Predictive Analytics - Regression and Classification
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Lecture - 30
Capital Asset Pricing Model

Welcome to the Predictive Analytics Regression Classification class, this is lecture 9 first video part a.

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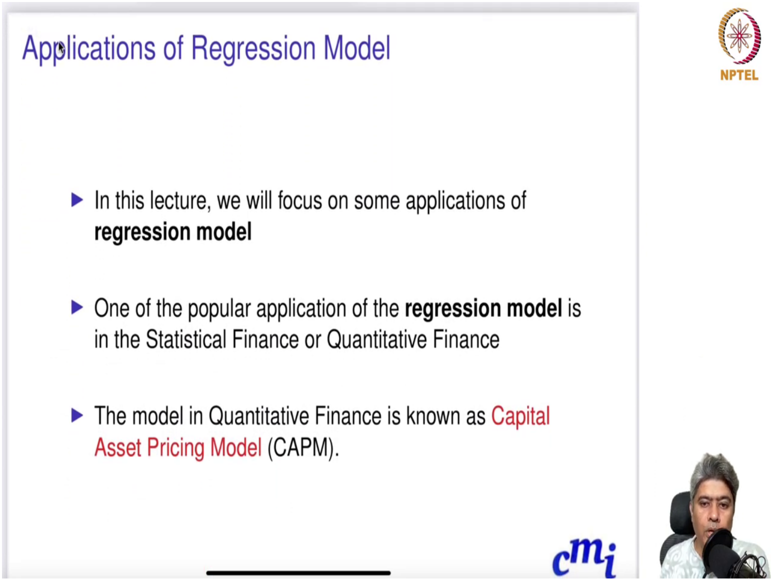

In this lecture, we are going to talk about an application of regression model. This is perhaps the most popular application of regression model. One of the popular application of this regression model is in the statistical finance or quantitative finance. The model in quantitative finance, this model is known as Capital Asset Pricing Model.

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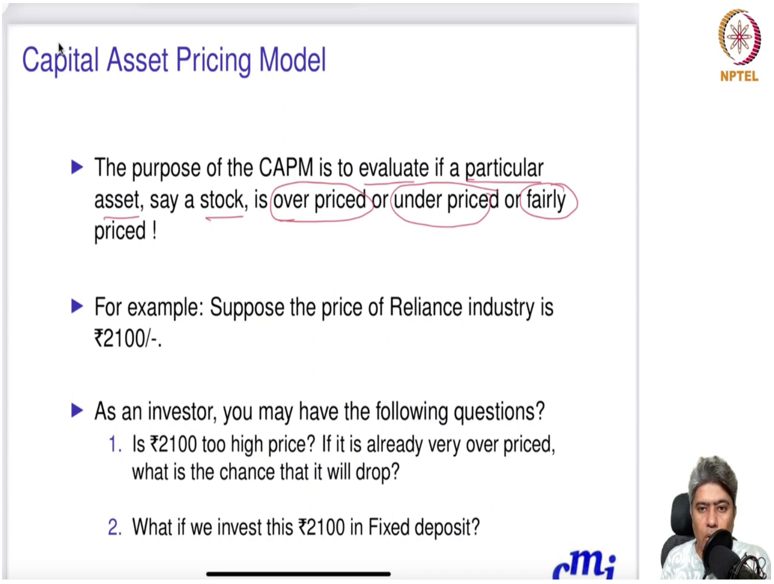

It was first developed by Eugene Farmer and later he was awarded Nobel Prize for his work. But when we see what is capital asset pricing model, we will see this is the beautiful application of regression model, a simple linear regression model. The purpose of the CAPM is to evaluate the if a particular asset say stock is overpriced or under priced or fairly priced. So, the purpose of capital asset pricing model is to evaluate if a asset say stock is overpriced or under priced or fairly priced.

For example, suppose the price of Reliance industry is rupees 2100. An investor as an investor, you may have the following questions. Number 1; Is rupees 2100, too higher price? If it is already very overpriced, what is the chance that it will drop? Second question, what if we invest these 2100 in rupees 2100 in fixed deposit? Am I going to like I have 2100 rupees in my pocket and I want to; I want to invest in Reliance industry. I am hoping that by the end of the year it will be it will go up.

But is it going to go up more than what is my return from fixed deposit? If it is it can go up, but if it is going up not more than fixed deposit, I will be better off. I will be better off invest my money in fixed deposit. So, this is what the idea these kind of questions capital asset pricing model tries to answer.

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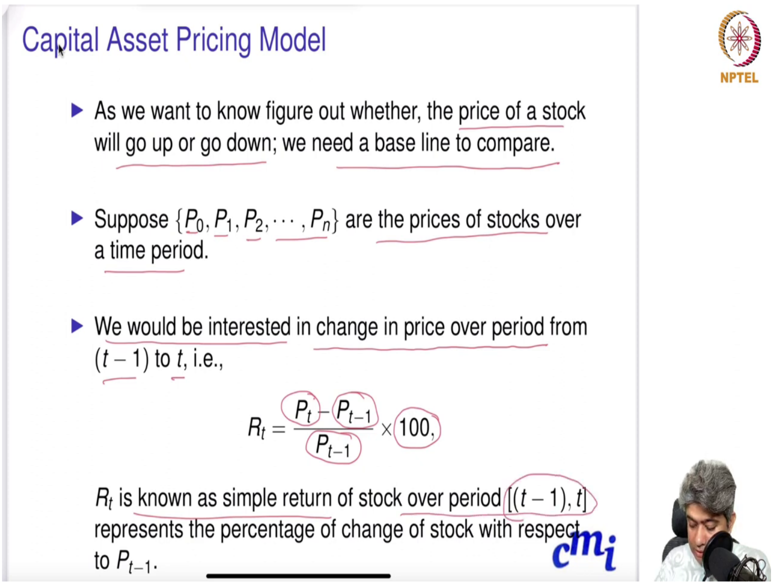

As we want to figure out whether the price of a stock will go up or go down, we need a baseline to compare. Suppose P 0, P 1, P 2 dot dot dot P n are the prices of stock over a time period, we would be interested in to figure out the change in price over period from t minus 1 to t. So, what we will do? We will just simply subtract P t minus 1 from P t and divide that by P t minus 1.

And if we multiply by 100 that will give me the simple return, R t is the simple return of stock over period t minus 1 to t. This represents the percentage of change of stock with respect to P t minus 1 with respect to P t minus 1 what percentage of change has taken place in the price of the stock.

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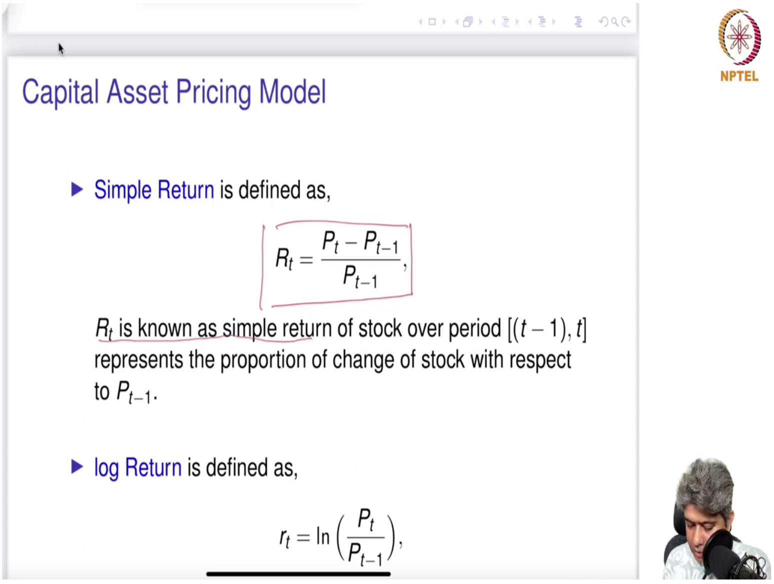

So, simple return is defined as simply P t minus P t minus 1 divided by P t. You can multiply it by 100 or without 100 is also ok an R t for the time being I am not multiplying it with 100. So, R t is the simple return. Similarly, log return is defined as just log of P t by P t minus 1 and it is defined with small r t. So, simple return I am denoting by big capital R t and the log return I am denoting by small r t.

