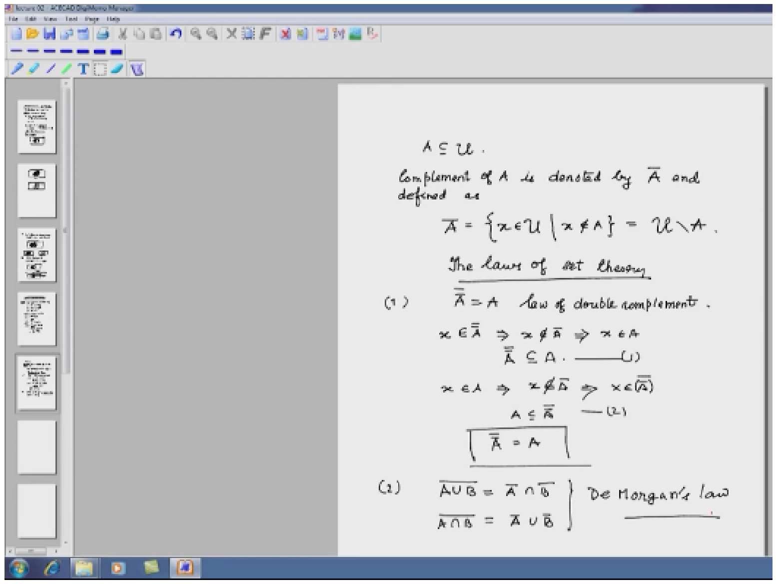

Compliment of A is denoted by A bar or A over line and defined as complement of A is a set consisting of all x belonging to u such that x is not in A or in other words by using the operations that we have defined already the complement of A is u set - A that is the elements in u which are not in A now we will consider the laws of set theory I will note down several laws here many of them are quite intuitive and extremely clear.

So I will not give the proofs in general but I will give sketches of proofs as we go on describing these laws only one or two laws we will check in details now the first one says that A complement is equal to A this is called a law of double compliment it is not difficult to prove this because we can always say that suppose I have got an element X which is inside a complement of complement this implies that X is not in a complement.

Now if X is not in a complement this automatically implies that X is inside a the reason is that a complement consists of exactly those elements which are not in A so if something is not in A then it is of course in A therefore we see that any element in a complement is going to be in a therefore A complement is a subset of A on the other hand if I have an X inside a this will mean that X definitely is not inside A complement but from the definition of complement I can take again a compliment because X is not in a complement and I can consider.

A complement as a set with which I am starting then I take complement of that and then X has to be inside complement of A complement and therefore A must be a subset of A complement and combining these two results we see that a complement is equal to a which is called the law of double compliment the next result is the de Morgan's law which states that A U B complement is equal to A complement intersection B complement and A ∩ B complement equal to a complement Union B complement is called de Morgan's law.

Now de Morgan's law needs a proof so let us start with the proof of de Morgan's law let us write de Morgan's law again.

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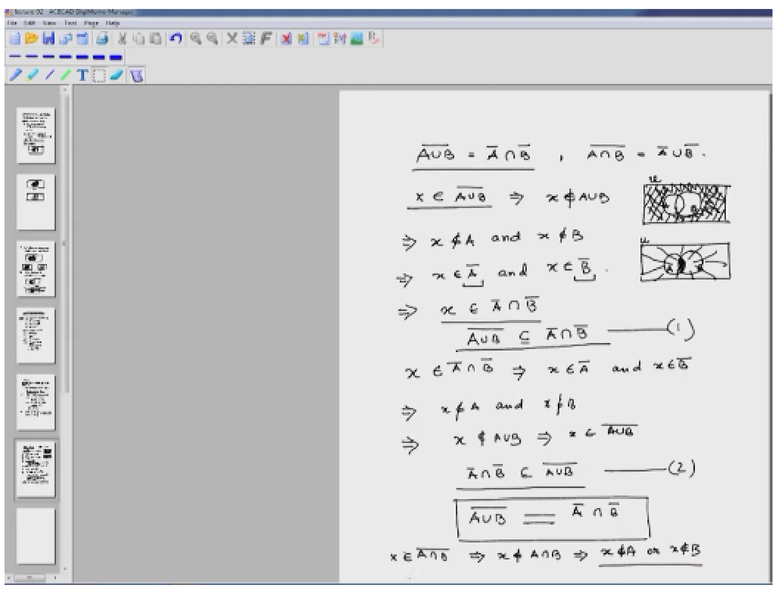

