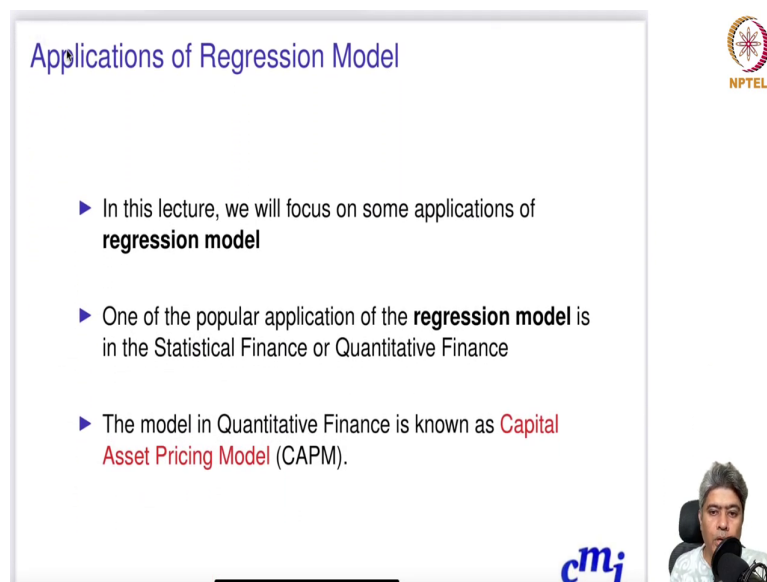


**Predictive Analytics - Regression and Classification**  
**Prof. Sourish Das**  
**Department of Mathematics**  
**Chennai Mathematical Institute**

**Lecture - 30**  
**Capital Asset Pricing Model**

Welcome to the Predictive Analytics Regression Classification class, this is lecture 9 first video part a.

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Applications of Regression Model

- ▶ In this lecture, we will focus on some applications of **regression model**
- ▶ One of the popular application of the **regression model** is in the Statistical Finance or Quantitative Finance
- ▶ The model in Quantitative Finance is known as **Capital Asset Pricing Model (CAPM)**.

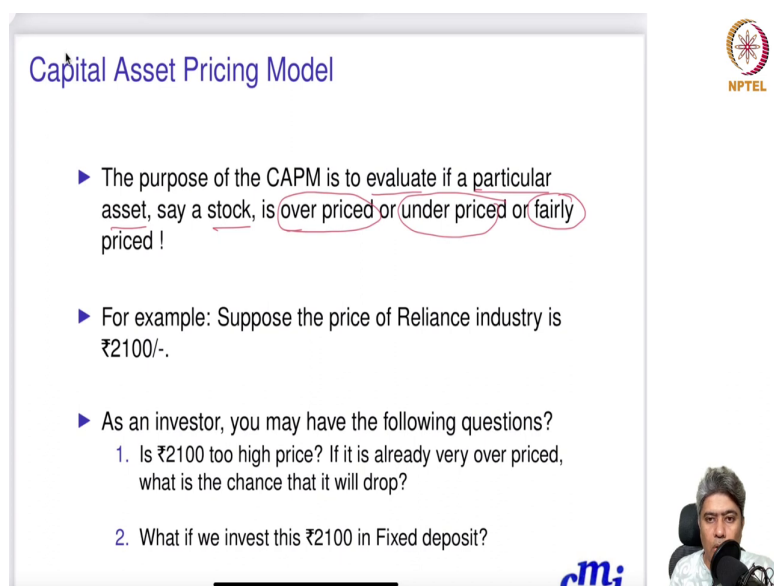
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In this lecture, we are going to talk about an application of regression model. This is perhaps the most popular application of regression model. One of the popular application of this regression model is in the statistical finance or quantitative finance. The model in quantitative finance, this model is known as Capital Asset Pricing Model.

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The slide is titled "Capital Asset Pricing Model" in blue text at the top left. It contains three main bullet points, each starting with a blue triangle. The first bullet point states the purpose of CAPM is to evaluate if a particular asset, such as a stock, is over priced or under priced or fairly priced. The terms "over priced", "under priced", and "fairly priced" are circled in red. The second bullet point gives an example: "Suppose the price of Reliance industry is ₹2100/-". The third bullet point asks, "As an investor, you may have the following questions?" followed by two numbered sub-questions: "1. Is ₹2100 too high price? If it is already very over priced, what is the chance that it will drop?" and "2. What if we invest this ₹2100 in Fixed deposit?". In the bottom right corner, there is a small video feed of a man speaking into a microphone, and the NPTEL logo is visible in the top right corner of the slide area.

- ▶ The purpose of the CAPM is to evaluate if a particular asset, say a stock, is over priced or under priced or fairly priced !
- ▶ For example: Suppose the price of Reliance industry is ₹2100/-.
- ▶ As an investor, you may have the following questions?
  1. Is ₹2100 too high price? If it is already very over priced, what is the chance that it will drop?
  2. What if we invest this ₹2100 in Fixed deposit?

It was first developed by Eugene Farmer and later he was awarded Nobel Prize for his work. But when we see what is capital asset pricing model, we will see this is the beautiful application of regression model, a simple linear regression model. The purpose of the CAPM is to evaluate the if a particular asset say stock is overpriced or under priced or fairly priced. So, the purpose of capital asset pricing model is to evaluate if a asset say stock is overpriced or under priced or fairly priced.

For example, suppose the price of Reliance industry is rupees 2100. An investor as an investor, you may have the following questions. Number 1; Is rupees 2100, too higher price? If it is already very overpriced, what is the chance that it will drop? Second question, what if we invest these 2100 in rupees 2100 in fixed deposit? Am I going to like I have 2100 rupees

in my pocket and I want to; I want to invest in Reliance industry. I am hoping that by the end of the year it will be it will go up.