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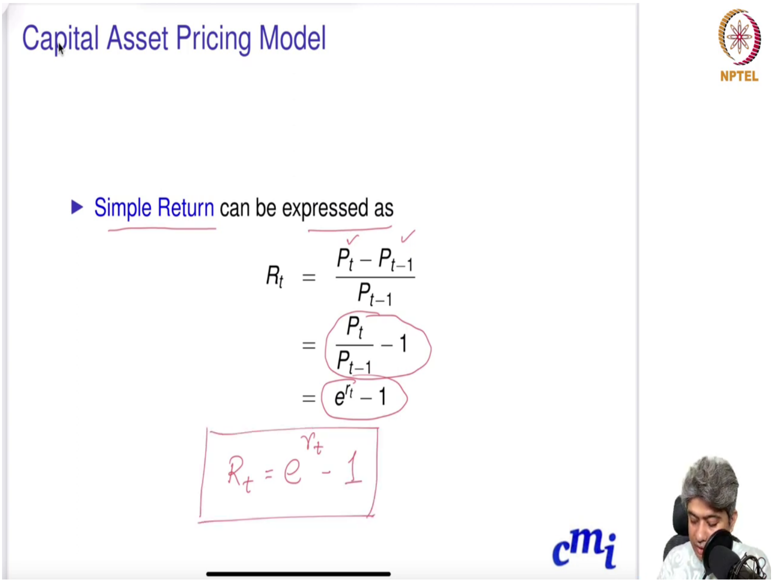

Now, one can show that simple return can be expressed as follows. So, you can divide the both P t and P t minus 1 by P t minus 1, then you can write it as a P t by P t minus 1 and then you can write R t equal to e to the power r t by minus 1. So, simple return and log return has a 1 to 1 correspondence, it is a 1 to 1 function effectively.

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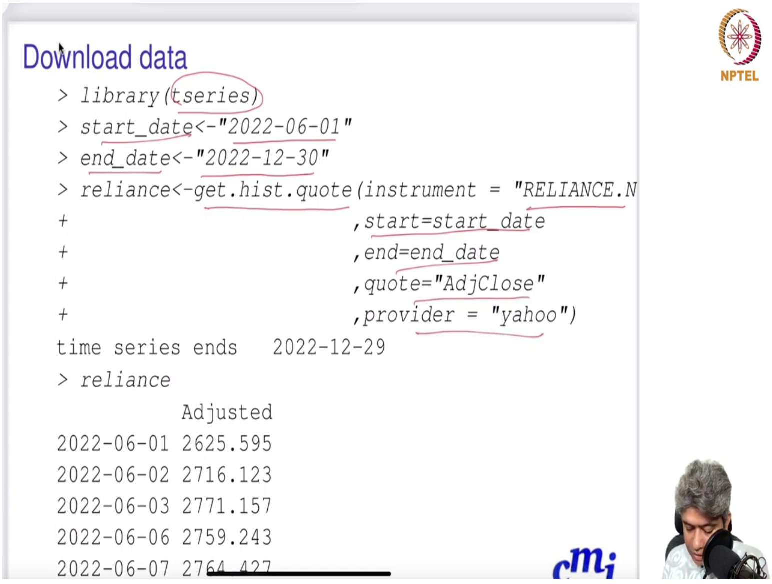

Now, if you here is a piece of code which shows how to download data. So, what we are doing here? From 1st June 2022 to 30th December 2022 for 6 month this is my start_date and this is my end_date. In the library tseries package there is a function get historical quote or get dot hist dot quote. If you just give RELIANCE dot NS then start_date end_date and give the adjusted close price and provider equal to yahoo if you give this values.

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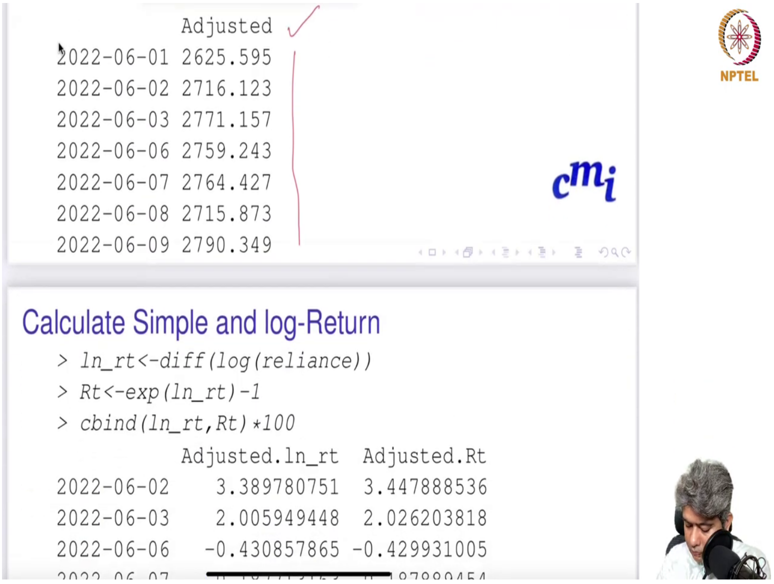

So, it will give you the dataset for adjusted close prices of reliance industries from these days for 6 months.

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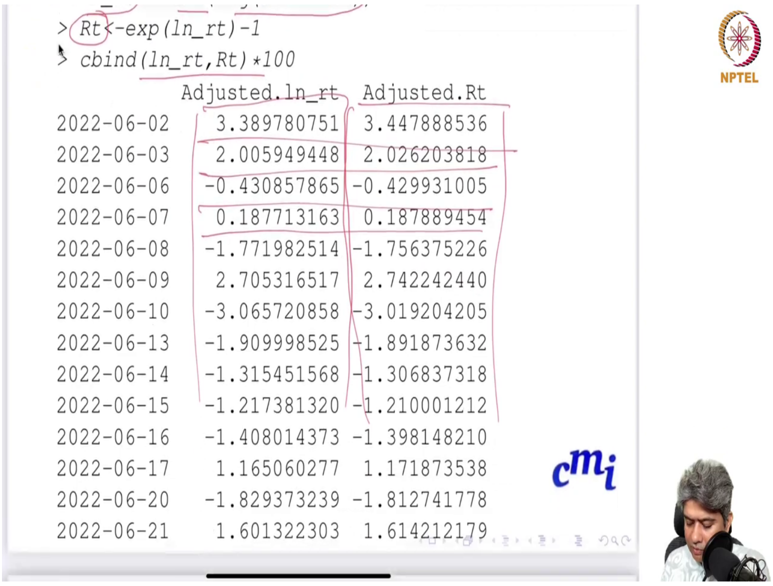

Now, what you can do? You can compute the log return just simply take the log of the reliance take the difference difference will take the consecutive difference and then compute the simple return and then just cbind them. Then this column is essentially the log return and this column is essentially the simple return and now if you can check it out that these simple return and log return are almost same.

There I mean the changes you will see after 3 decimal places or typically 3 decimal places or up to 2 decimal places. So, there is a significant similarity between the log return and the simple return.

