INDIAN INSTITUTE OF TECHNOLOGY
ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)

Discrete Mathematics

Module-02
Logic
Lecture-06
Rules of influence for quantified propositions

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In today's lecture we will discuss the rules of inference for quantified propositions.

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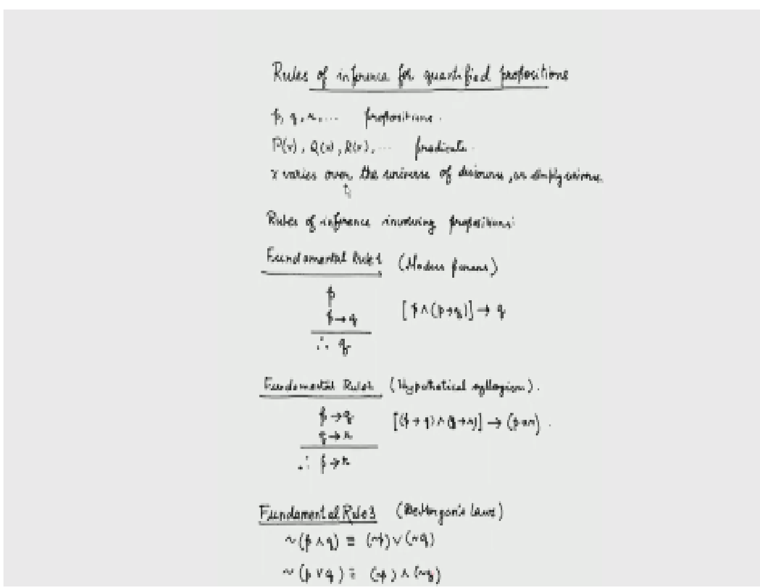

Now let us discuss briefly whatever we have done in the previous lectures as a recall. So we know what a proposition is and we will be denoting propositions by small letters p, q, r, and so on. So these are propositions which are essentially statements having a truth value. Now we also know what are predicates or open propositions. Predicates will be denoted by P(x), Q(x), R(x), and so on.

Where X where is over the universe of discourse or simply universe. We have seen some rules of inferences. So rules of inference involving propositions, we list some rules and refer to them as fundamental rules. So fundamental rule one, we have seen this under the name of modisponence which states that P and P→Q, therefore Q. The tautological form of this rule is P→Q→Q, and we have already checked that this rule is a tautology and therefore is valid inference.

The next rule is fundamental rule two, which is what we have studied in the name of hypothetical syllogism. This rule states that P→Q, Q→R, therefore P→R or in the tautological form P→Q and Q→R, P→R. As before we can check that this is indeed a valid inference. Now the fundamental rule three, this is known as De Morgan's laws state that not of p and q is equivalent to not of p and not of q and not of p or q is equivalent to not p and not of q finally the last law that we state here involving the propositions is fundamental law for.

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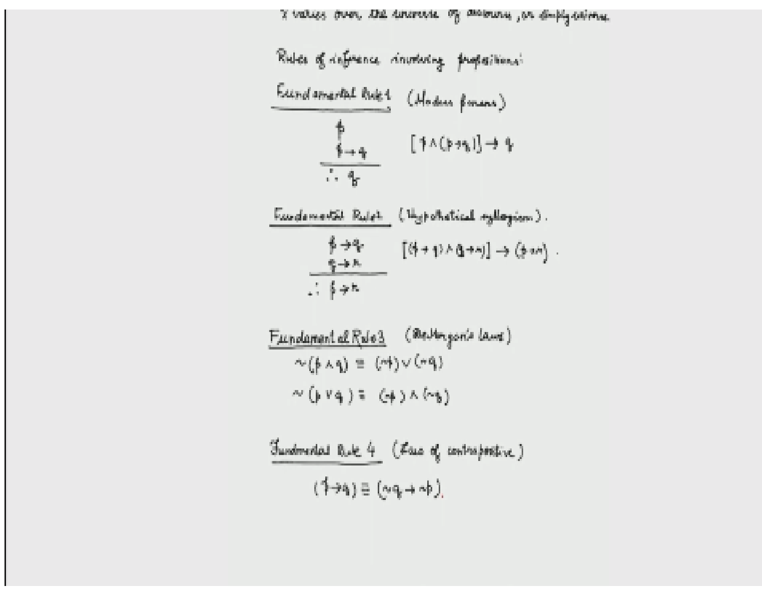

