**INDIAN INSTITUTE OF TECHNOLOGY
ROORKEE**

**NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)
Discrete Mathematics**

**Module-04
Discrete Mathematics**

**Lecture-01
Sample space,events**

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So today's last lecture starts with the topic random experiment statisticians use the word experiment.

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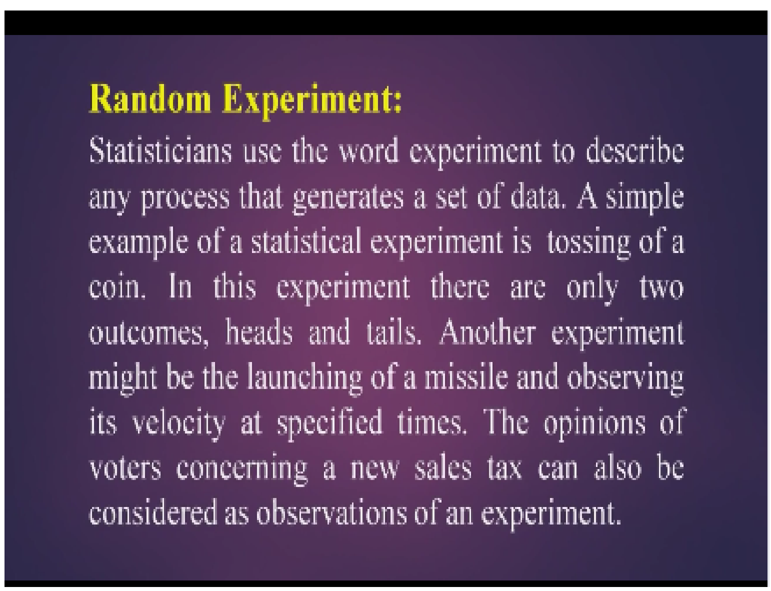

To describe any process that generates a set of data a simple example of a statistical experiment is tossing of a coin in this experiment there are only two outcomes heads and tails another experiment might be the launching of a missile and observing its velocity at specified times the opinions of voters concerning a new sales tax can also be considered as observations of an experiment.

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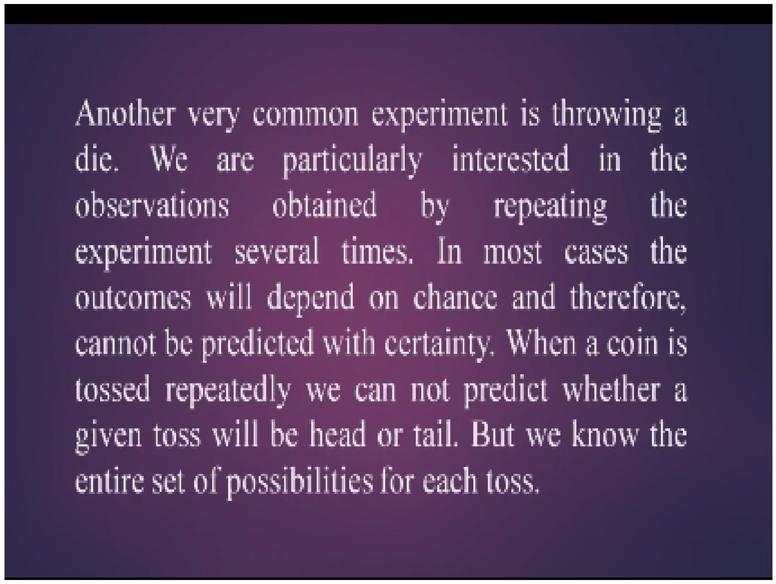

Another very common experiment is throwing a die. We are particularly interested in the observations obtained by repeating the experiment several times in most cases the outcomes will depend on chance and therefore cannot be predicted with certainty when a coin is tossed repeatedly we cannot predict whether are given to us will be head or tail but we know the entire set of possibilities for each toss next I would like to talk about sample space.

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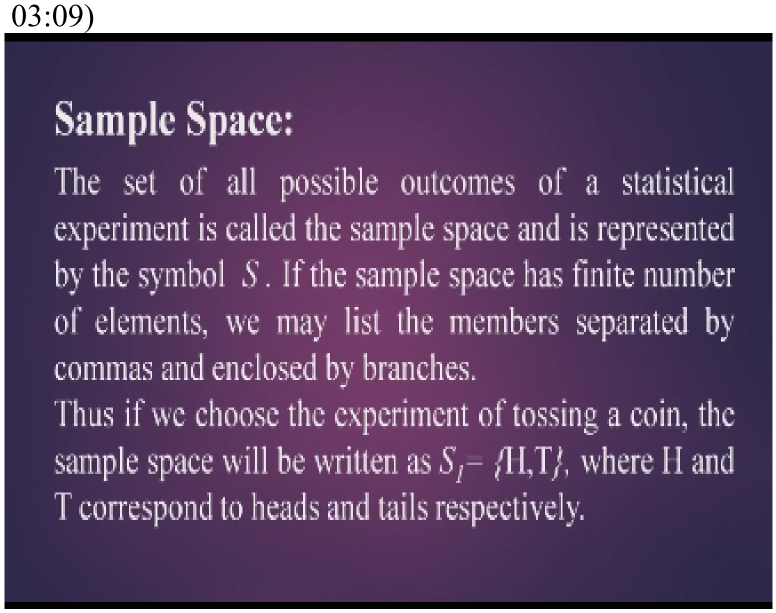

The set of all possible outcomes of a statistical experiment is called the sample space and is represented by the symbol capital S if the sample space has finite number of elements we may list the members separated by commas and enclosed by branches. Thus if we choose the experiment of tossing a coin the sample space will be written as S1 here it is like this S 1 = the (H,T) where H and T correspond to heads and tails respectively.