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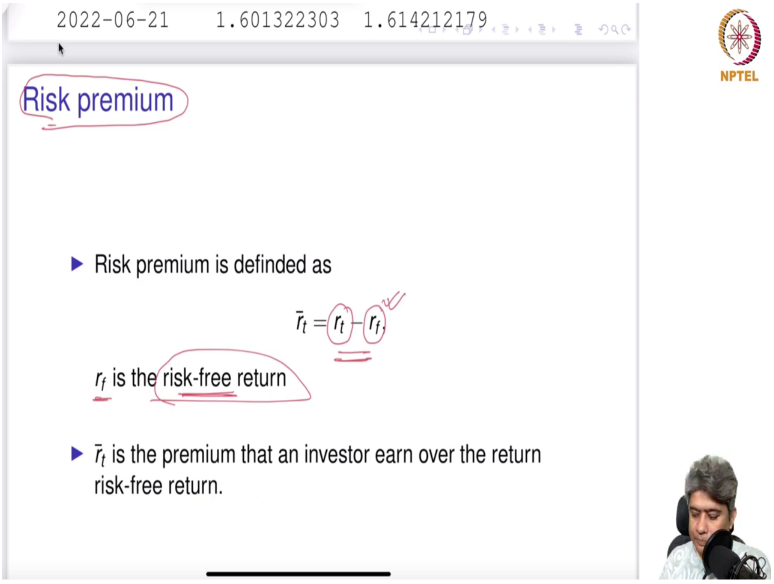

Now, we will define another concept called risk premium. Now, what is risk premium? Risk premium is whatever the return minus r f or r f is risk free rate of return because during the same period you would have invest the same money in say fixed deposit of State Bank of India. Then the return that you will I mean if for sure you know that if State Bank of India says that ok at the end of the time period, I will give you 5 percent return. So, these 5 percent annualized return is a guaranteed return.

So, naturally you would like to have that return at the if you want to averse completely the risk because most likely State Bank of India will not default on its promise. So; obviously, you can have the you would this risk-free return r f is kind of guaranteed. Then r t minus r f is the premium that you are getting because you are taking risk for investing your money in Reliance industries ok.

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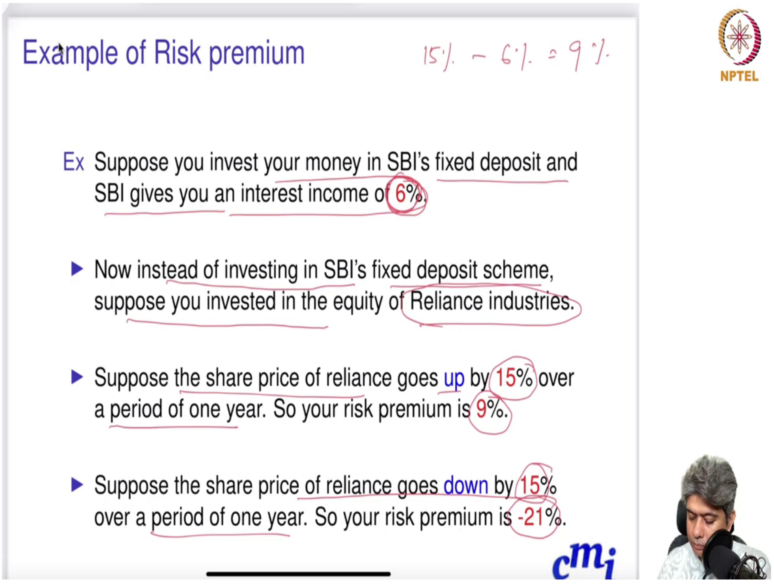

So, here let us try to understand the example of risk premium. Suppose you invest your money in SBI's fixed deposit and SBI gives you interest income of 6 percent ok. By the end of the year SBI will give you guaranteed 6 percent. Now, instead of investing in SBI's fixed deposit scheme suppose you invest in the equity of Reliance industries ok. So, suppose the share price of reliance goes up by 15 percent over a period of one year.

So, the risk premium is 9 percent because 15 percent minus 6 percent is 9 percent is the risk premium. Now, suppose share price goes down by 15 percent then what is the risk premium? The risk premium is 21 percent because you are not only losing 15 percent on your investment you are also losing the 6 percent you could have earned if you would have invested in the SBI's fixed deposit. So, risk premium is total negative 21 percent.

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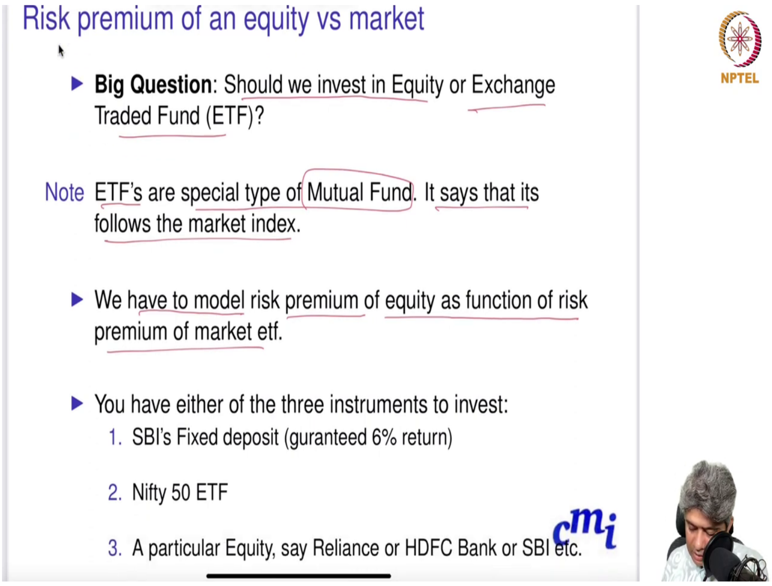

Now, big question is should we invest in equity or exchange traded fund? What is exchange traded fund? ETF's are special kind of mutual fund typically it says that its a it follows a market index ok. So, we have to model risk premium of equity as a function of the risk premium of the market.

You have either three instrument to invest you can you have three decision to make either you can invest in SBI's fixed deposit that will be guaranteed 6 percent return or you can invest in nifty 50 ETF; that means, if you the this nifty 50 ETF guarantees you are return of nifty 50 the nifty 50 is a index of the entire market if it goes up by 5 percent it guarantees you return of 5 percent.

If it goes down by 3 percent it guarantees you return of negative 3 percent. So, whatever the nifty 50. So, either you can overall invest in overall market or a particular equity say you want to invest in Reliance or HDFC Bank or State Bank of India in these functions.

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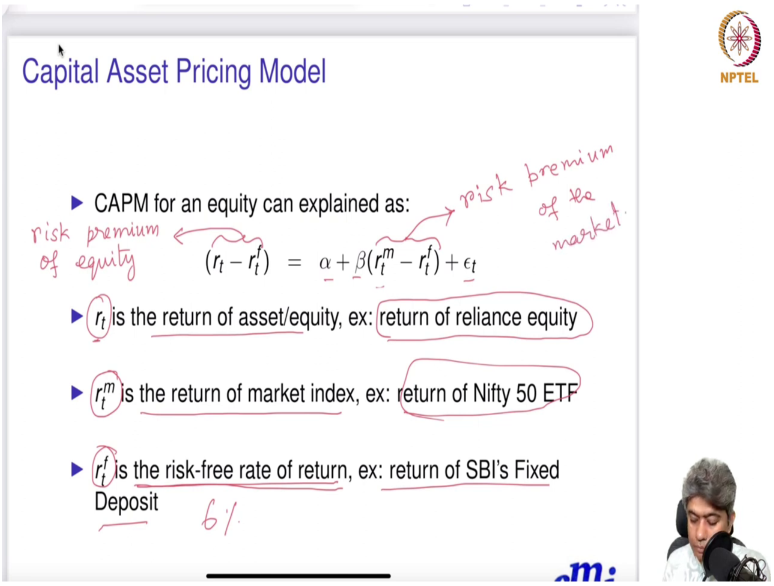

Now, capital asset pricing model is essentially can be explained as r t minus r t f equal to alpha plus beta times r t m minus r t f plus epsilon t. Now, let me explain you what is r t; r t this is r t is the return of the asset or equity or return of the reliance let us say for example, return of the reliance equity. What is r t m? Here it is what is r t m? r t m is the return of the market index return of the nifty 50 ok.