Fundamental rule 4 which is the law of counter positive states that p implies q is equivalent to not q implies not p therefore in order to prove p implies q we may as well prove not q implies not p.

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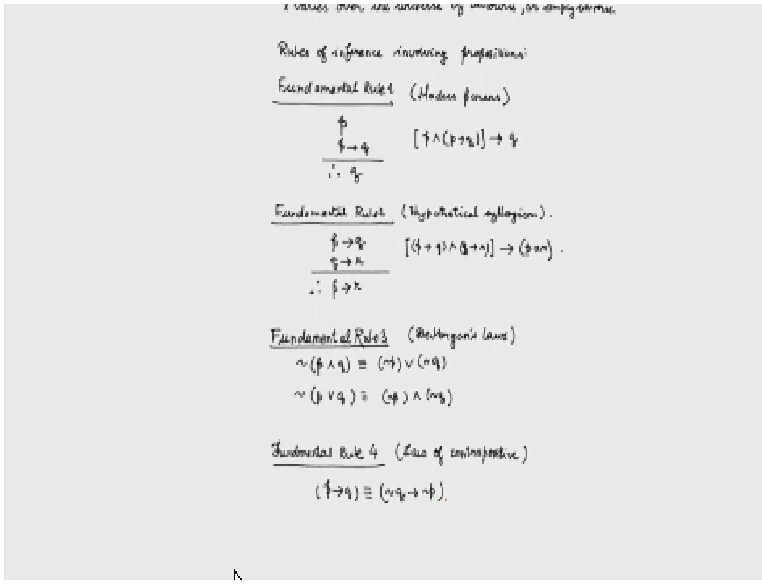

What is seen at this point is that if we move on to the first order logic from the propositional logic then we will have predicates and quantifiers in particular existence existential and universal quantifiers in this framework of the first order logic the rules of inferences that we have derived for propositional logic are not sufficient therefore we need additional rules in what follows I will state 4 additional rules which are used in propositional logic along with the rules of the 4 additional rules which are used in predicate logic or the first order logic along with the rules that we have derived from the propositional logic.

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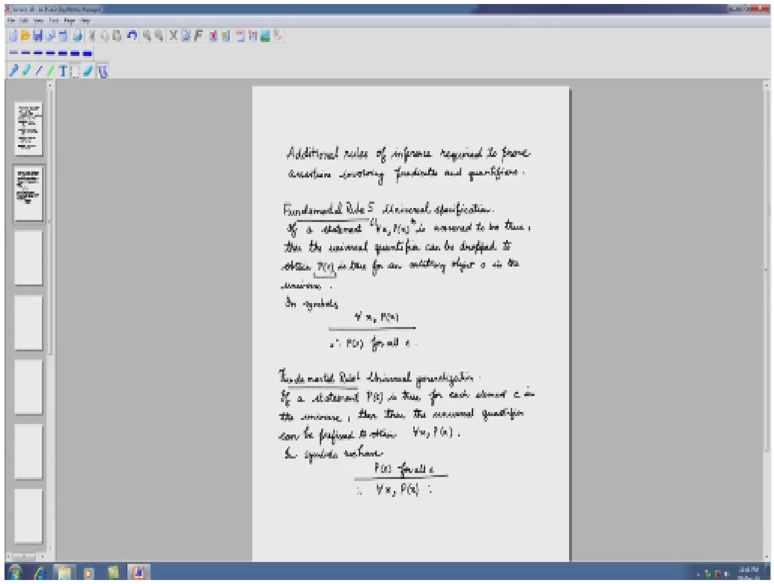

