INDIAN INSTITUTE OF TECHNOLOGY
ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)
Discrete Mathematics

Module-01
Set theory

Lecture-01
Introduction to the theory of sets

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In today's lecture we will start with the definition of a set the question is that what is a set?

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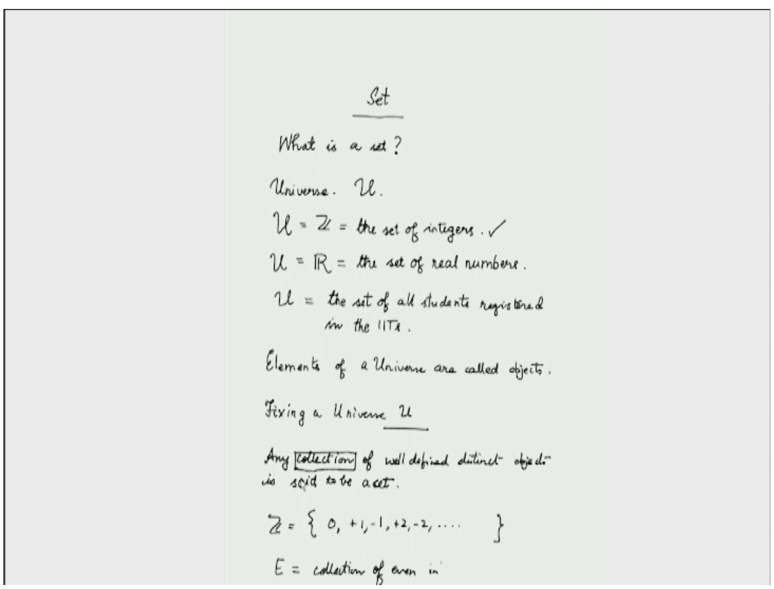

Although it is, is it to ask this question it is not so easy to answer what has been realized that this simple question leads to several more difficult questions in this lecture we are not going to discuss the difficulties which leads to eventually introduction of estimate of set theory instead of that we will discussed a sum what working or operational understanding of a set.

What we note that in mathematics or in several other applications when we talk about things we somehow understand a collection of things which we are interstitial for example when we have talking about integers that is 0 + or -1 + -2, +-3 and all that we really think of the set of elephants or set of cows or set of students on the other hand when we consider set of students we usually do not consider the set of complex numbers in the same time or we do not consider set of l possible subsets of integers.

Thus there is an idea of universe which we will denote by script U so when we are talking about set of integers then we will simply say that our universe U is the set of integers + Z which is the set of integers remember we are talking about real numbers then U will be R the set of real numbers or we may be talking of the set of all students in IIT the set of all students registered in the IIT's so these are not numbers these are human beings of course we can label them any numbers.

We can label them by names but there is a trouble that names of two students may be same so possibly we will label them by their enrollment number and the name of the IIT and like that but of course they are not set of integers not that set of all subsets of integers so like that there are different scenarios were we have different universals the elements of an the elements of an universe is called objects.

That is the things that make up the universe in case of integers the object will be integers in case of real numbers the objects will be real numbers in case of the set of students the objects will be individual students here we will use elements and objects synonymously now once we fix the universe then any collection of well-defined objects inside that universe will be called as set provided that the universe is not too large.

We are not going to discuss the issues that when will the universe be too large and all the other complicated questions we will a simply take the examples that I have discussed already and many such examples were this description of a set works nicely so we can even say that will be fix universe fixing universe the any collection of, any collection of well-defined distinct objects is said to be a set.

Now of course this is not a definition it is just giving an idea of what we will mean by set it is not a definition because the question that what is a collection will be raised if we call this a definition and then we have to define some of the collection the main idea over here is that if we fix a universe.

So we fix the type of objects that we are dealing with and within that type of objects we specify certain objects carefully so that when we encountered an object in that universe we are able to say whether the object that we are encountered has that specified property and this collection or ensemble of these objects within an universe with specified properties is called set. For example, if we consider the set of integers Z which is given by 0, +1, -1, +2, -2, and so on.