What is r t f? r t f is here these two are r t f ok. So, r t f is the risk premium of the return risk free rate of return sorry risk-free rate of return say return from the SBI's fixed deposit say 6 percent guaranteed return ok. So, what is it let us try to understand. So, this means this is the risk this is risk premium of reliance index or equity risk premium of equity, particular equity where you want to invest and this is risk premium of the market ok; risk premium of the market ok.

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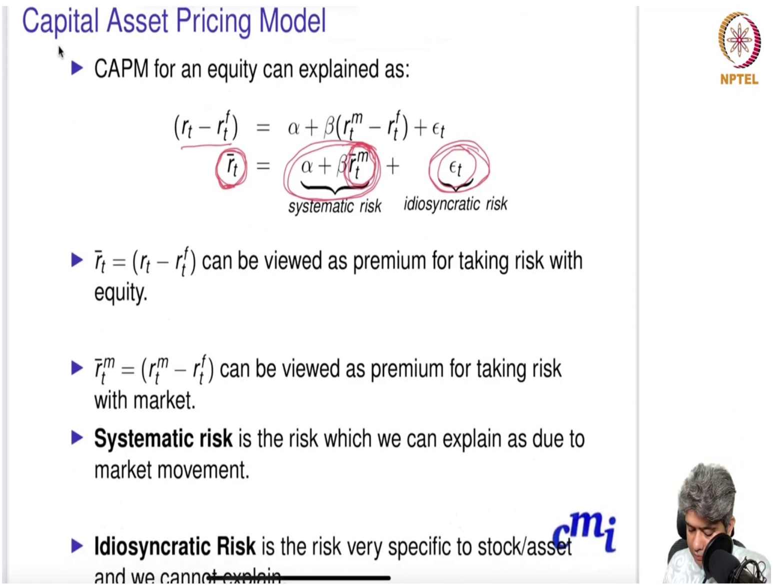

Now, what happens is we can call this is the risk premium of the equity, this is the risk premium of the market, this alpha plus. So, this is my response I am calling it this risk premium of equity is the response and risk premium of the market is my independent variable. So, I am we are defining it as alpha plus beta times risk premium of the market plus epsilon t.

In finance this alpha plus beta r t m is called systematic part of the return and epsilon t is called idiosyncratic part of the return and the risk because of this part is called systematic risk and this because of this part called idiosyncratic risk or the risk which we cannot explain ok.

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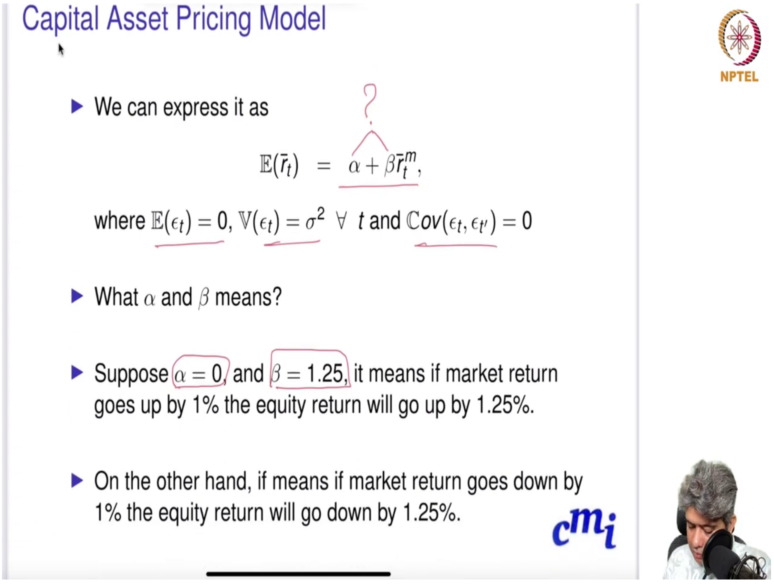

So, we can express expected return risk free rate of return as alpha plus beta times r t m where expected value of epsilon t is 0, variance of epsilon t is sigma square and covariance of epsilon t and epsilon t dash is 0. Now, what alpha and beta means in this case? Suppose alpha is 0 and beta is 1.25.

So, it means if market return goes up by 1 percent the equity return will go up by 1.25 percent. On the other hand, if market return goes down by 1 percent, equity return will go down by 1.25 percent.

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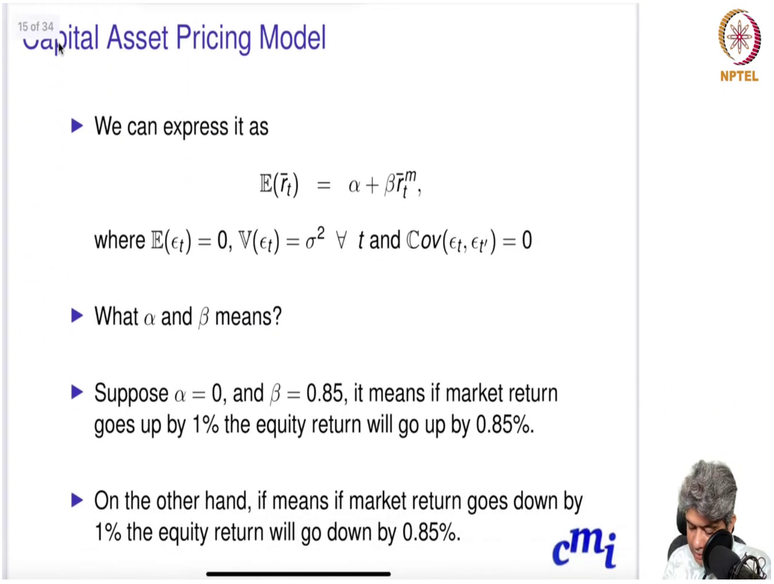

On the other hand, let us take alpha is 0, but beta is now 0.85, it means if the market return goes up by 1 percent, your equity return will go up by 0.85 percent. On the other hand, if it means the if the market return goes down by 1 percent the equity return will go down by 0.85 percent.

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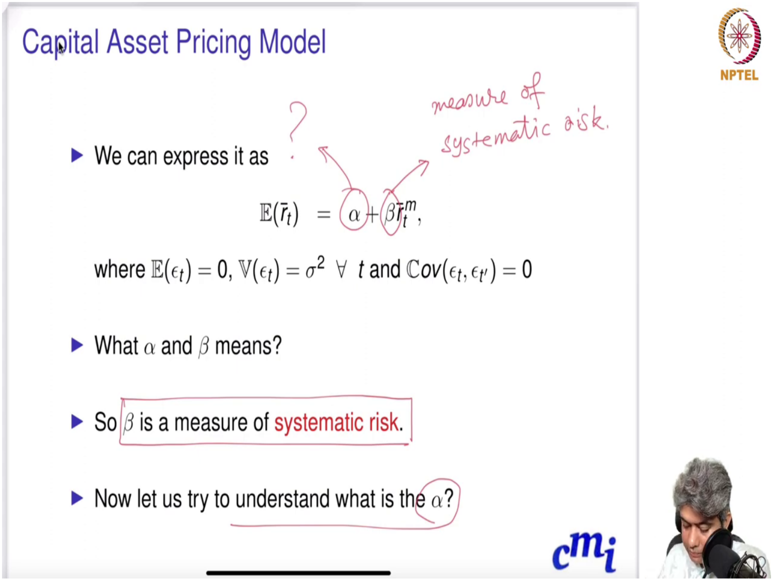

So, beta is measure of systematic risk; beta is the measure of systematic risk. It is a measure of systematic risk. Now, let us try to understand what is alpha, what is alpha. Let us try to understand what is alpha ok.