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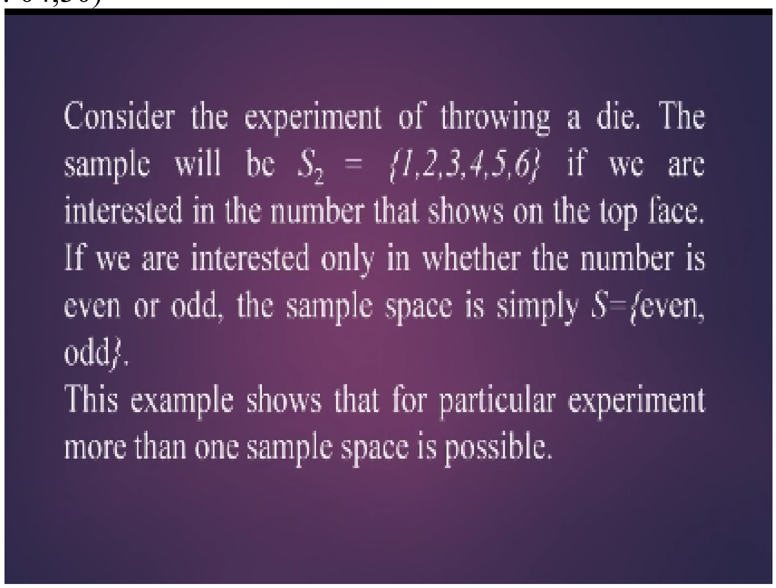

Consider the experiment of throwing a die the sample will be S 2 which is set of six numbers {1,2,3,4,5,6} if we are interested in the number that shows on the top face if we are interested only in whether the number is even or odd the sample space is simply, if the set containing even and odd this example shows that for particular experiment more than one sample space is possible so the sample space for a particular experiment depends on the situation or some particular problem now let us take one example here an experiment of flipping a coin.

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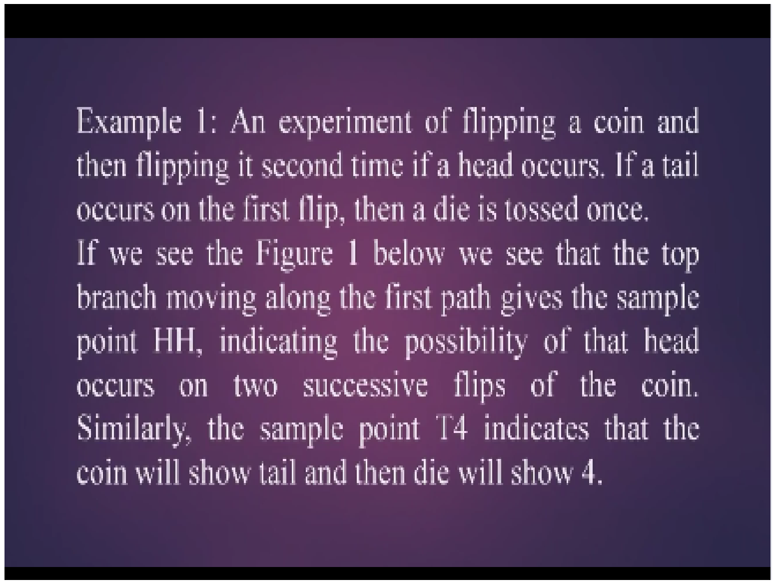

And then flipping it second time if a head occurs if a tail occurs on the first flip then a die is tossed once if we see the figure one below that is this one.

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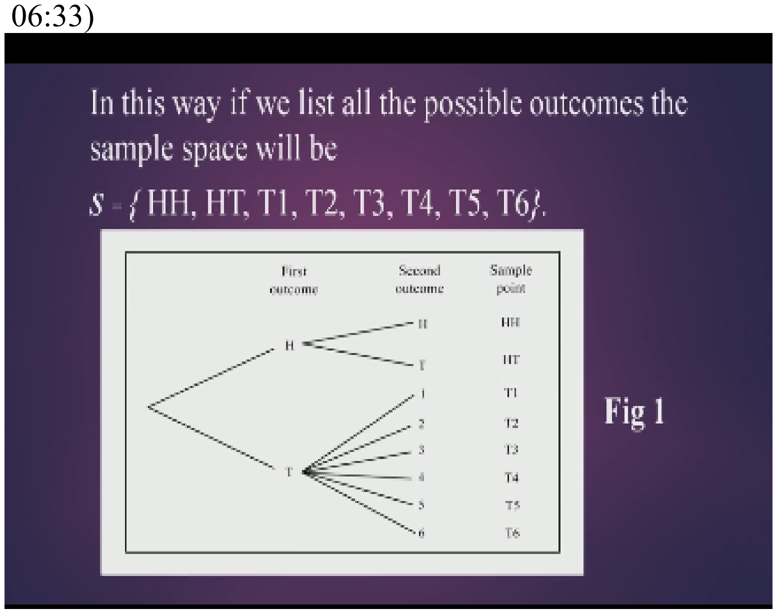

We see that the top branch moving along the first path gives the sample point h/h indicating the possibility of that head occurs on two successive clips of the coin so let us see the figure here the first outcome is head so if the coin is tossed we have two outcomes head and tail so there is if head occurs then the coin is tossed second time so if the first one is head then from this node we have two branches head and tail so finally we will have two sample points HH and HT.

Now according to the problem if tail occurs for the first time then a die is thrown so from this tail node we will have six possibilities 1, 2, 3,4,5,6 so finally we will have six of sample points t1 t2 t3 t4 t5 and t6 so for this particular experiment we will have the sample space s which is the set containing the sample points HH, HT t1 t2 t3 t4 t5 and t6.