It states that A U B complement is equal to A complement intersection B complement and A ∩ B complement equal to A complement Union B complement first we prove this part in order to prove this equality we start by considering an element X inside A U B complement which implies that x is not in A U B now let us draw a small Venn diagram over here and see what happens.

So here we have got of course this is a universal set u here we have got A and B and here somebody tells me that x is in A U B complement I know that a A U B covers the interiors of the circles corresponding to A and B therefore A ∩ B definitely is a region outside A U B so this shaded region is A U B complement so I am saying that A x ∈ A U B complement implies that X does not ∈ A U B but what does it mean it means that X cannot be inside A neither can be X inside B.

Therefore this means that X is not in A and X is not in B now we know that if somebody tells me that X is not in a this means definitely X is inside A compliment and X not in B means definitely X is inside B complement now this means that X is inside A complement intersection B complement because this is the definition of intersection that if I have an element which is inside let us say set C and also inside set D then that element is inside C ∩ D and here those two sets are A complement and B complement.

Therefore x ∈ A complement and x ∈ B complement means x ∈ A complement intersection B complement now we have proved here that X belonging to a complement Union sorry X belonging to A U B complement implies X ∈ A complement Union sorry X ∈ A complement intersection B complement therefore we can say that A U B complement is a subset of A compliment intersection B compliment.

Let us write this equation as the equation 1 next we start from the last line of the previous chain of arguments so we start from the point that X is an element of A compliment intersection B compliment suppose this happens then this implies that X is an element of A compliment and X is an element of B complement this means X is not in A and X is not in B now we again refer back to the Venn diagram that I have drawn a while back and we see that if I have a scenario where X is not in A and X not in B.

That means X will be in the region outside exactly outside A U B therefore I can safely say that X does not ∈ A U B which means that X ∈ A U B complement therefore we can write that A complement intersection B complement is a subset of A U B complement let us denote this by 2.

Now here we see that we are considering two sets that one is A U B complement and the other is a complement intersection B complement and we have proved that the set in the left hand side is a subset of the set in the right hand side and the right hand side is a subset of the set left hand side because of these two subsets equality relations and therefore we can safely say that these two sets are equal and this is the first part of de Morgan's theorem.

Now let us try to prove the second part of the de Morgan's theorem here our starting point is X is inside $A \cap B$ complement now this means that X is not in $A \cap B$ now X is not in $A \cap B$ means that X is not in A or X not in B let us see that Venn diagram corresponding to this situation this is A this is B so let us write the sets A B and this is $A \cap B$.
Now I am told that X is not in $A \cap B$ so that means X is not here this means that X cannot be both in A and B because a F X is in A and B both then it is inside $A \cap B$ so that means either one is correct either X is not in A and in B or X is in B but not in A or X is in either therefore X can be anywhere over here.

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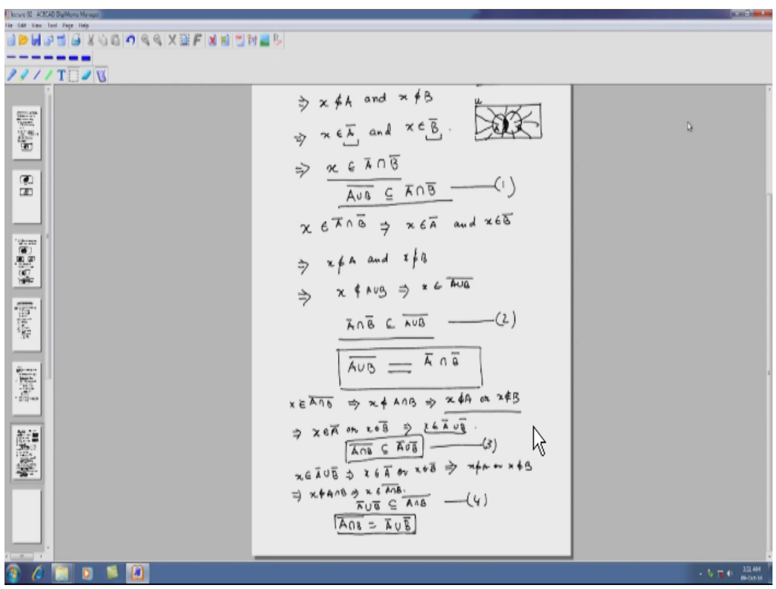