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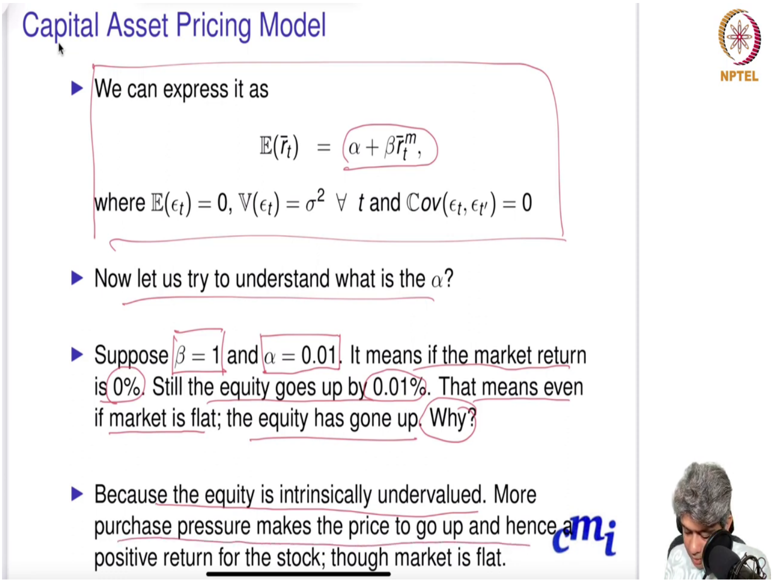

We can express it as like this. Let us now keep this part is same. Now, let us try to understand what is alpha. Let us fix beta as 1 beta equal to 1. Now, if alpha equal to 0.01, it means if the market return is 0 percent, still equity goes up by 0.01 percent. That means even if market is flat, the equity has gone up. Why? Just pause the video and think about it why it can why it will happen?

If the market is flat; that means, if market return is 0; that means, market is flat, market does not go up, but equity goes up. Why? Because the equity is intrinsically undervalued, more purchase pressure makes the price go up and hence positive return for the stock price though market is flat.

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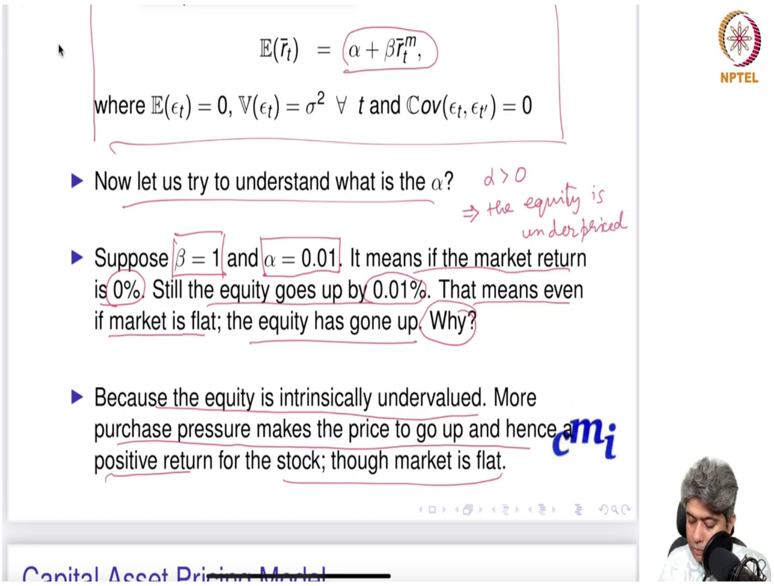

So, alpha strictly greater than 0 means the equity is under priced, the equity is under priced ok.

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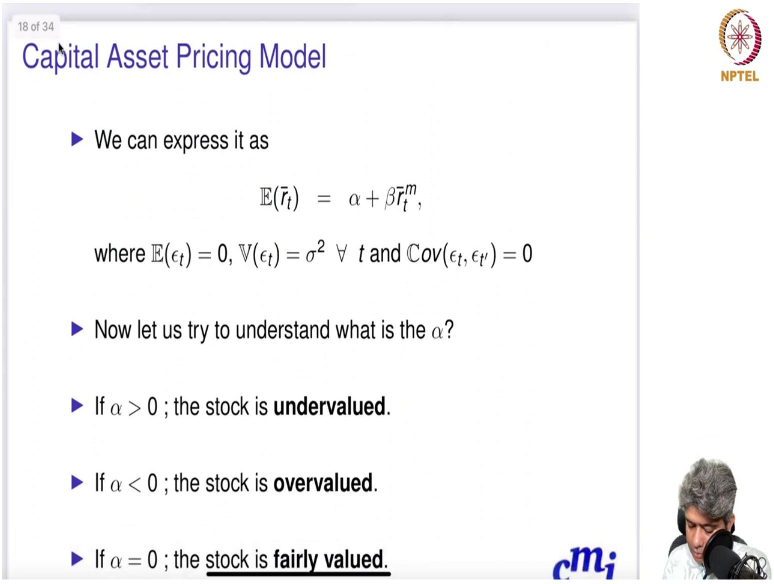

Similarly, if the alpha is greater than 0, then the stock is undervalued. If alpha is less than 0, then the stock is overvalued and if alpha equal to 0, then the stock is fairly valued ok.

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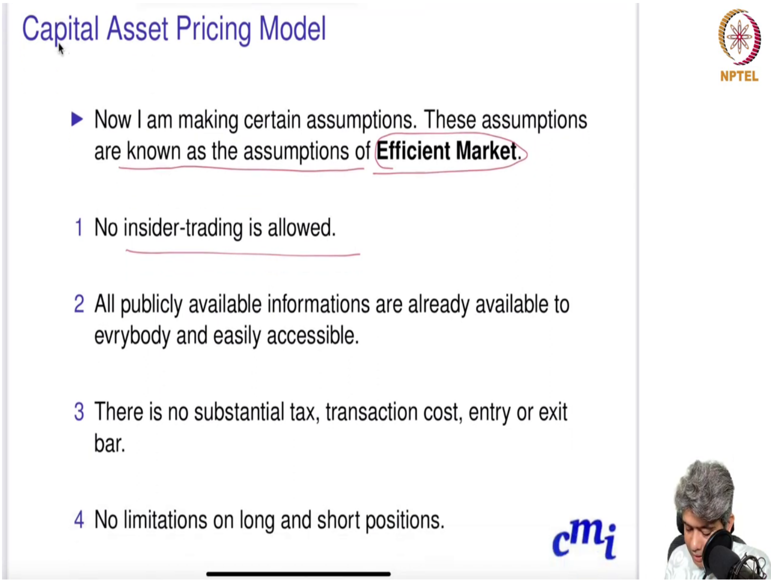

Now, I am making certain assumptions. These assumptions are known as assumptions of efficient market. So, no insider trading is allowed, like nobody has any internal information and doing any trading with internal information. All publicly informations are already available everybody and easily accessible. There is no substantial tax transaction cost, entry or exit bar. No limitations on long and short positions.