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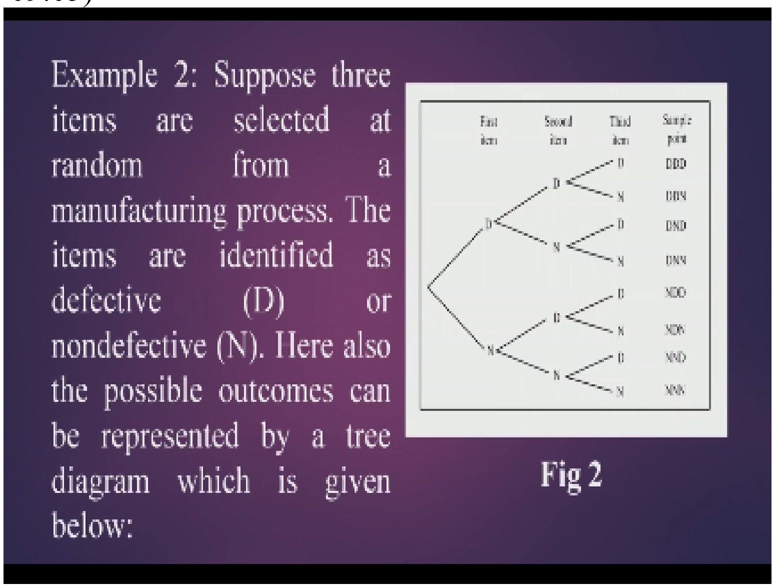

Another example suppose three items are selected at random from a manufacturing process the items are identified as defective which is denoted by T or non-defective which is denoted by N here also the possible outcomes can be represented by a three diagram which is given here so once the first item is chosen we will have two outcomes D and N now if it is D then the second item is chosen and it will have two branches D and N again so we will have D ,D or D N now suppose the second item is also defective.

So we will follow this path so third item is chosen again we will have two outcomes D and N so if we have the first item defective second item also defective then finally the samples per sample point will be D, D ,D, D, D in now if we go along this path that means the first item is defective second item is non-defective so the third item is chosen so it will have two outcomes D and N so ultimately we will have two sample points D and D DN n in this way if the first item is non defective then the second item is chosen.

And we will have two outcomes D and N first and first one is non defective and second one is defective so we are going along this path we will have again two outcomes D and N so finally we will have two sample points N DD and ND N now the last case that is the first item is non-defective second item is also non defective ,so we are going along this path third item is selected it will have two outcomes D and N so finally we will have two outcomes again NND,NNN so from this experiment.

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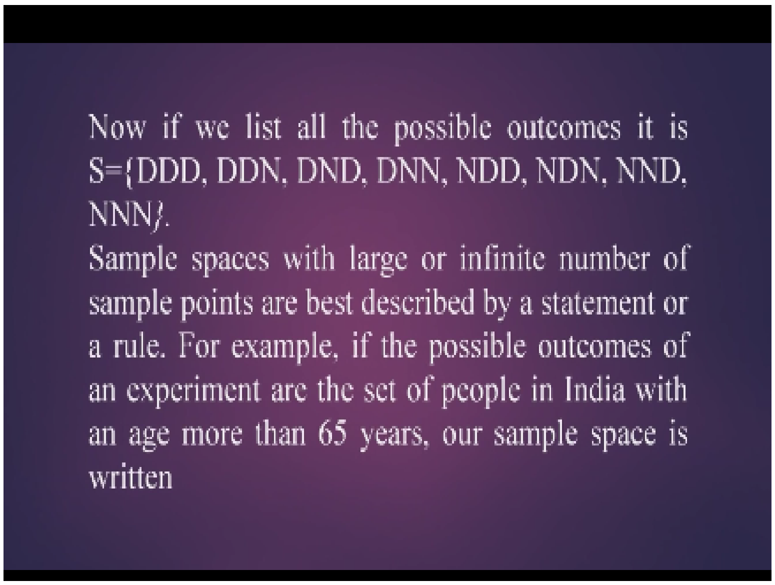

We have the sample space is which is the set of the sample point S= {DDD, DDN, DND, DNN, NDD, NDN.NND, and NNN} sample spaces with large or infinite number of sample points are best described by a statement or a rule for example if the possible outcomes of an experiment are the set of people in India with an age more than 65 years our sample space is written as S.

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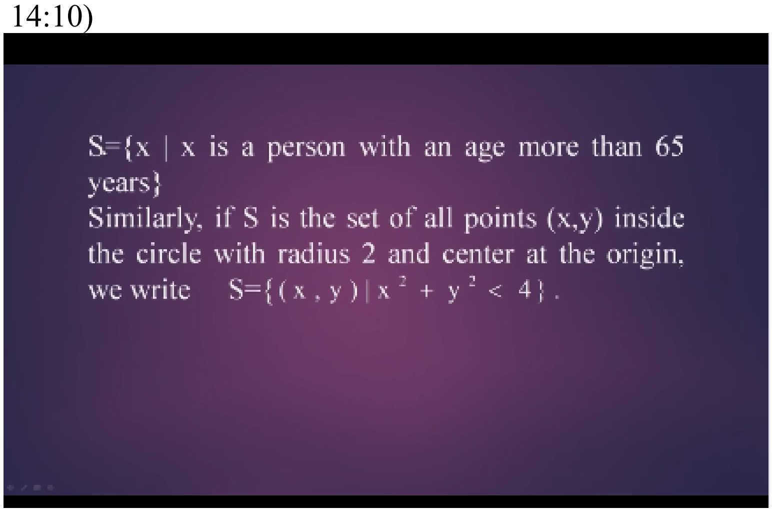

Which is the set of x such that x is a person with an age more than 65 years similarly if s is the set of all points (x,y) inside the circle with radius 2 and Center at the origin we write is that is the sample space is the set of the points (x,y) such that ($X^2 + y^2 < 4$).