But is it going to go up more than what is my return from fixed deposit? If it is it can go up, but if it is going up not more than fixed deposit, I will be better off. I will be better off invest my money in fixed deposit. So, this is what the idea these kind of questions capital asset pricing model tries to answer.


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### Capital Asset Pricing Model


- ▶ As we want to know figure out whether, the price of a stock will go up or go down; we need a base line to compare.
- ▶ Suppose  $\{P_0, P_1, P_2, \dots, P_n\}$  are the prices of stocks over a time period.
- ▶ We would be interested in change in price over period from  $(t - 1)$  to  $t$ , i.e.,

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100.$$

$R_t$  is known as simple return of stock over period  $[(t - 1), t]$  represents the percentage of change of stock with respect to  $P_{t-1}$ .

  
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As we want to figure out whether the price of a stock will go up or go down, we need a baseline to compare. Suppose  $P_0, P_1, P_2 \dots P_n$  are the prices of stock over a time period, we would be interested in to figure out the change in price over period from  $t$  minus 1 to  $t$ . So, what we will do? We will just simply subtract  $P_{t-1}$  from  $P_t$  and divide that by  $P_{t-1}$ .

And if we multiply by 100 that will give me the simple return,  $R_t$  is the simple return of stock over period  $t-1$  to  $t$ . This represents the percentage of change of stock with respect to  $P_{t-1}$  with respect to  $P_{t-1}$  what percentage of change has taken place in the price of the stock.

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Capital Asset Pricing Model

► Simple Return is defined as,

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

$R_t$  is known as simple return of stock over period  $[(t-1), t]$  represents the proportion of change of stock with respect to  $P_{t-1}$ .

► log Return is defined as,

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

So, simple return is defined as simply  $P_t$  minus  $P_{t-1}$  divided by  $P_{t-1}$ . You can multiply it by 100 or without 100 is also ok an  $R_t$  for the time being I am not multiplying it with 100. So,  $R_t$  is the simple return. Similarly, log return is defined as just log of  $P_t$  by  $P_{t-1}$  and it is defined with small  $r_t$ . So, simple return I am denoting by big capital  $R_t$  and the log return I am denoting by small  $r_t$ .




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Capital Asset Pricing Model

▶ Simple Return can be expressed as


$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$
$$= \frac{P_t}{P_{t-1}} - 1$$
$$= e^{r_t} - 1$$

$R_t = e^{r_t} - 1$





Now, one can show that simple return can be expressed as follows. So, you can divide the both  $P_t$  and  $P_{t-1}$  by  $P_{t-1}$ , then you can write it as  $\frac{P_t}{P_{t-1}}$  and then you can write  $R_t$  equal to  $e^{r_t} - 1$ . So, simple return and log return has a 1 to 1 correspondence, it is a 1 to 1 function effectively.

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```
Download data
> library(tseries)
> start_date<-"2022-06-01"
> end_date<-"2022-12-30"
> reliance<-get.hist.quote(instrument = "RELIANCE.N
+                               ,start=start_date
+                               ,end=end_date
+                               ,quote="AdjClose"
+                               ,provider = "yahoo")
time series ends 2022-12-29
> reliance
      Adjusted
2022-06-01 2625.595
2022-06-02 2716.123
2022-06-03 2771.157
2022-06-06 2759.243
2022-06-07 2764.427
```



Now, if you here is a piece of code which shows how to download data. So, what we are doing here? From 1st June 2022 to 30th December 2022 for 6 month this is my start\_date and this is my end\_date. In the library tseries package there is a function get historical quote or get dot hist dot quote. If you just give RELIANCE dot NS then start\_date end\_date and give the adjusted close price and provider equal to yahoo if you give this values.

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Adjusted ✓


2022-06-01	2625.595
2022-06-02	2716.123
2022-06-03	2771.157
2022-06-06	2759.243
2022-06-07	2764.427
2022-06-08	2715.873
2022-06-09	2790.349

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### Calculate Simple and log-Return

```
> ln_rt<-diff(log(reliance))
> Rt<-exp(ln_rt)-1
> cbind(ln_rt,Rt)*100
```

	Adjusted.ln_rt	Adjusted.Rt
2022-06-02	3.389780751	3.447888536
2022-06-03	2.005949448	2.026203818
2022-06-06	-0.430857865	-0.429931005
2022-06-07	0.018712152	0.018712152






So, it will give you the dataset for adjusted close prices of reliance industries from these days for 6 months.

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```
> Rt <- exp(ln_rt) - 1
> cbind(ln_rt, Rt) * 100
```

	Adjusted.ln_rt	Adjusted.Rt
2022-06-02	3.389780751	3.447888536
2022-06-03	2.005949448	2.026203818
2022-06-06	-0.430857865	-0.429931005
2022-06-07	0.187713163	0.187889454
2022-06-08	-1.771982514	-1.756375226
2022-06-09	2.705316517	2.742242440
2022-06-10	-3.065720858	-3.019204205
2022-06-13	-1.909998525	-1.891873632
2022-06-14	-1.315451568	-1.306837318
2022-06-15	-1.217381320	-1.210001212
2022-06-16	-1.408014373	-1.398148210
2022-06-17	1.165060277	1.171873538
2022-06-20	-1.829373239	-1.812741778
2022-06-21	1.601322303	1.614212179



Now, what you can do? You can compute the log return just simply take the log of the reliance take the difference difference will take the consecutive difference and then compute the simple return and then just cbind them. Then this column is essentially the log return and this column is essentially the simple return and now if you can check it out that these simple return and log return are almost same.