This is this short position is bit questionable because in any market, its typically they do not allow unlimited short positions. They do allow some kind of short positions, but not unlimited short positions.

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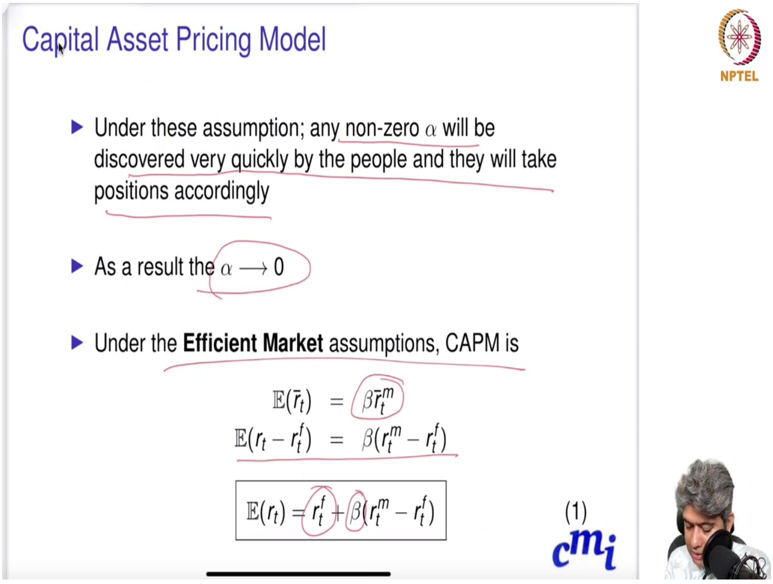

So, under these assumptions, any non-zero alpha will be discovered very quickly by the people and they will position accordingly. As a result, what will happen? Alpha will go to 0. What it says that as soon as people will figure out that this particular stock is undervalued, people will start buying or as soon as people figure out this particular stock is overvalue, people will start selling.

So, as a result, very quickly alpha will go to 0 and so and the prices will be become efficient. So, under efficient market assumption, what happens is alpha is always 0, so expected r t becomes beta r t m or expected r t minus r t f equal to beta r t m r t f or you can write it as expected r t equal to r t f plus beta times r t m minus r t f.

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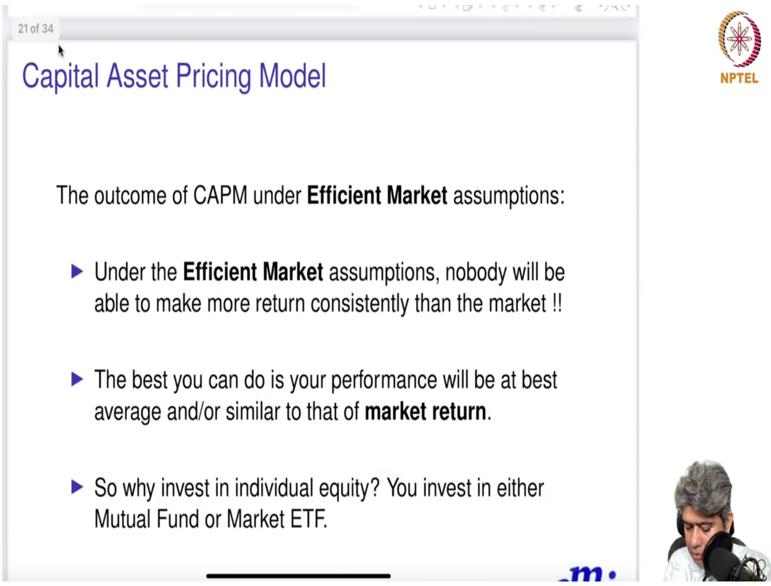

So, the outcome of the CAPM under efficient market assumptions: under the efficient market assumption, nobody will be able to make more return consistently than the market, this is the first thing. The best you can do is your performance will be at best average and similarly similar to that of market return. So, why invest in individual equity? You should invest in either mutual fund or big market ETF. So, that is what the market, efficient market typically propose.

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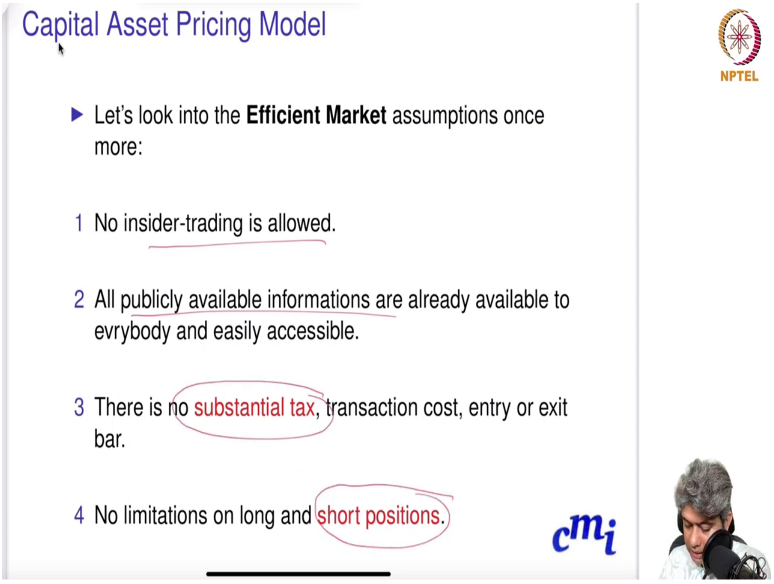

Let us look into efficient market assumption once more. So, no insider trading allowed all publicly available information already available. So, there is no substantial tax. This is not right and short positions allowed that is also typically not right.

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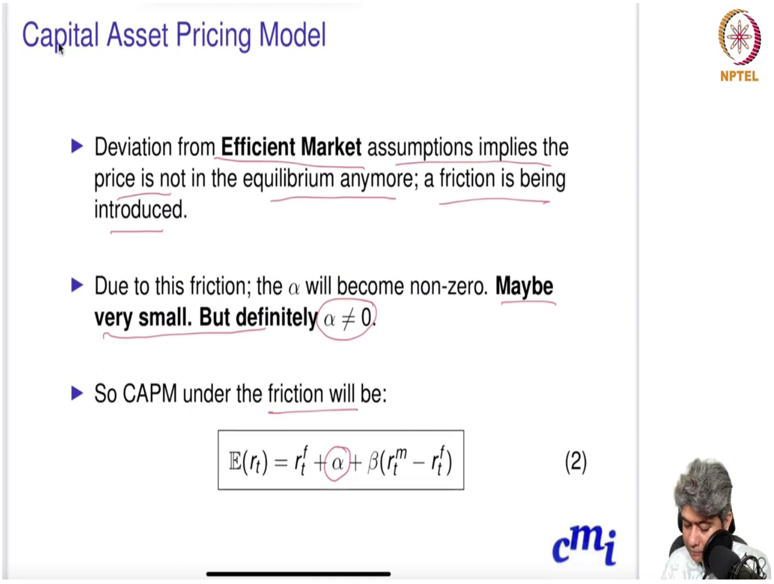

So, because of the deviation from the efficient market assumption implies price is not equilibrium anymore; a friction is being introduced because if you remove those two, some friction is there. So, maybe they are very small, but definitely alpha non-zero. So, CAPM under the friction will be something like that r t f plus unknown alpha plus beta times r t m minus r t f.