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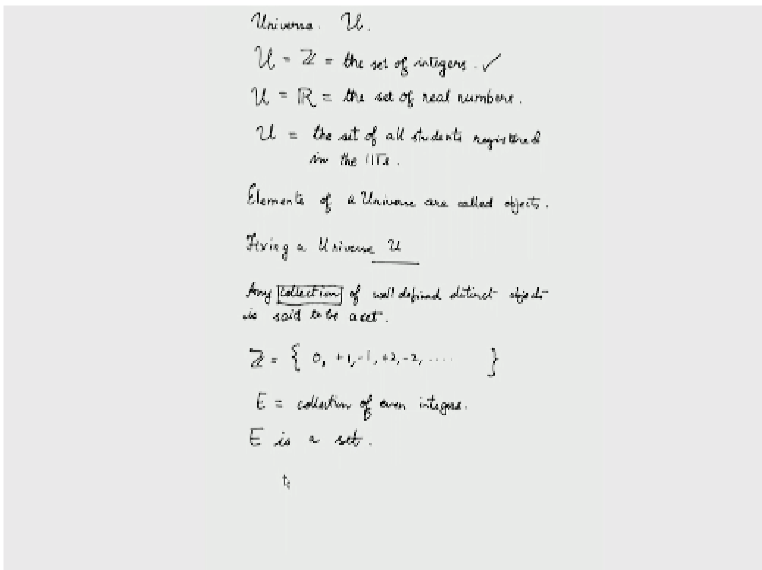

If we consider the set of integers which are multiples of 2 that is set of even integers we can easily determined given an element in Z were there is even or not. So let us call E has the collection of even integers this collection E is a set, the reason is that we have fixed the universe and once we have fixed the universe, if I say that I am interested in the set of even integers, then given a integer I can determine whether it is even or not.

And then I can perceive of the collection of even integers. In case of objects which are not numbers, if I consider the set of all students registered in IITs, then we can consider the students registered in IIT Roorkee. So the students registered in IIT Roorkee forms a set. In the universe of all the students in registered in IITs. So we have more or less understood what we mean by a set.

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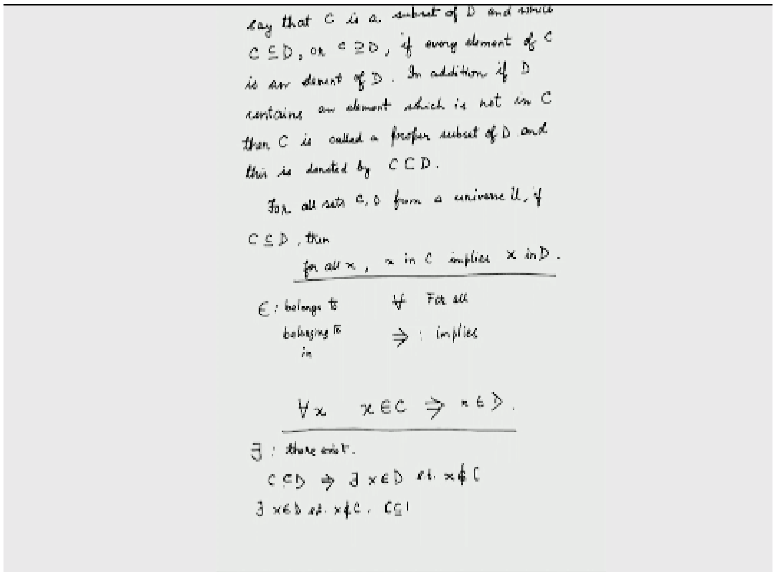

Now we move on to subsets. If CD is sets from a universe U we say that C is a subset of D and write C or C if every element of C is an element of D. In addition, if D contains an element which is not in C, then C is called a proper subset of D. And this is denoted by C properly contained in D. Now this symbolically will mean that for all sets CD from a universe U, if C is a subset of D, then for all X, X in C implies X in D.

We introduce some more symbols here that is the symbol this and the symbol inverted A. This symbol which looks like ε means it belongs to or belonging to or simply in. This means for all, thus the sentence that I have written here can be rewritten as for all X, X belonging to C, here we have another symbol for implies which is this one, so I write over here, this means implies X belonging to D.

If we use these symbols very often so, it is good to get use to these symbols. Now there is another symbol that is used frequently which is this, this means there exist. Thus, we will see and say that C is a proper subset of D that is C, a proper subset of D implies there exist X belonging to D such that X does not belong to the set C. The converse is also true that is if there exist X belonging to D, such that X does not belong to C and of course here C must be a subset particular of D, then C is said to be a proper subset of D. Now the question that what is the idea of equality of sets that is when do we say that two sets are equal.