There I mean the changes you will see after 3 decimal places or typically 3 decimal places or up to 2 decimal places. So, there is a significant similarity between the log return and the simple return.



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2022-06-21 1.601322303 1.614212179



## Risk premium

- ▶ Risk premium is defined as

$$\bar{r}_t = r_t - r_f$$

$r_f$  is the risk-free return

- ▶  $\bar{r}_t$  is the premium that an investor earn over the return risk-free return.






Now, we will define another concept called risk premium. Now, what is risk premium? Risk premium is whatever the return minus  $r_f$  or  $r_f$  is risk free rate of return because during the same period you would have invest the same money in say fixed deposit of State Bank of India. Then the return that you will I mean if for sure you know that if State Bank of India says that ok at the end of the time period, I will give you 5 percent return. So, these 5 percent annualized return is a guaranteed return.

So, naturally you would like to have that return at the if you want to averse completely the risk because most likely State Bank of India will not default on its promise. So; obviously, you can have the you would this risk-free return  $r_f$  is kind of guaranteed. Then  $r_t$  minus  $r_f$  is the premium that you are getting because you are taking risk for investing your money in Reliance industries ok.

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Example of Risk premium  $15\% - 6\% = 9\%$



Ex Suppose you invest your money in SBI's fixed deposit and SBI gives you an interest income of 6%.

- ▶ Now instead of investing in SBI's fixed deposit scheme, suppose you invested in the equity of Reliance industries.
- ▶ Suppose the share price of reliance goes up by 15% over a period of one year. So your risk premium is 9%.
- ▶ Suppose the share price of reliance goes down by 15% over a period of one year. So your risk premium is -21%.

So, here let us try to understand the example of risk premium. Suppose you invest your money in SBI's fixed deposit and SBI gives you interest income of 6 percent ok. By the end of the year SBI will give you guaranteed 6 percent. Now, instead of investing in SBI's fixed deposit scheme suppose you invest in the equity of Reliance industries ok. So, suppose the share price of reliance goes up by 15 percent over a period of one year.

So, the risk premium is 9 percent because 15 percent minus 6 percent is 9 percent is the risk premium. Now, suppose share price goes down by 15 percent then what is the risk premium? The risk premium is 21 percent because you are not only losing 15 percent on your investment you are also losing the 6 percent you could have earned if you would have invested in the SBI's fixed deposit. So, risk premium is total negative 21 percent.




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**Risk premium of an equity vs market**

- ▶ **Big Question:** Should we invest in Equity or Exchange Traded Fund (ETF)?

**Note** ETF's are special type of Mutual Fund. It says that its follows the market index.

- ▶ We have to model risk premium of equity as function of risk premium of market etf.
- ▶ You have either of the three instruments to invest:
  1. SBI's Fixed deposit (guaranteed 6% return)
  2. Nifty 50 ETF
  3. A particular Equity, say Reliance or HDFC Bank or SBI etc.



Now, big question is should we invest in equity or exchange traded fund? What is exchange traded fund? ETF's are special kind of mutual fund typically it says that its a it follows a market index ok. So, we have to model risk premium of equity as a function of the risk premium of the market.

You have either three instrument to invest you can you have three decision to make either you can invest in SBI's fixed deposit that will be guaranteed 6 percent return or you can invest in nifty 50 ETF; that means, if you the this nifty 50 ETF guarantees you are return of nifty 50 the nifty 50 is a index of the entire market if it goes up by 5 percent it guarantees you return of 5 percent.

If it goes down by 3 percent it guarantees you return of negative 3 percent. So, whatever the nifty 50. So, either you can overall invest in overall market or a particular equity say you want to invest in Reliance or HDFC Bank or State Bank of India in these functions.

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### Capital Asset Pricing Model


▶ CAPM for an equity can explained as:

$$(r_t - r_t^f) = \alpha + \beta(r_t^m - r_t^f) + \epsilon_t$$

*risk premium of equity* ←  $(r_t - r_t^f)$        $\beta(r_t^m - r_t^f)$  → *risk premium of the market*

- ▶  $r_t$  is the return of asset/equity, ex: return of reliance equity
- ▶  $r_t^m$  is the return of market index, ex: return of Nifty 50 ETF
- ▶  $r_t^f$  is the risk-free rate of return, ex: return of SBI's Fixed Deposit 6%





Now, capital asset pricing model is essentially can be explained as  $r_t$  minus  $r_t^f$  equal to alpha plus beta times  $r_t^m$  minus  $r_t^f$  plus epsilon t. Now, let me explain you what is  $r_t$ ;  $r_t$  this is  $r_t$  is the return of the asset or equity or return of the reliance let us say for example, return of the reliance equity. What is  $r_t^m$ ? Here it is what is  $r_t^m$ ?  $r_t^m$  is the return of the market index return of the nifty 50 ok.

What is  $r_t^f$ ?  $r_t^f$  is here these two are  $r_t^f$  ok. So,  $r_t^f$  is the risk premium of the return risk free rate of return sorry risk-free rate of return say return from the SBI's fixed deposit say 6 percent guaranteed return ok. So, what is it let us try to understand. So, this means this is the

risk this is risk premium of reliance index or equity risk premium of equity, particular equity where you want to invest and this is risk premium of the market ok; risk premium of the market ok.