So we move on to additional rules of inference required to prove assertions in involving predicates and quantifiers fundamental rule 5 this rule is called universal specification states that if a statement for all x P(x) is assumed to be true then the universal quantifier can be dropped to obtain P© is true for an arbiter object c in the universe apparently this rule is simple it just says if we have a statement for all x P(x) where P(x) is a predicate and if we assume that it is true then we will be able to given any arbiter object c in the universe the proposition P(c) that we get by replacing x by c in the predicate P(x)

The preposition p(c) that we get by replacing x / c in the predicate P (x) is going to be true in symbols we can represent this rule as for all x P (x) therefore you see for all c next we move to the fundamental rule 6 this is called universal generalization, universal generalization takes that if a statement we see is true for each element c in the universe when that universal quantifier can be fixed and we obtain the prepositions for all x P (x) again in symbols we have P(c) for all c therefore for all x P(x) we move on two more rules involving predicates and quantifiers fundamental.

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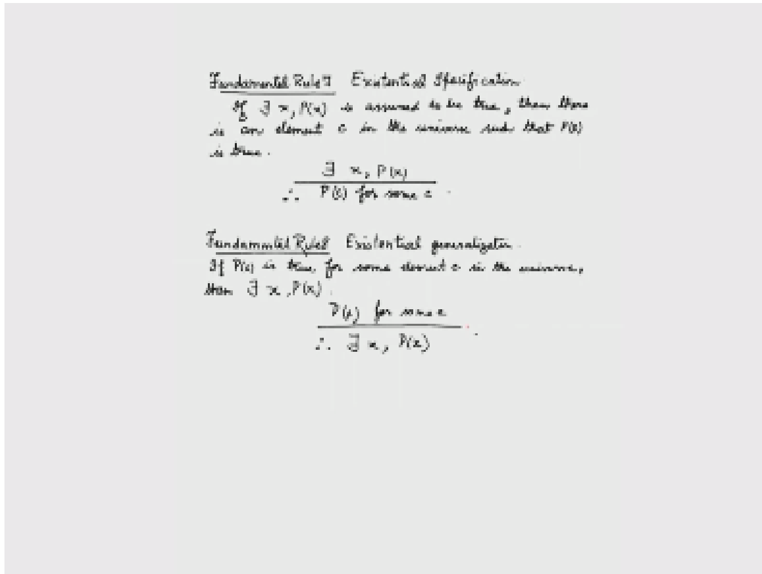

Rule 7 this is called Existential specification states that if they are exist x P(x) is assume which is then they are is an element c in that universe such that you see is true by using symbols like before we can represent this rule has where exist x P(x) therefore P(c) for some c lastly we look at fundamental rule 8 this rule is called the Existential generalization if P(c) is true for some element c in the universe then there exist x px and again in symbols we can write pc for some c therefore there exist x ex, these are the rules that we will be using to prove prepositions involving predicates and quantifiers let us look at some examples.

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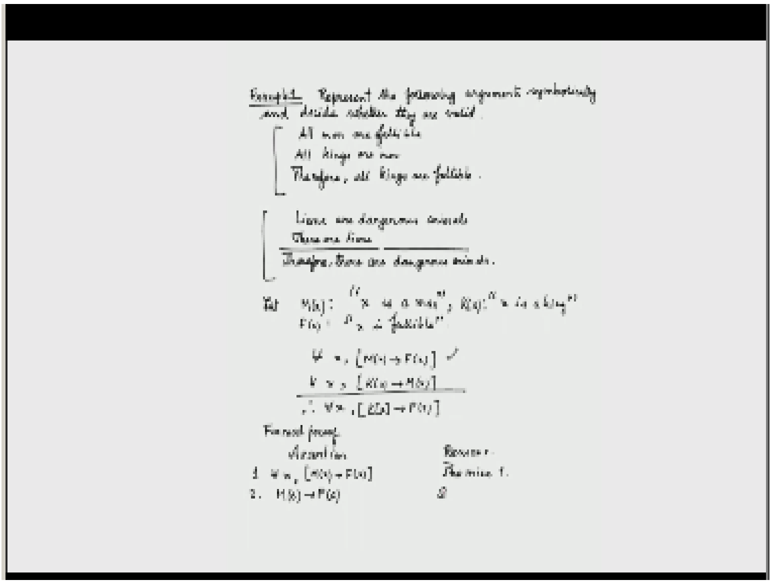