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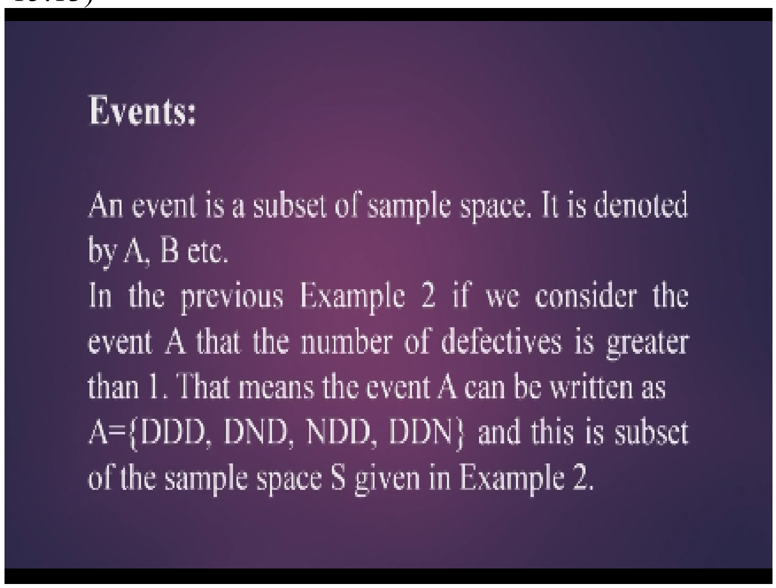

Next is events what is an event an event is a subset of sample space it is denoted by A B etc so we can define random experiment sample space events in this way also an experiment is a process that yields outcome an event is an outcome or combination of outcomes, Robban experiment of course and what is the sample space the sample space is an event consisting of all possible outcomes, so sample space is the event consisting of all possible outcomes now in the previous example two.

If we consider the event a that the number of defectives is greater than 1 that means the event a can be written as d the set containing {DDD,DND,NDD,DDN} so this is an event because this is a subset of the sample space is given in example two that is this one let us consider another example here given the sample space is that is the set containing X such that X is a person with an age more than 65 years the event.

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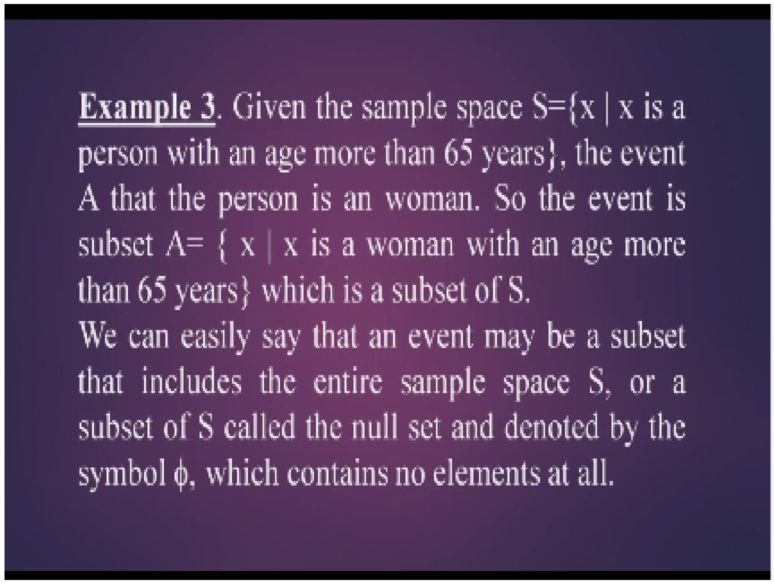

A that the person is an woman so the event is subset a that is the set containing X such that X is a woman with an age more than 65 years. So this is the event here this is also a subset of the sample space S which is this one here we can easily say that an event may be a subset that includes the entire sample space is or a subset of is called the null set and did not a denoted by the symbol ϕ which contains no elements at all. So let us consider more examples here.

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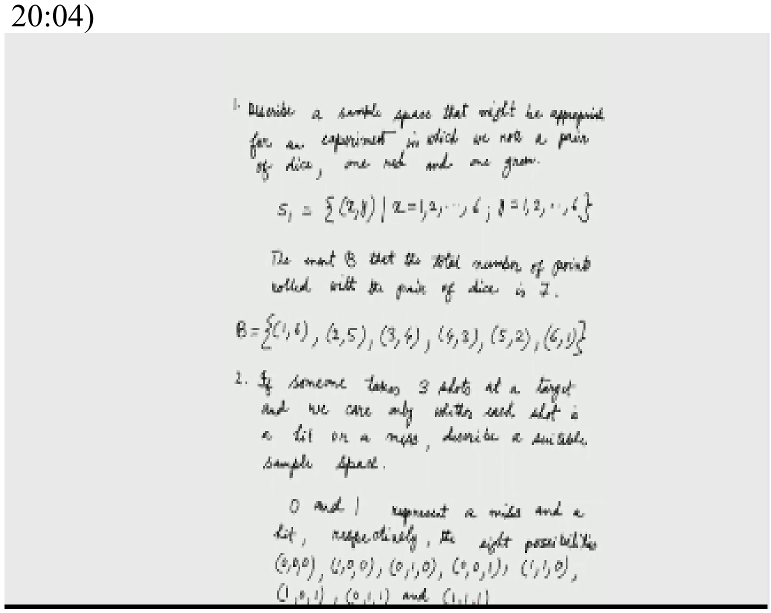

Describe sample space that might be appropriate for an experiment in which we roll a pair of dice ,one rate and one gream, so the solution will be like this the sample space that provides the most information consists of the 36 points given by S1 which is the set containing the points { X, Y }such that X will take the value from 1,2…, 6 Y will take the value from 1 ,2…,6 so the sample space is like this the sample space will contain 36 points because x is from 1 to 6 and y is from 1 to 6 where its represents the number turned up by the red die and y represents the number turned up by the green die.

Now for this particular experiment let us describe the event B that the total number of points rolled with the pair of dice is 7 so in this case among 36 possibilities only (1,6).(2,5),(3.4),(4,3),(5,2),(6,1) so this six cases are favorable so the event B will contain the six points see this six sample points another example is like this if someone takes three shots at a target and we care only whether each short is a hit or a miss describe suitable sample space for this case.

So let us try the solution in this way if we let 0 & 1 represent miss and a hit respectively the eight possibilities are (0,0,0),(1,0,0),(0,1,0),(0,0,1),(1,1,0),(1,0,1),(0,1,1) and (1,1,1) may be displayed as in the following figure.