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### Capital Asset Pricing Model

- ▶ CAPM for an equity can explained as:

$$(r_t - r_t^f) = \alpha + \beta(r_t^m - r_t^f) + \epsilon_t$$

$$\bar{r}_t = \underbrace{\alpha + \beta \bar{r}_t^m}_{\text{systematic risk}} + \underbrace{\epsilon_t}_{\text{idiosyncratic risk}}$$

- ▶  $\bar{r}_t = (r_t - r_t^f)$  can be viewed as premium for taking risk with equity.
- ▶  $\bar{r}_t^m = (r_t^m - r_t^f)$  can be viewed as premium for taking risk with market.
- ▶ **Systematic risk** is the risk which we can explain as due to market movement.
- ▶ **Idiosyncratic Risk** is the risk very specific to stock/asset and we cannot explain

Now, what happens is we can call this is the risk premium of the equity, this is the risk premium of the market, this alpha plus. So, this is my response I am calling it this risk premium of equity is the response and risk premium of the market is my independent variable. So, I am we are defining it as alpha plus beta times risk premium of the market plus epsilon t.

In finance this alpha plus beta r t m is called systematic part of the return and epsilon t is called idiosyncratic part of the return and the risk because of this part is called systematic risk and this because of this part called idiosyncratic risk or the risk which we cannot explain ok.

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**Capital Asset Pricing Model**

▶ We can express it as




$$E(\bar{r}_t) = \alpha + \beta \bar{r}_t^m$$

where  $E(\epsilon_t) = 0$ ,  $V(\epsilon_t) = \sigma^2 \forall t$  and  $Cov(\epsilon_t, \epsilon_{t'}) = 0$

▶ What  $\alpha$  and  $\beta$  means?

▶ Suppose  $\alpha = 0$ , and  $\beta = 1.25$ , it means if market return goes up by 1% the equity return will go up by 1.25%.

▶ On the other hand, if market return goes down by 1% the equity return will go down by 1.25%.





So, we can express expected return risk free rate of return as alpha plus beta times  $r_t^m$  where expected value of epsilon  $t$  is 0, variance of epsilon  $t$  is sigma square and covariance of epsilon  $t$  and epsilon  $t$  dash is 0. Now, what alpha and beta means in this case? Suppose alpha is 0 and beta is 1.25.

So, it means if market return goes up by 1 percent the equity return will go up by 1.25 percent. On the other hand, if market return goes down by 1 percent, equity return will go down by 1.25 percent.

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15 of 34 Capital Asset Pricing Model

- ▶ We can express it as
$$\mathbb{E}(\bar{r}_t) = \alpha + \beta \bar{r}_t^m,$$
where  $\mathbb{E}(\epsilon_t) = 0$ ,  $\mathbb{V}(\epsilon_t) = \sigma^2 \forall t$  and  $\text{Cov}(\epsilon_t, \epsilon_{t'}) = 0$
- ▶ What  $\alpha$  and  $\beta$  means?
- ▶ Suppose  $\alpha = 0$ , and  $\beta = 0.85$ , it means if market return goes up by 1% the equity return will go up by 0.85%.
- ▶ On the other hand, if means if market return goes down by 1% the equity return will go down by 0.85%.



On the other hand, let us take alpha is 0, but beta is now 0.85, it means if the market return goes up by 1 percent, your equity return will go up by 0.85 percent. On the other hand, if it means the if the market return goes down by 1 percent the equity return will go down by 0.85 percent.

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### Capital Asset Pricing Model

▶ We can express it as ?




$$\mathbb{E}(\bar{r}_t) = \alpha + \beta \bar{r}_t^m,$$

where  $\mathbb{E}(\epsilon_t) = 0$ ,  $\mathbb{V}(\epsilon_t) = \sigma^2 \forall t$  and  $\text{Cov}(\epsilon_t, \epsilon_{t'}) = 0$

▶ What  $\alpha$  and  $\beta$  means?

▶ So  $\beta$  is a measure of systematic risk.

▶ Now let us try to understand what is the  $\alpha$ ?






So, beta is measure of systematic risk; beta is the measure of systematic risk. It is a measure of systematic risk. Now, let us try to understand what is alpha, what is alpha. Let us try to understand what is alpha ok.



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### Capital Asset Pricing Model


- ▶ We can express it as
$$E(\bar{r}_t) = \alpha + \beta \bar{r}_t^m,$$
where  $E(\epsilon_t) = 0$ ,  $V(\epsilon_t) = \sigma^2 \forall t$  and  $Cov(\epsilon_t, \epsilon_{t'}) = 0$
- ▶ Now let us try to understand what is the  $\alpha$ ?
- ▶ Suppose  $\beta = 1$  and  $\alpha = 0.01$ . It means if the market return is 0%. Still the equity goes up by 0.01%. That means even if market is flat; the equity has gone up. Why?
- ▶ Because the equity is intrinsically undervalued. More purchase pressure makes the price to go up and hence a positive return for the stock; though market is flat.



We can express it as like this. Let us now keep this part is same. Now, let us try to understand what is alpha. Let us fix beta as 1 beta equal to 1. Now, if alpha equal to 0.01, it means if the market return is 0 percent, still equity goes up by 0.01 percent. That means even if market is flat, the equity has gone up. Why? Just pause the video and think about it why it can why it will happen?



If the market is flat; that means, if market return is 0; that means, market is flat, market does not go up, but equity goes up. Why? Because the equity is intrinsically undervalued, more purchase pressure makes the price go up and hence positive return for the stock price though market is flat.

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$$\mathbb{E}(\bar{r}_t) = \alpha + \beta \bar{r}_t^m$$

where  $\mathbb{E}(\epsilon_t) = 0$ ,  $\mathbb{V}(\epsilon_t) = \sigma^2 \forall t$  and  $\text{Cov}(\epsilon_t, \epsilon_{t'}) = 0$

- ▶ Now let us try to understand what is the  $\alpha$ ?  *$d > 0$   
 $\rightarrow$  the equity is underpriced*
- ▶ Suppose  $\beta = 1$  and  $\alpha = 0.01$ . It means if the market return is 0%. Still the equity goes up by 0.01%. That means even if market is flat; the equity has gone up. Why?
- ▶ Because the equity is intrinsically undervalued. More purchase pressure makes the price to go up and hence a positive return for the stock; though market is flat.