One represent the following arguments symbolically and decide whether they are valid the arguments are all men are fallible all kings are men therefore all kings are fallible. Now this is one argument and another argument that we will discuss is lions are dangerous animals there are lions, therefore there are dangerous animals this is another one.

Now let us look at the first argument, let mx is the predicate x is a man and kx is the predicate x is a king and fx is a predicate x is fallible, now we symbolize the argument the first argument in this way for all x mx implies fx for all x kx implies mx therefore for all x kx implies fx. Now we see the formal proof of this argument step wise, so we write at each step an assumption and the reason. One for all x mx fx so this is premise one that is this one two mc implies fc now this is what we get by fundamental rule five and step one rule five three for all x ax implies x.

Now this is true four mc implies fc this is step 3 and rule five now here we see that we can use I am sorry we can use the rule two that is hypothesis on step two and step four so by using that we have fc implies fc this is step two and four along with rule two which is hypothetical syllogism and then we use rule six to obtain kx implies fx step five rule six now this is the next involving lines so here the symbols we write let lx

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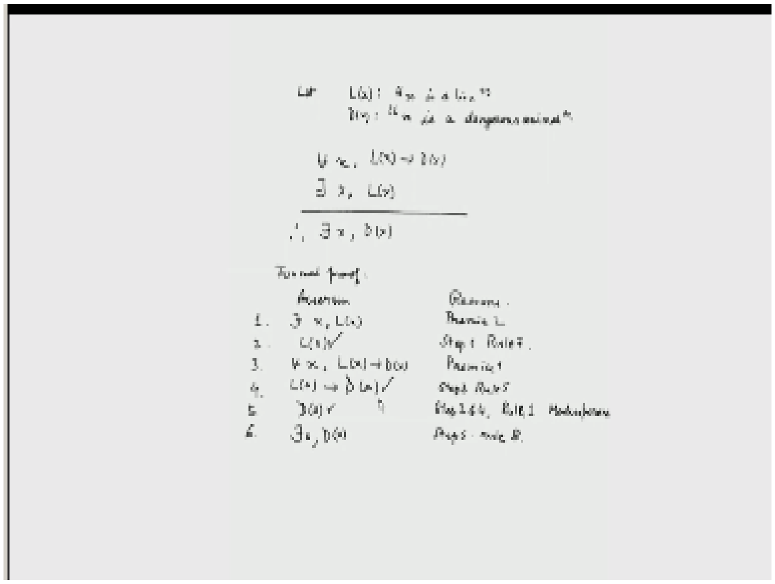

X is a line dx is a predicate that x is dangerous animal now the statement that we have a lion is a dangerous animal if we write it is a predicate and universal quantifier say it is all x lx implies that dx and we can use systemically quantifier there exist lx for which it is true and in a consequence therefore there exist x let us see valid or not again we want to follow this assertion reasons step one there exists x lx which premises to reduce 7 to write la step one rule seven step three.

Now it premises one that is for all x lx implies ex that is one for rule five write la implies pa step three and rule five step two and four rule one is corresponds so I am combining step two and step four I am using corresponds and then six there exist x dx this is step five and rule eight first we have to evaluate the that we have stated just now this is the end of this lecture thank you.

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