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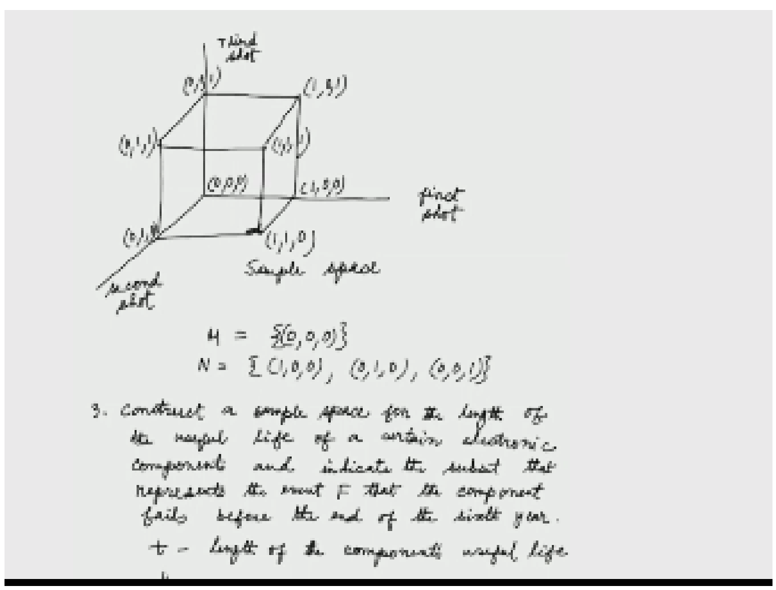

So this axis is corresponding to first short this one is corresponding to second shot and this one is corresponding to third short so these are the sample points so the samples point here can be displayed by the following figure so this is the sample space for this particular experiment. Now let us find the event M that the person will miss the target three times in a row and the event in that the person will hit the target once and miss it twice, so the event M will be so it will contain only the sample point M={0,0,0} because it will miss.

So it is corresponding to the event that the person will miss the target three times so that is why it will be {(0,0,0)} now the next event N will be {(1,0,0),(0,1,0),(0,0,1)} so the set containing these three points because it is the event that the person will heat the target once and miss it twice let us consider one more example, in this context construct is sample space for the length of the useful life of a certain electronic components and indicate the subset that represents duh event if that the component fails before the end of the sixth year.

So here the sample space we have to find so let us write this in this way if t, is the length of the components useful life in years.

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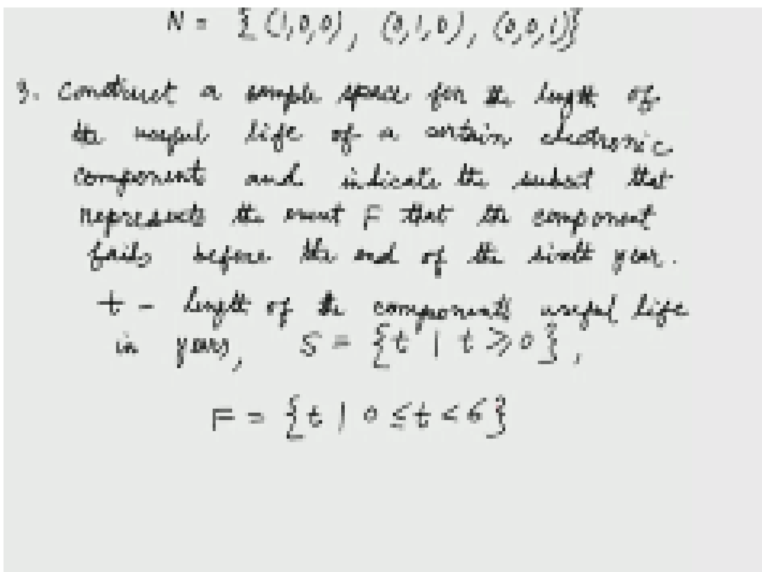

The sample space can be written as S {t1 t >=,0},the subset F we will be the {t10<=t<6} this is the event that the component fails before the end of the sixth year so in this way we can find the sample space and events and some events for a particular experiment, now let us consider some more events the complement of an event.

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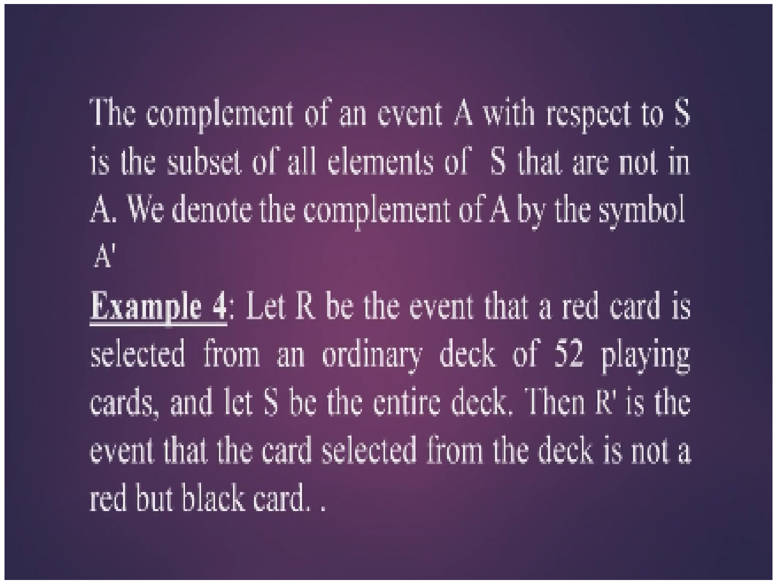

A with respect to S is the subset of all elements of S that are not in A we denote the complement of A by the symbol A' it is like this so in this context let us discuss one example let R be the event that a red card is selected from an ordinary deck of 52 playing cards and let S be the entire take then R' is the event that the card selected from the deck is not a red but black card because we know that in a deck of cards there are only two possibilities either it will be red or black so that is why the complement of the event R will be will be a set of cards.