Capital Asset Pricing Model




So, alpha strictly greater than 0 means the equity is under priced, the equity is under priced ok.

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### Capital Asset Pricing Model

- ▶ We can express it as
$$\mathbb{E}(\bar{r}_t) = \alpha + \beta \bar{r}_t^m,$$
where  $\mathbb{E}(\epsilon_t) = 0$ ,  $\mathbb{V}(\epsilon_t) = \sigma^2 \forall t$  and  $\text{Cov}(\epsilon_t, \epsilon_{t'}) = 0$
- ▶ Now let us try to understand what is the  $\alpha$ ?
- ▶ If  $\alpha > 0$ ; the stock is **undervalued**.
- ▶ If  $\alpha < 0$ ; the stock is **overvalued**.
- ▶ If  $\alpha = 0$ ; the stock is **fairly valued**.






Similarly, if the alpha is greater than 0, then the stock is undervalued. If alpha is less than 0, then the stock is overvalued and if alpha equal to 0, then the stock is fairly valued ok.

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**Capital Asset Pricing Model**

▶ Now I am making certain assumptions. These assumptions are known as the assumptions of **Efficient Market**.

- 1 No insider-trading is allowed.
- 2 All publicly available informations are already available to evrybody and easily accessible.
- 3 There is no substantial tax, transaction cost, entry or exit bar.
- 4 No limitations on long and short positions.



Now, I am making certain assumptions. These assumptions are known as assumptions of efficient market. So, no insider trading is allowed, like nobody has any internal information and doing any trading with internal information. All publicly informations are already available everybody and easily accessible. There is no substantial tax transaction cost, entry or exit bar. No limitations on long and short positions.

This is this short position is bit questionable because in any market, its typically they do not allow unlimited short positions. They do allow some kind of short positions, but not unlimited short positions.

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**Capital Asset Pricing Model**

- ▶ Under these assumption; any non-zero  $\alpha$  will be discovered very quickly by the people and they will take positions accordingly
- ▶ As a result the  $\alpha \rightarrow 0$
- ▶ Under the **Efficient Market** assumptions, CAPM is

$$\mathbb{E}(\bar{r}_t) = \beta \bar{r}_t^m$$
$$\mathbb{E}(r_t - r_t^f) = \beta(r_t^m - r_t^f)$$

$$\mathbb{E}(r_t) = r_t^f + \beta(r_t^m - r_t^f)$$

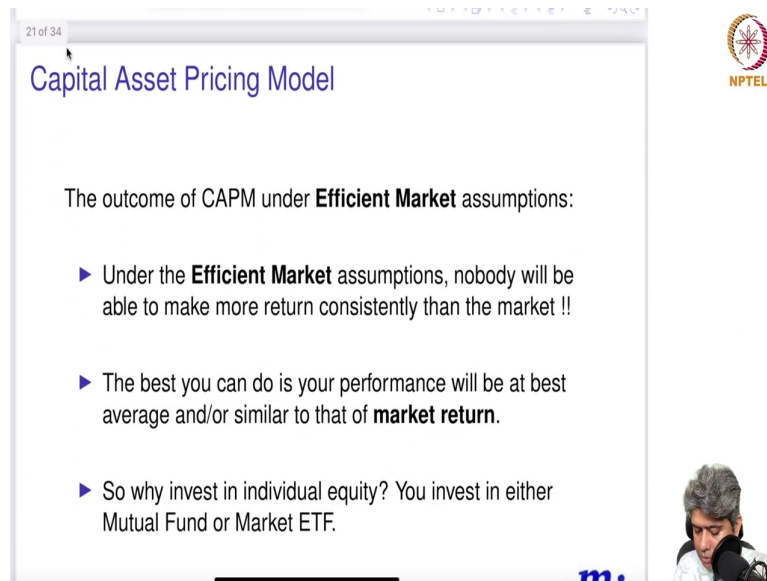
(1)  
**cmj**



So, under these assumptions, any non-zero alpha will be discovered very quickly by the people and they will position accordingly. As a result, what will happen? Alpha will go to 0. What it says that as soon as people will figure out that this particular stock is undervalued, people will start buying or as soon as people figure out this particular stock is overvalue, people will start selling.

So, as a result, very quickly alpha will go to 0 and so and the prices will be become efficient. So, under efficient market assumption, what happens is alpha is always 0, so expected  $r_t$  becomes  $\beta r_t^m$  or expected  $r_t$  minus  $r_t^f$  equal to  $\beta(r_t^m - r_t^f)$  or you can write it as expected  $r_t$  equal to  $r_t^f$  plus  $\beta$  times  $r_t^m$  minus  $r_t^f$ .

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## Capital Asset Pricing Model

The outcome of CAPM under **Efficient Market** assumptions:

- ▶ Under the **Efficient Market** assumptions, nobody will be able to make more return consistently than the market !!
- ▶ The best you can do is your performance will be at best average and/or similar to that of **market return**.
- ▶ So why invest in individual equity? You invest in either Mutual Fund or Market ETF.



So, the outcome of the CAPM under efficient market assumptions: under the efficient market assumption, nobody will be able to make more return consistently than the market, this is the first thing. The best you can do is your performance will be at best average and similarly similar to that of market return. So, why invest in individual equity? You should invest in either mutual fund or big market ETF. So, that is what the market, efficient market typically propose.