And that is expressed by this but this means that $X \in$ A complement or $X \in$ B complement which means that $X \in$ A complement Union B complement therefore like before we can write that $A \cap B$ complement is a subset of a complement Union B complement let us write this as 3 we can start again from the last line taking X to be belonging to a complement Union B complement this means that $X \in$ A complement or $X \in$ B complement which in turn means that X is not in A or X not in B which in turn means X not in $A \cap B$ which in turn means $X \in A \cap B$ complement.

Which gives us a complement Union B complement is a subset of A ∩ B whole complement call it for combining 3 & 4 we have A ∩ B complement equal to A complement Union B complement this completes the proof of de Morgan's law now let us quickly list all the other laws which are used very frequently.

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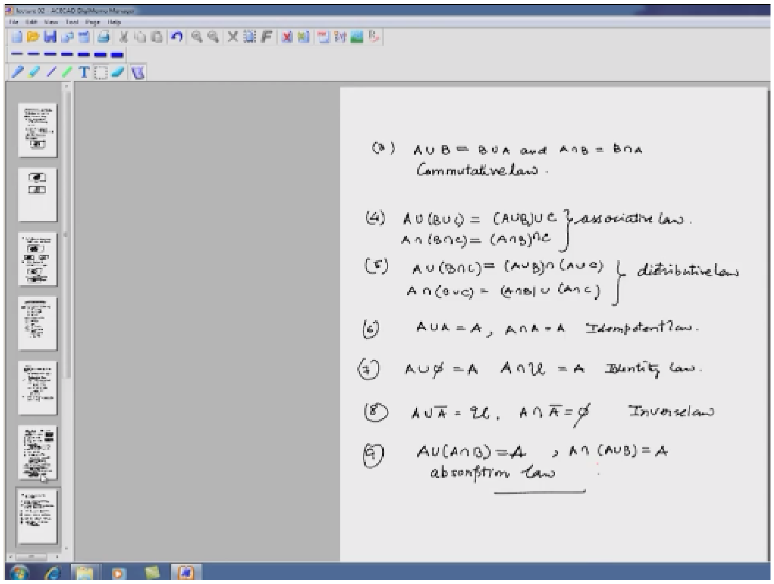

A U = B U A and A ∩ B = B ∩ A this is called commutative law 4) A U (B U C) = (A U B) U C and A ∩ (B ∩ C) = (A ∩ B) ∩ C these together is called associative law 5) A U (B ∩ C) = (A U B)∩ A U C and A ∩ B U C = A ∩ B U A ∩ C these two together is called distributive law 6)now A U A = A and A ∩ A = A this is the idempotent law 7) A U ɸ =A that is of course if I take union of A to the element which has no to the set which has no element and the result is a similarly if I take A and take the intersection with the universe then also I will get A.

This is called identity law 8) A U A complement is U and A ∩ A compliment is ɸ this is called inverse law lastly we end today's lecture by a law which is called absorption law and which is useful very often which goes like this A U(A ∩ B) is A and A ∩(A U B) is also A this is called absorption law these laws more or less covers the most important properties of set operations which are used to manipulate sets we have proved some initial laws and later on we have given a complete proof of de Morgan's law which is not very easy to see and the rest of the other laws are quite intuitive and I leave it as exercise for the participants we stop the lecture today here thank you.

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