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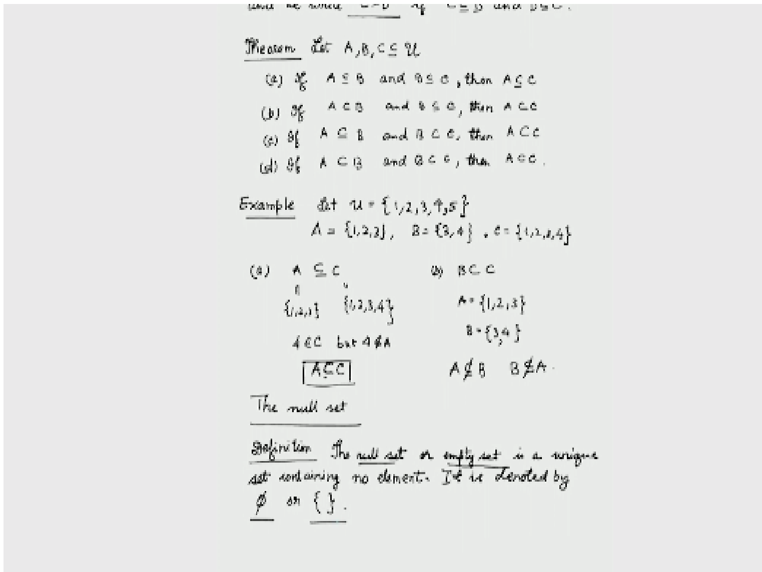

For a given universe U, the sets C and D taken from you are said to be equal and we write C=D, if c is a subset of D and D is a subset of C. So here we note the chain of arguments, first we give an idea of a set and we give the idea of a universe. And then, once we give the idea of a set, then we give the definition of the subset relation that is we take two sets from the universe and then we say that the set.

The first set is a subset of the second set if all the elements of the first set are elements of the second set, so that is subset equal and if it so happens that the second set has some elements or at least one element which is not in the first set then we will say the first set is a proper subset of the second set. And then we say that if we have got two sets C and D of course, consisting of the elements of the universe is which we have fixed before starting on these discussions.

Then if it so happens that C is a subset of D and, and D is a subset of C that is all the elements of C are elements of D, and all the elements of D are elements of C, then we say that these two sets are equal. This leads to some results related to subsets which are somewhat easy. Therefore, I will just write the results and lead the proof to the audience. So the first theorems is that let A, B, C are subsets of U, that is I have fixed here, and A, B, C are sets.

Then if a is a subset of B, and B is a subset of C, then A is a subset of C. B if A is a proper subset of B, and B is a subset of C, then A is a proper subset of C. C, if A is a subset of B, and B is a proper subset of C, then A is a proper subset of C, D if A is a subset of B, and B is a subset of C, then A is a subset of C. These are extremely state for the results, and as I have said that I leave it for exercise.

Now let us look at some examples, suppose we fix our U to something very small that is 1, 2, 3, 4, 5. And consider two sets A 1, 2, 3 and B 3, 4. Now we see that A well another set C 1, 2, 3, 4. So we see that A is a subset of C, because A consists of 1, 2, 3 and C consists of 1, 2, 3, 4. Further we see that 4 is in C, but 4 is not in A. Therefore, A is a proper subset of C. Similarly, we see that B is a proper subset of C, but if we consider A and B, A is 1, 2, 3 and B is 3, 4, then A is not a subset of B, and B is not a subset of A.

Next we introduced another very special set that is called the null set. So the null set are the empty set is a unique set containing the element as I have told before that when we have fixed the universe, then a set in that universe contains some specific elements of the universe or it may as well contain all the elements. The only thing that we expect that when we say that A is a set in the universe U, then given an element or an object in U, I should be able to decide whether that object is inside A or not.

Now if A is whole of U, then the decision is easy, because if you give me any object, then I know the default that it is in A, because contains all the objects of U. If A has some objects in U and some objects in U are not in A, then also hypothetically we can have some kind of rule or listing by with which you would be able to say that whether an object is using A or not. But this thing when extended to the other extreme where A does not have any object in U, then also A is a well defined collection of objects of U.

Because when we take any object in U, we know that it is not in A, so A contains no object. And this is a very special set called the null set ort the empty set and it is denoted by 5 or just two bases without anything inside. One result let it to the empty set which again is very obvious.