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### Capital Asset Pricing Model

▶ Let's look into the **Efficient Market** assumptions once more:

- 1 No insider-trading is allowed.
- 2 All publicly available informations are already available to evrybody and easily accessible.
- 3 There is no substantial tax, transaction cost, entry or exit bar.
- 4 No limitations on long and short positions.






Let us look into efficient market assumption once more. So, no insider trading allowed all publicly available information already available. So, there is no substantial tax. This is not right and short positions allowed that is also typically not right.

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### Capital Asset Pricing Model

- ▶ Deviation from **Efficient Market** assumptions implies the price is not in the equilibrium anymore; a friction is being introduced.
- ▶ Due to this friction; the  $\alpha$  will become non-zero. **Maybe very small. But definitely  $\alpha \neq 0$ .**
- ▶ So CAPM under the friction will be:

$$\mathbb{E}(r_t) = r_t^f + \alpha + \beta(r_t^m - r_t^f) \quad (2)$$


So, because of the deviation from the efficient market assumption implies price is not equilibrium anymore; a friction is being introduced because if you remove those two, some friction is there. So, maybe they are very small, but definitely alpha non-zero. So, CAPM under the friction will be something like that  $r_t^f$  plus unknown alpha plus beta times  $r_t^m$  minus  $r_t^f$ .



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Capital Asset Pricing Model



▶ The problem has now turned into a testing of hypothesis problem:

$$H_0 : \alpha = 0 \text{ vs } H_a : \alpha \neq 0$$

▶  $H_0 : \alpha = 0$  means the stock is fairly priced and the market is efficient.

▶  $H_a : \alpha \neq 0$  means the stock is not fairly priced and the market is not efficient.

▶ Let's look into the data.



So, now what we do, we just do a test, we propose a test that null hypothesis alpha equal to 0 versus alternative alpha not 0. Alpha equal to 0 means stock is fairly priced and the market is efficient. Alpha non-zero means stock is not fairly priced and market is not efficient. Let us look into the data ok.

(Refer Slide Time: 22:34)

```
Capital Asset Pricing Model
> library(tseries)
> start_date<-"2022-06-01"
> end_date<-"2022-12-30"
> rel<-get.hist.quote(instrument = "RELIANCE.NS"
+                       ,start=start_date,end=end_date
+                       ,quote="AdjClose",provider = "y
time series ends 2022-12-29
> nifty<-get.hist.quote(instrument = "^NSEI"
+                       ,start=start_date,end=end_date
+                       ,quote="AdjClose",provider = "y
time series ends 2022-12-29
> data <-merge(nifty,rel)
> rt<-diff(log(data))
> head(rt*100)

                Adjusted.nifty Adjusted.rel
2022-06-02      0.63498023      3.3897808
```



So, we I have taken the data reliance industries data between the 1st June 2022 and the 30th December 2022, so about 6 month of data I have taken. During the same time, I have taken nifty data also during the exact same time and then I merge the two data set and calculated their log return ok.

(Refer Slide Time: 23:06)

```
> rel<-get.hist.quote(instrument = "RELIANCE.NS"  
+ , start=start_date, end=end_date  
+ , quote="AdjClose", provider = "y  
time series ends 2022-12-29  
> nifty<-get.hist.quote(instrument = "^NSEI"  
+ , start=start_date, end=end_date  
+ , quote="AdjClose", provider = "y  
time series ends 2022-12-29  
> data <-merge(nifty, rel)  
> rt<-diff(log(data))  
> head(rt*100)
```

	Adjusted.nifty	Adjusted.rel
2022-06-02	0.63498023	3.3897808
2022-06-03	-0.26315096	2.0059494
2022-06-06	-0.08897911	-0.4308579

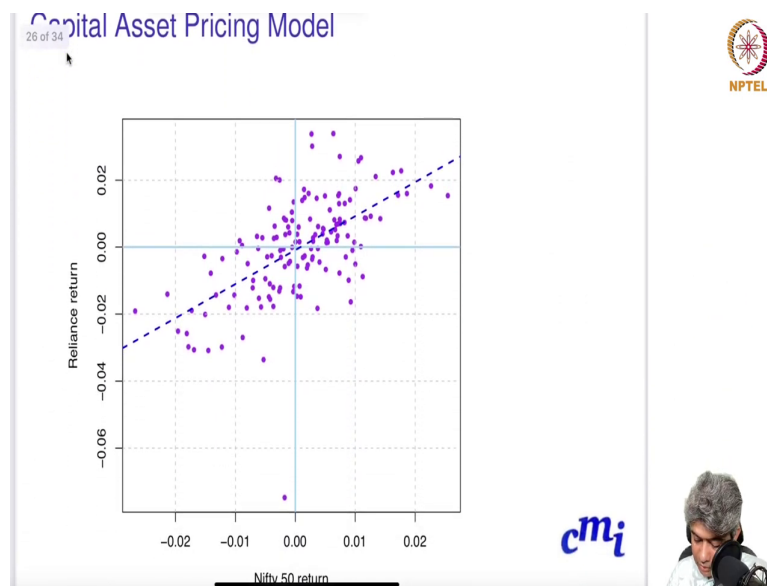
cmj

Capital Asset Pricing Model



And here is the few first few days of data of nifty and the reliance industry, log return of nifty and log return of reliance industry and I plotted them over the period.

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So, we over the x axis I plotted nifty 50 return log return and on the y axis I plotted reliance log return. So, there is a big outlier here where nifty 50 is 0, but reliance industry dropped significantly and, but most of the data you can see is sort of on a straight-line kind of behavior we are seeing.