So it will be the event that the card selected from the deck is not a red part black card.

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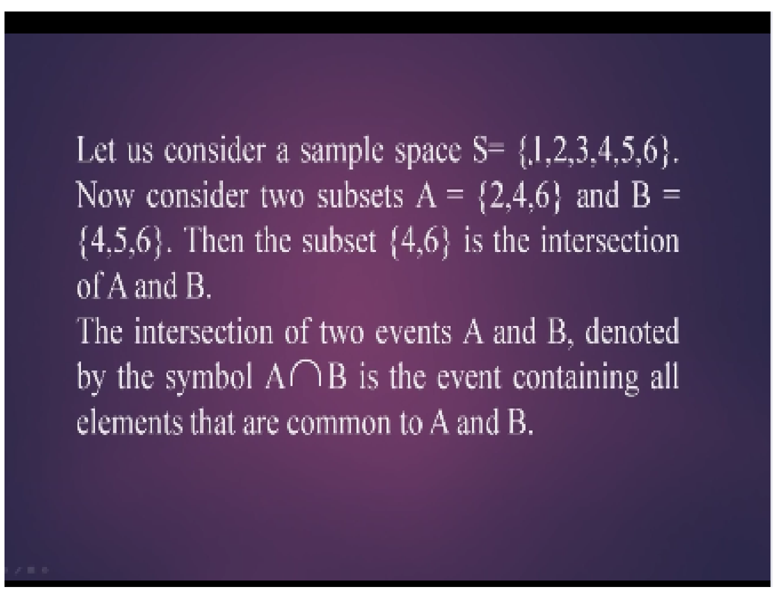

Let us consider a sample space is that is the set of six integers S={1, 2 ,3 ,4, 5, 6} now consider two subsets A {2,4,6} and B = {4,5,6} the set {4,5, 6 } so here the subset {4 ,6} so{ 4 ,6}this is a subset of A this is a subset of B so this subset 4 6 that is the set containing 4 & 6 is the ∩ of A and B so it is common to a as well as B so this is called the ∩ of A and B so if we have a sample space s and we have two A and B then a ∩ B is another event which contains all elements that are common to a and B.

So in this way combining various events we can form many events let us consider another example here let P be the event that a person selected.

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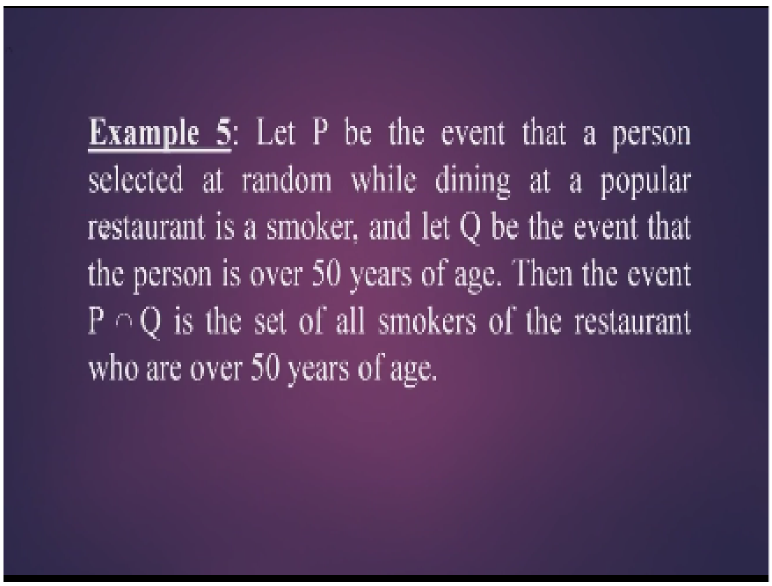

At random while dining at a popular restaurant is a smoker and let Q be the event that the person is a smoker his O my sorry let Q be the event that the person is over 50 years of age so P and Q event that a person selected at random while dining at a popular restaurant is a smoker and QB the event that the person is over 50 years of age then the event B ∩ Q is the set of all smokers of the restaurant who are over 50 ,50 years of age so P ∩ Q is it should be in P as well as in Q so that is why it will have both the cases that is they should be smokers.

They should be over 50 years of age consider the random experiment of throwing a die the event is of occurring odd numbers.

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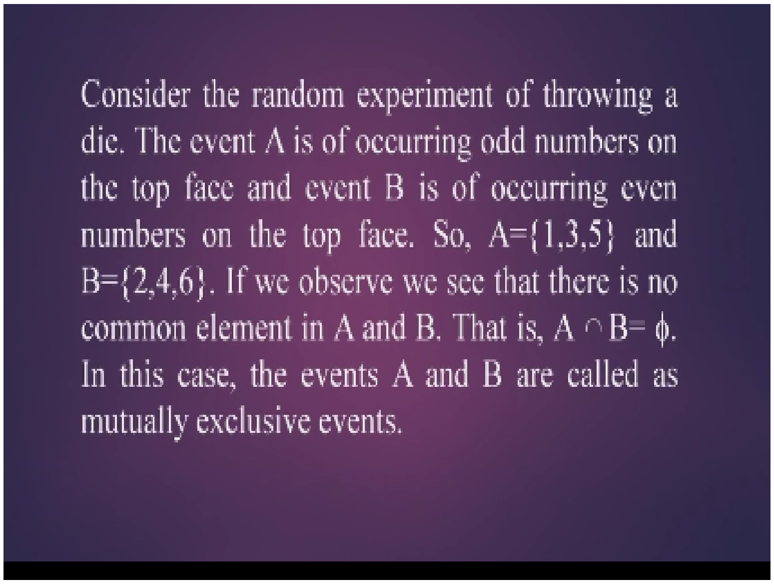

On the top face and event B is of occurring even numbers on the top face so if A that is A is the set event A that will contain three points {1,3,5} and B will contain {2,4,6} if we observe we see that there is no common element because we have considered two cases one is the first one is of odd numbers second one is of even numbers so that is why there is no common element in A ∩ B and that is why we can say that A ∩ B = ϕ in this case the events A ∩ B are called as mutually exclusive events.