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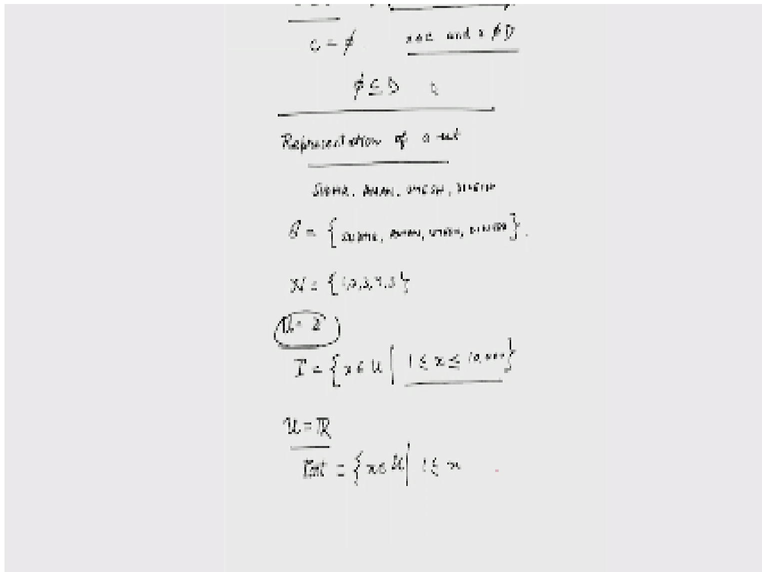

For any universe U let A is a subset of U then 5 that is a empty set is a subset of A and if A is not equal to ϕ then ϕ is a proper subset of A so this basically says a empty set ϕ is a subset of any set A and if A itself is not ϕ then ϕ is a proper subset of A. The reason behind this is that we say that a set C is a subset of another set D subset of D if X belonging to C in + X belongs to B now as long as this statement is true that is any X belonging to C belongs to D then C is a subset of D now given here any C and D if we want to show that C is a not subset of D then we have to find out an element X and C which is not in D.

So that this statement is false the problem here is that when C is equal to ϕ then C has no element therefore we cannot find an element for which X belongs to C and X not belonging to D holds therefore we cannot prove that this is false and therefore we have to take that ϕ is a subset of D or in the case of the theorem this is a so therefore what we see is that the empty set ϕ is a subset of any set we will do this theorem one once more when we formally study logic in after some lectures.

Now we will briefly look at the problem of representing a set so one form of representation we have been doing so far rather intuitively is to put the braces around certain objects for example if we have certain names of students like let us say Sudhir, Amin, umesh, dinesh then the set containing these students possibly may be denoted by capital S and written within braces sudhir, umesh, dinesh of course we understand that these students are coming from some universe that we have fixed before with numbers we have already seen and we have been doing that suppose we want numbers from one to ϕ that is integers from one to five.

We have just written something like this suppose I denote it by N and I write 1,2,3,4,5 so in this way we can collect the objects and put braces around them and represent a set but as is evident to all of us that if we have an infinite set or a very large set it will be very difficult to write all the elements or more often impossible to write all the elements within braces then we take some other alternatives for example what we can do is that first we fix the universal suppose I want all integers from 1 to 10,000 then fixing the universal.

I can write the set let us say I as X belonging to U such that 1 less than or equal to X less than or equal to 10,000 thus we can avoid listing down 10,000 integers by writing this inequality and understanding that U is the set of integers but we have to be careful here because it depends on how we choose the integers how we choose the universe because suppose we choose the universe to be set of real numbers then instead of a set of integers this same looking set will be an interval which I am writing as int between 1 to 10,000 you of course.

It is possible to write other sets in the same way for example suppose you would like we would like to write all the matrices all the two-by-two matrices with real entries whose determinant values is non-zero then we may write it like this that G is equal to a b c d.

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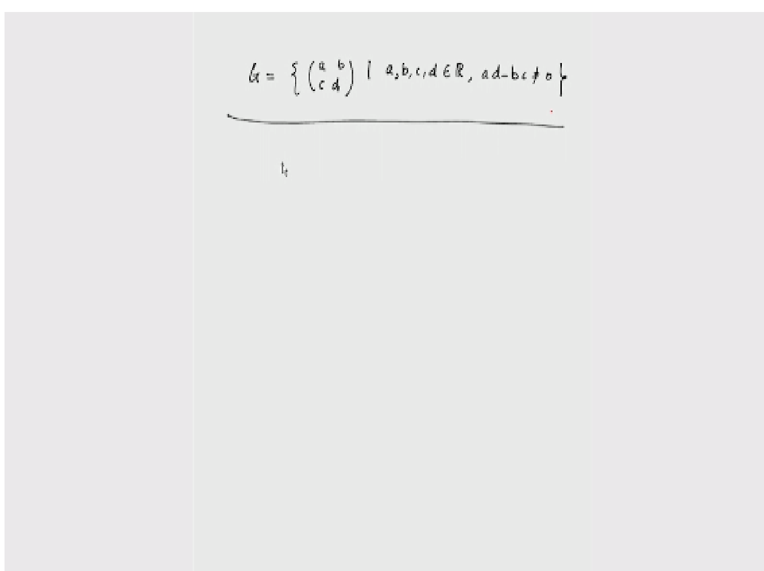

Such that a, b, c, d belongs to R and ad - BC is not = 0 of course this is an infinite set so I cannot write all the two-by-two matrices with real entries with non-zero determinant I cannot list them but with this notation I can specify them quite precisely next we move on to the idea of the power set of a set.