(Refer Slide Time: 23:46)

```
Call:
lm(formula = Adjusted.rel ~ Adjusted.nifty, data =

Residuals:
    Min       1Q   Median       3Q      Max
-0.072041 -0.006527 -0.000111  0.007411  0.031941

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.000893    0.001014  -0.881    0.3
Adjusted.nifty  1.015036    0.115677   8.775 4.68e-1
---
Signif. codes:  0

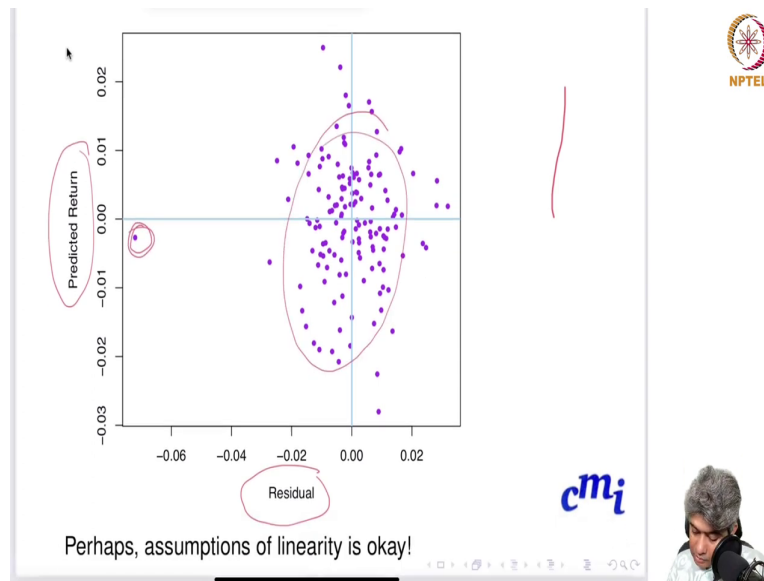
Note that the test is conducted under the assumption
 $\epsilon \sim N(0, \sigma^2 I_n)$  Let's check these out
H0:  $\alpha = 0$  vs H1:  $\alpha \neq 0$ 
```



And then we fit the we calculated the you know log return the fit the model and this is the alpha and this is the this is alpha and this is beta ok. And what we found that alpha p value is 0.3 t value is negative 0.881. So, and we have assumed that a epsilon follow normal 0 sigma square. So, based on that so, our my taste was not alpha equal to 0 versus alternative alpha not equal to 0.

So, if I assume alpha is follow normal 0 sigma square the residuals follow normal 0 sigma square under that assumption our p value is too high 0.3, so we cannot reject the null we cannot reject this null. So, we can say that reliance in industry's price is fairly valued it is neither overvalued nor undervalued ok.

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But we have to check whether the assumptions are correct or not. So, the first assumption is what we check that residual versus predicted. So, there is a big outlier, but overall, there is no much formation.

(Refer Slide Time: 25:20)

**Rank test for Randomness**

*Ho:  $\epsilon_i$  are random vs. Ha: Non-random*

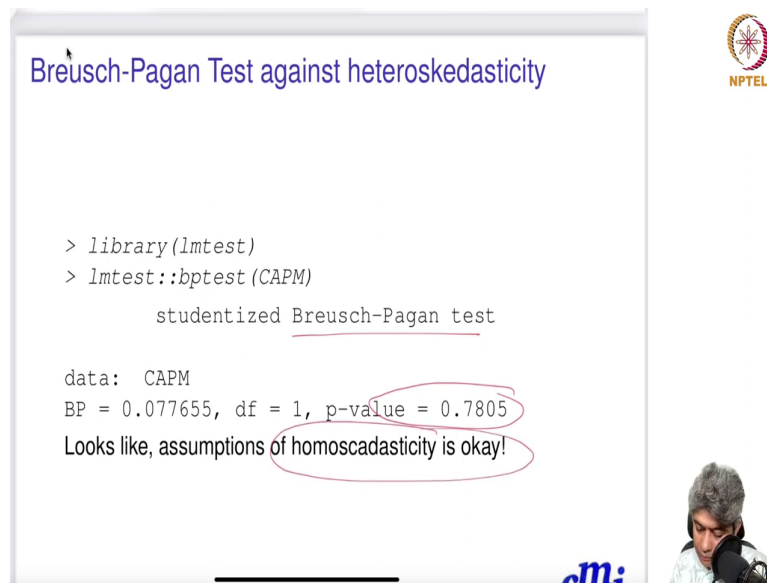
```
> library(randtests)
> randtests::bartels.rank.test(resid)
Bartels Ratio Test

data: resid
statistic = 0.96037, n = 145, p-value = 0.3369
alternative hypothesis: nonrandomness
Looks like, assumptions of randomness is okay!
```

cm;

So, we ran a random test for randomness we ran a randomness and p-value is very high. So, null hypothesis is that the epsilons are random and epsilons are random versus alternate non-random. So, here is a alternate hypothesis non-random and we fail to reject the randomness. So, looks like assumption of randomness is ok.

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Breusch-Pagan Test against heteroskedasticity

```
> library(lmtest)
> lmtest::bptest (CAPM)

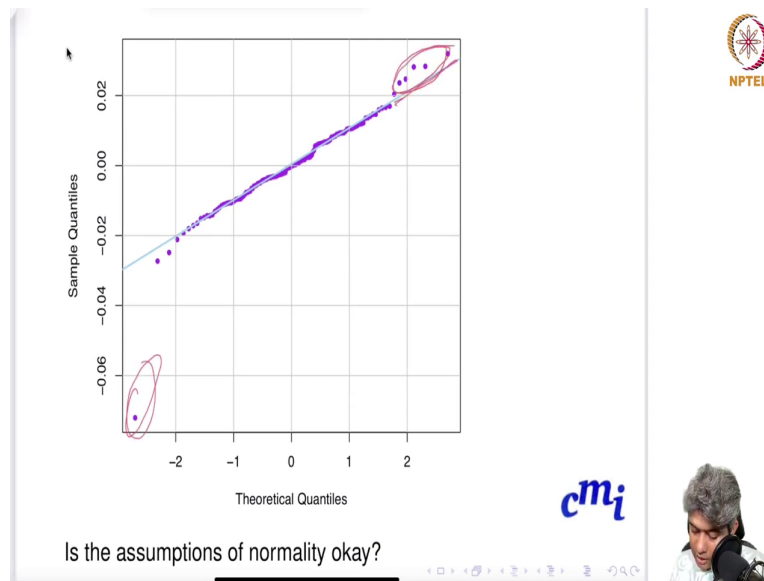
studentized Breusch-Pagan test

data: CAPM
BP = 0.077655, df = 1, p-value = 0.7805
Looks like, assumptions of homoscedasticity is okay!
```