So we can define the mutually exclusive events in this way two events A ∩ B are mutually exclusive events if they do not have any common element or another way we can say it in this way that if A ∩ B is equal to five the ∪ of the two events A and B denoted by the symbol.

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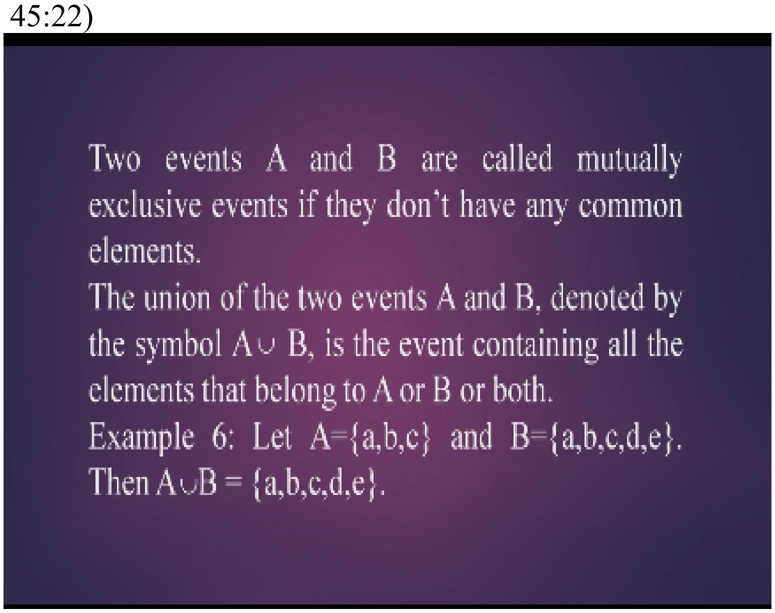

A ∪ B is the event containing all the elements that belong to A or B or both so this is another kind of event another kind of combination so we have two events A and B so we are combining these two events in this way that the resulting event contains all the elements that belong to a or B or both so this is denoted as A∪ B so ∪ between these two events so let us consider this example here let a equal to the set ABC and B the set containing a b c d e then A ∪ B will B={ a ,b, c, d, e} because it will contain all the event and all the elements.

That belong to a or b or both so that is why it will be the set containing {a, b ,c, d, e} the relationship between events and the corresponding sample space.

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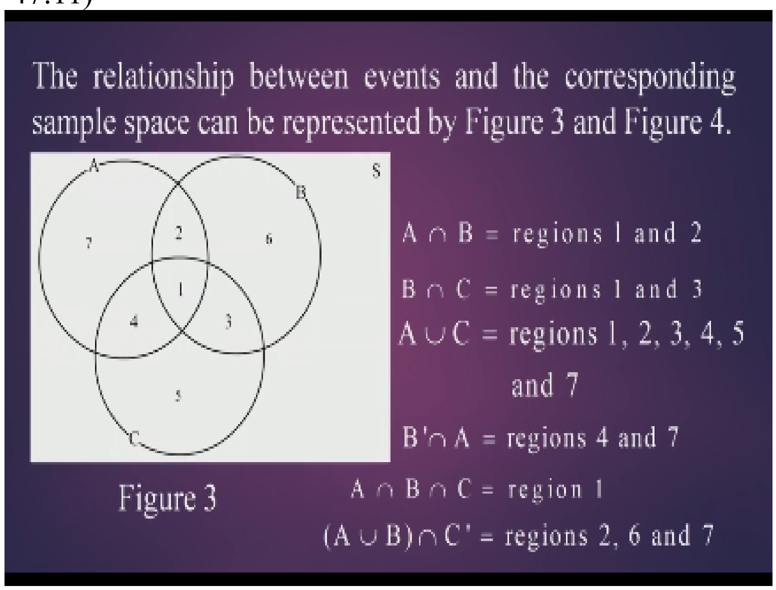

Can be represented by figure three and figure four let us consider figure three first here we have a sample spaces and in this sample space we have three events A B and C, so we have specified the regions different regions this is one region which is denoted by one this is another region which is denoted by 1,2,3,4,5,6,7 like this so A ∩ B if we have to find this particular event so this event corresponds to the regions 1 & 2 because ∩ B is the common part between A and B so that is why if we see this here this is the common portion between A and B

So that is why it will correspond to regions 1 & 2 if we have to find B ∩ C so we have to find the common portion between B and C and this is the common region here so this will be regions 1 & 3 in this way if we have to obtain A ∪ C so the event A ∪ C will correspond to the regions 1,2,.3,4,5,7 because A ∪ C so this whole portion we have to consider so it will be 1 2 3 4 5 & 7 now let us find B ‘∩ A part is B ‘ this is complement of the event B so complement of event B will be this portion this whole portion.

So B ′∩so we have to take the ∩ of this with A so B ′∩ A so this will correspond to the region's 4 & 7 so this is the common between B ′and A now let us find A ∩ B ∩ C so that means we have to find the common region between a b and c between a and b we have this common region between b and c we have this common region between a and c we have this common region so if we take this 3 this is the common between A B and C so this region one is the region corresponding to A∩ B ∩ C this event now let us find another event that is a ∪ B ∩ C ′ so a ∪ B is this whole region and we have to take the ∩ with C ′ so C ′ is this portion so a ∪ B ∩ C ′ will correspond to regions 2 6 & 7 in figure 4.