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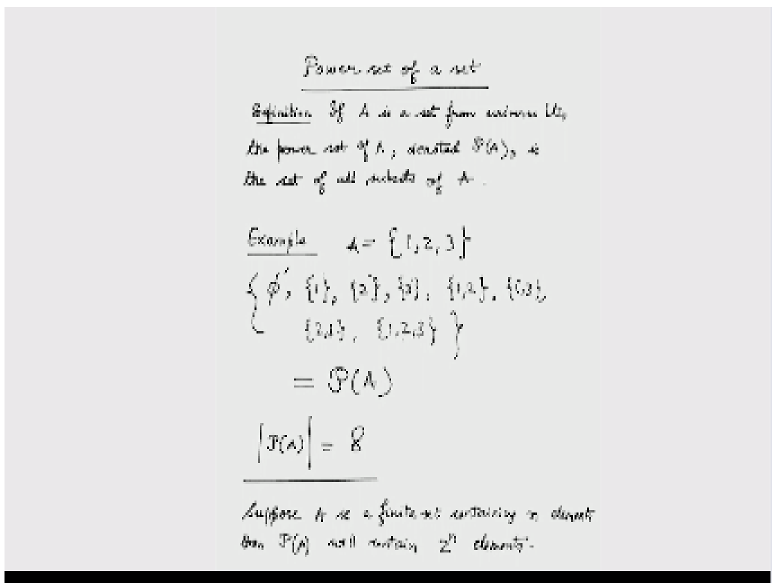

You now suppose a is A set of objects from a universe you the power set of A is a set of all subsets of A now let us look at an example suppose a is a finite set containing only three elements now we start writing the set of all subsets of a the first and the most obvious subset which is a subset of any set is ϕ that is a set containing no element and then we have the sets containing just one element that is 1,2,3 and then we have the sets containing two elements 1, 2, 1,3 and 2, 3 and finally we have the set containing 3 elements that is 1, 2, 3 and if we put braces around all these sets.

Then we have the power set of a will be denote by P(A) now if you count the number of elements in P(A) we see that this is 1,2,3,4,5,6,7 and 8 we write that as simply writing P(A) within two horizontal lines and this is eight by very easy counting argument we can prove that if we have a set containing n elements then its power set will contain two to the power n elements suppose a is a finite set containing n elements then P(A) will contain two to the power n elements and the notation that we have used here is quite genera.

So if we have a finite set then by writing that set symbol within two vertical lines we denote the number of elements in that set suppose s is a finite set then is the number of elements in s in today's lecture we have started with the definition or not really the definition but a description of the idea of a set.

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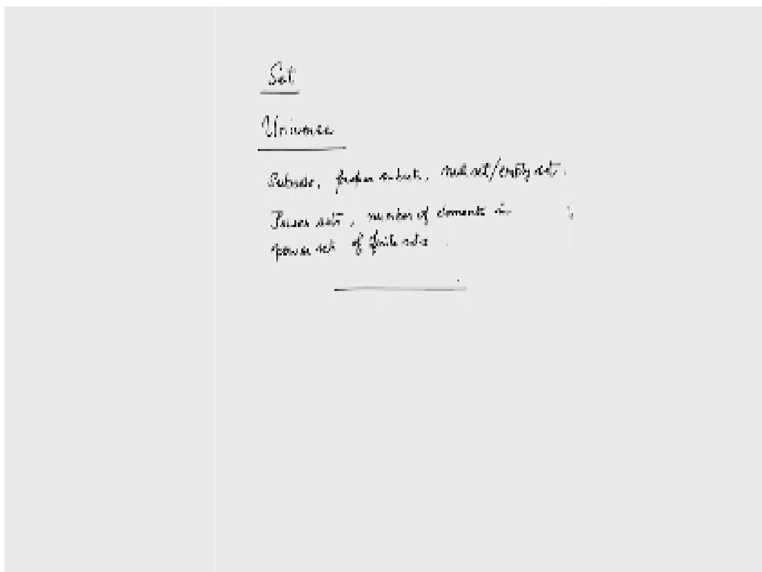

We have talked about how to conceive of a universe and then conceive of some the some collections of specific objects within the universe and after that basic description we have defined the terms such as subsets proper subsets null sets null set or empty set and lastly we have talked about power set and number of elements in a set in the next lectures we will discuss more on sets such as operations on sets and laws of set operations but this is all for today thank you.

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