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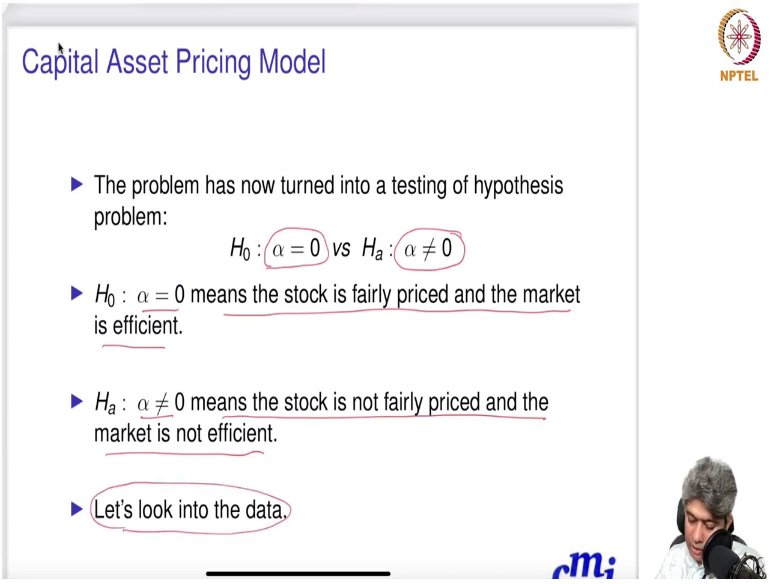

So, now what we do, we just do a test, we propose a test that null hypothesis alpha equal to 0 versus alternative alpha not 0. Alpha equal to 0 means stock is fairly priced and the market is efficient. Alpha non-zero means stock is not fairly priced and market is not efficient. Let us look into the data ok.

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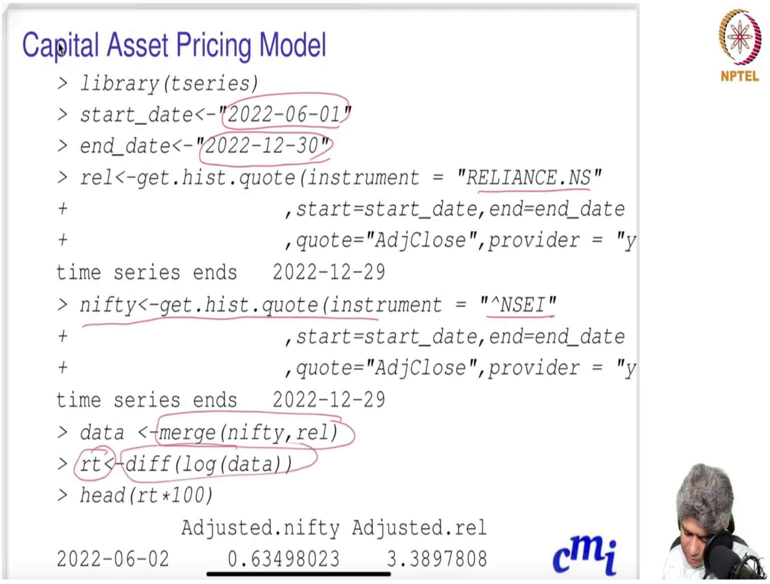

So, we I have taken the data reliance industries data between the 1st June 2022 and the 30th December 2022, so about 6 month of data I have taken. During the same time, I have taken nifty data also during the exact same time and then I merge the two data set and calculated their log return ok.

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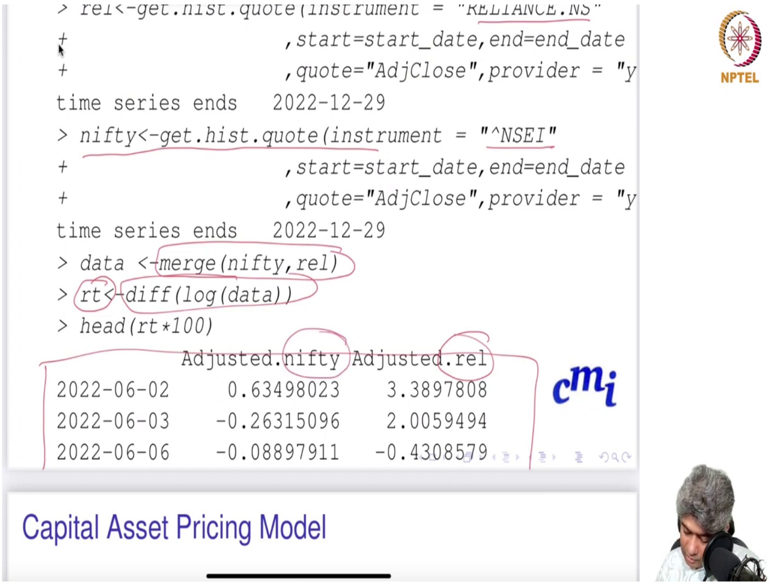

And here is the few first few days of data of nifty and the reliance industry, log return of nifty and log return of reliance industry and I plotted them over the period.

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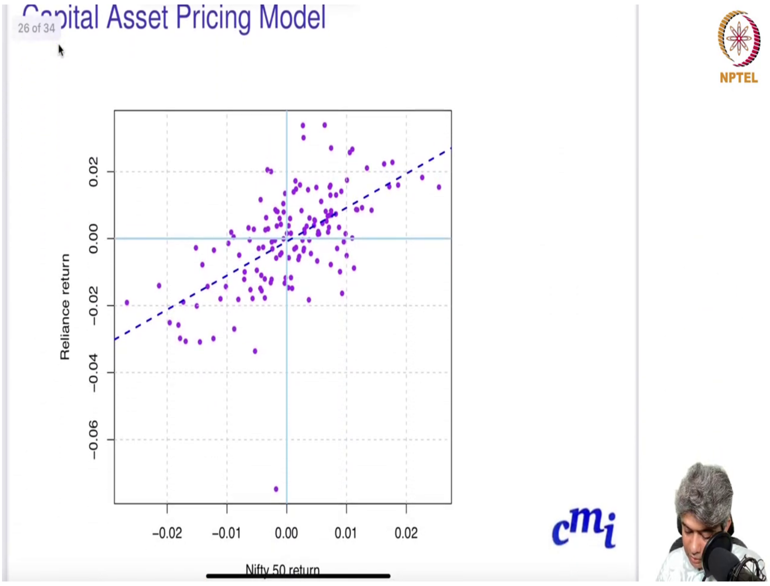

So, we over the x axis I plotted nifty 50 return log return and on the y axis I plotted reliance log return. So, there is a big outlier here where nifty 50 is 0, but reliance industry dropped significantly and, but most of the data you can see is sort of on a straight-line kind of behavior we are seeing.

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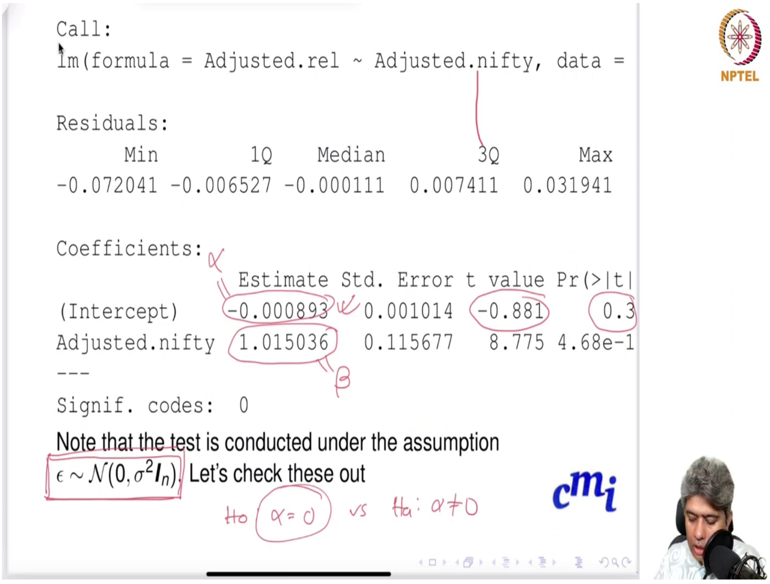

And then we fit the we calculated the you know log return the fit the model and this is the alpha and this is the this is alpha and this is beta ok. And what we found that alpha p value is 0.3 t value is negative 0.881. So, and we have assumed that a epsilon follow normal 0 sigma square. So, based on that so, our my taste was not alpha equal to 0 versus alternative alpha not equal to 0.