Next, we tried test for heteroscedasticity. We just simply ran Breusch-Pagan test and looks like p-value is too quite large. So, we can get heteroscedasticity assumption is also ok.

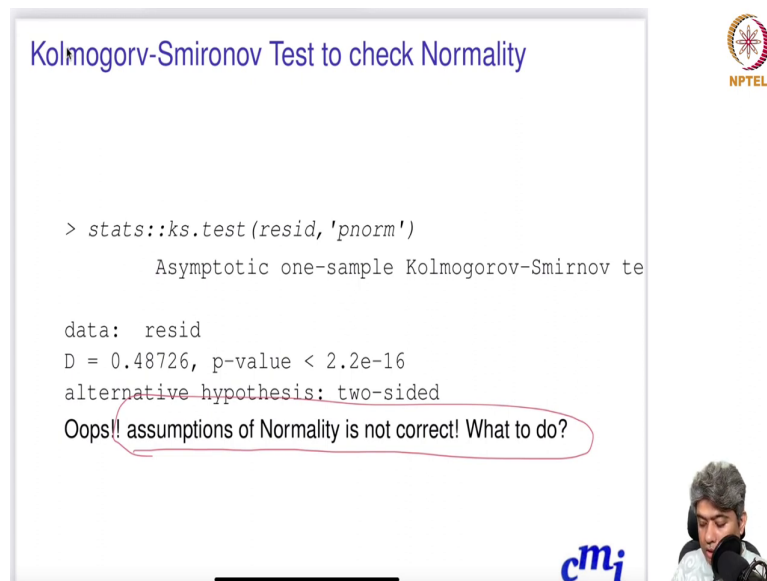


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Then we looked into normality. So, here is one big outlier and there are quite a few points which is away from the qq line.

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Kolmogorov-Smirnov Test to check Normality

```
> stats::ks.test(resid, 'pnorm')
Asymptotic one-sample Kolmogorov-Smirnov test

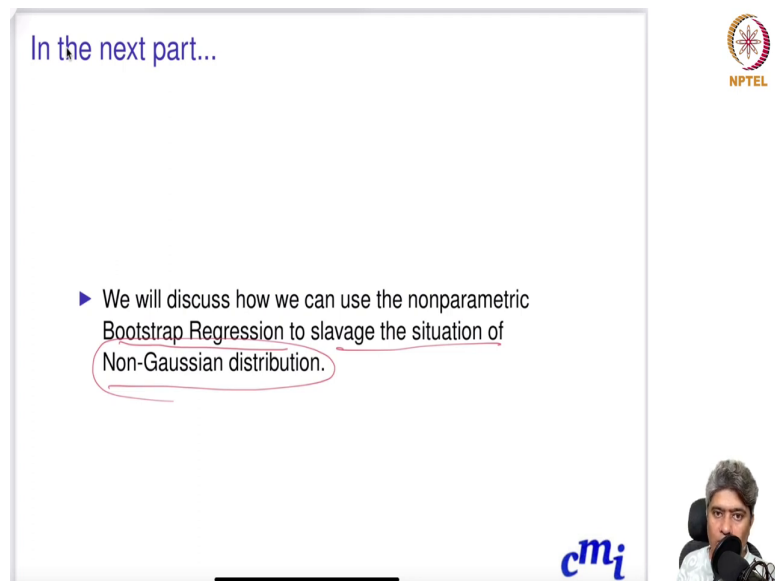
data:  resid
D = 0.48726, p-value < 2.2e-16
alternative hypothesis: two-sided
Oops!! assumptions of Normality is not correct! What to do?
```

The slide features the NPTEL logo in the top right corner, the CMi logo in the bottom right corner, and a small inset image of a woman speaking into a microphone.

So, we do not know. So, we just ran a Kolmogorov-Smirnov of test and oops assumption of normality is not correct. If the assumption of the normality is not correct then we cannot do the test for the whether the it is correctly priced or fairly priced or under priced or over priced that test is not valid anymore.

So, simple linear regression though its a application capital asset pricing model is a simple application of simple linear regression model, but we cannot do the test for alpha because underlying assumption of normality does not hold good here.

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In the next part...

- ▶ We will discuss how we can use the nonparametric Bootstrap Regression to salvage the situation of Non-Gaussian distribution.

cmj



So, how we solve this issue? In the next video, in the next lecture we will discuss how can we use non-parametric bootstrap regression to salvage the situation of non-Gaussian distribution. And next video we will do hands-on on CAPM and after that we will talk about bootstrap regression to salvage the situation of non-Gaussian distribution.

CAPM is one of the celebrated model in the economics and finance and this is the one of the most beautiful application of linear regression in economics. It was done by Eugene Farmer and because of that he won Nobel Prize. So, with this beautiful model, I will stop this video. I hope you enjoyed capital asset pricing model.

Thank you very much. See you soon.