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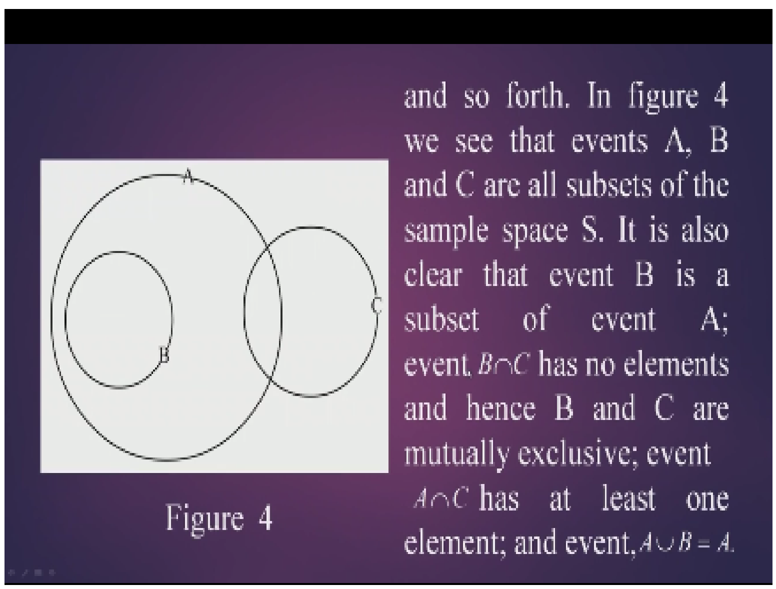

We see that events a B and C are all subsets of the sample space is so this is the sample space is and ABC these three events are subsets of this sample spaces it is also clear that event B is a subset of the event A so B is the subset so it is inside a it is a subset of A even B ∩ C has no elements they are disjoint and hence B and C are mutually exclusive event A ∩ C has at least one element and A ∪ B =A because B is A subset of A.

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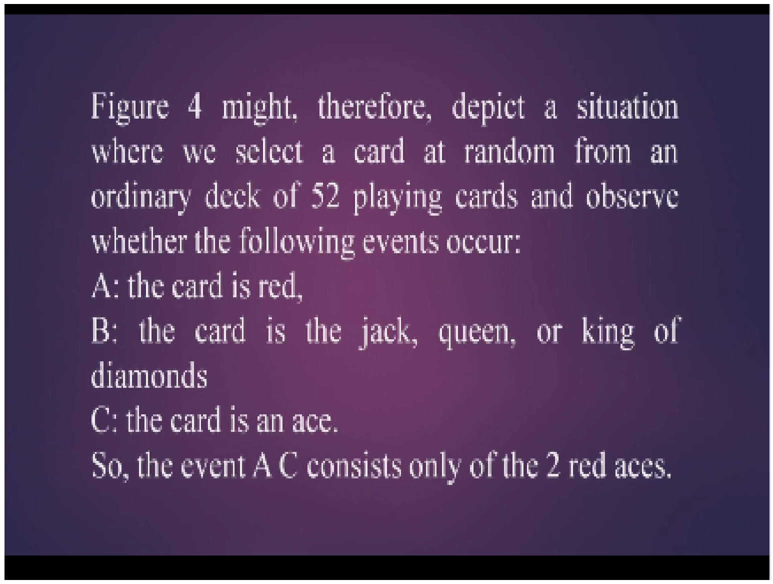

Figure 4 might therefore they picked a situation where we select a card at random from an ordinary deck of 52 playing cards and observe whether the following events occur the event A is the event that the card is rate B is the event that the card is the Jack ,queen ,or king of diamonds C is the event that the card is an S so the event $a \cap C$ consists only of the two red aces.

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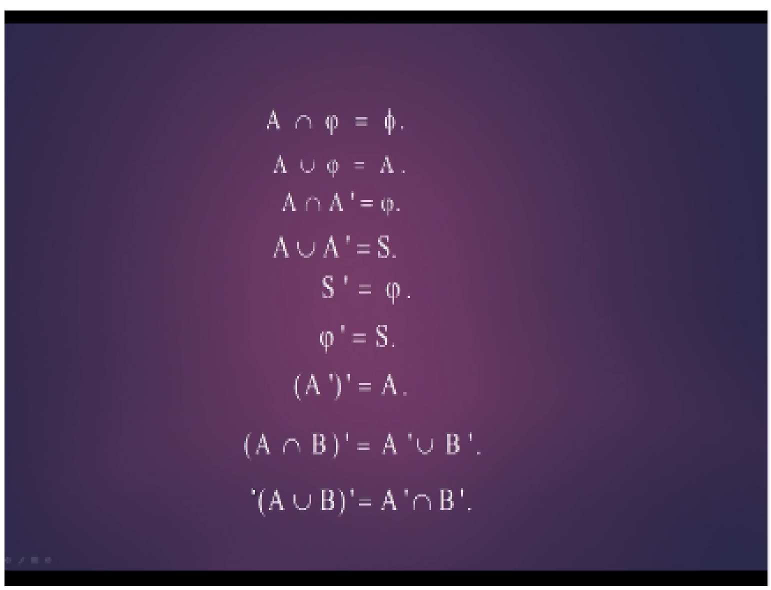

So finally I would like to discuss some results which are related to this kind of combinations if we have two events A and ϕ then $A \cap \phi = \phi$ so because this is the common portion between A and ϕ the null set so that is why the resulting event will be a null set again if we combine A and ϕ in this way that is by ∪ that is A ∪ ϕ again ϕ is the null set so A ∪ ϕ will become a only so the resulting event will be A ∩ A ′ so A ∩ A compliment so if we consider the common portion between a and A ′ naturally.

The resulting event will be the null set ϕ A ∪ A ′ ∪ between A and complement of A so we will have the whole sample space as the resulting event is ′ that is the compliment of the sample space so it will be the null set now if we consider ϕ ′ so it will be S now if we consider a complement of the complement of A that is A ′ ′ so first we are taking the complement of a then again we are taking the complement of that so we will get back A ∩ B ′ A ∩ B whole it is an event A ∩ B and then we take the complement of A ∩ B.

So this will be same as a complement A ∪ B complement so complement of A ∩ B will be same as the event a complement A ∪ B complement A ∪ B complement so complement of the event A ∪ B so this is same as a complement A∩ B complement so the resulting event will be A ∩ B.