So, if I assume alpha is follow normal 0 sigma square the residuals follow normal 0 sigma square under that assumption our p value is too high 0.3, so we cannot reject the null we cannot reject this null. So, we can say that reliance in industry's price is fairly valued it is neither overvalued nor undervalued ok.

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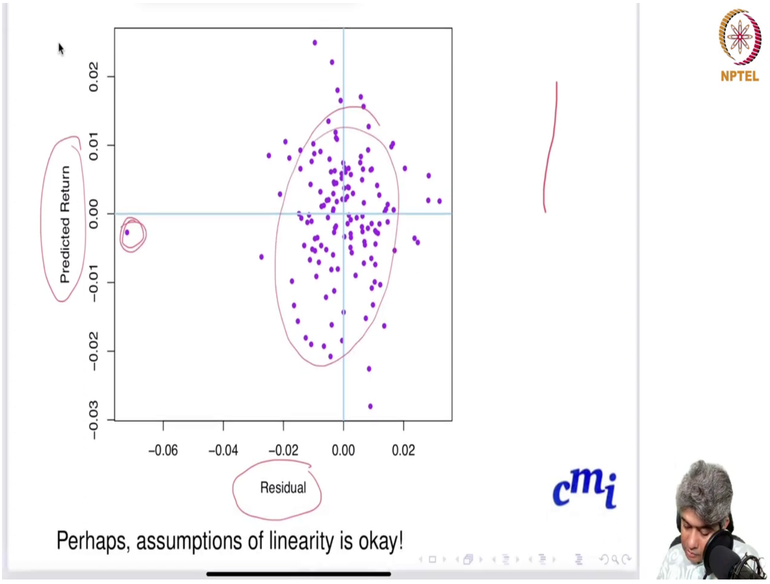

But we have to check whether the assumptions are correct or not. So, the first assumption is what we check that residual versus predicted. So, there is a big outlier, but overall, there is no much formation.

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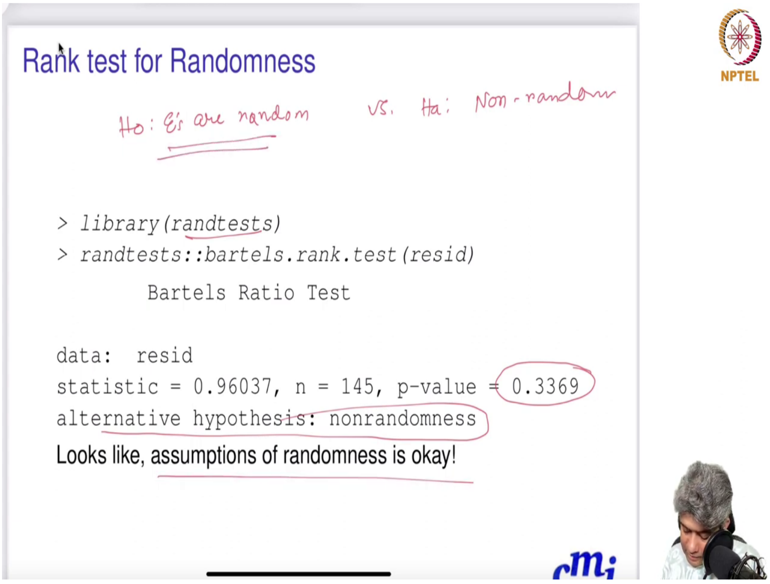

So, we ran a random test for randomness we ran a randomness and p-value is very high. So, null hypothesis is that the epsilons are random and epsilons are random versus alternate non-random. So, here is a alternate hypothesis non-random and we fail to reject the randomness. So, looks like assumption of randomness is ok.

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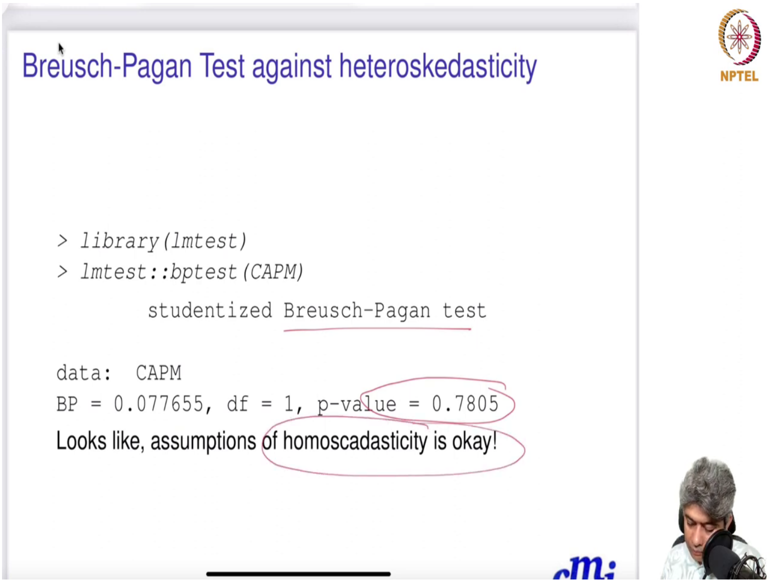

Next, we tried test for heteroscedasticity. We just simply ran Breusch-Pagan test and looks like p-value is too quite large. So, we can get heteroscedasticity assumption is also ok.

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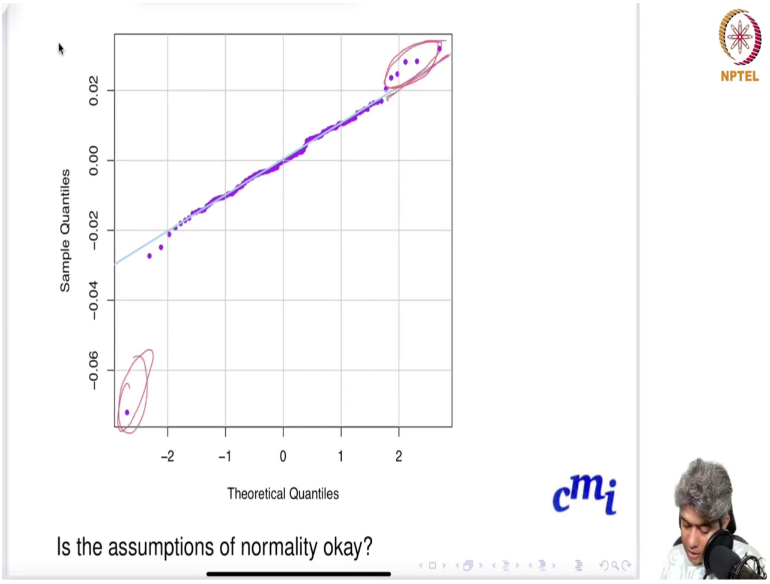

Then we looked into normality. So, here is one big outlier and there are quite a few points which is away from the qq line.

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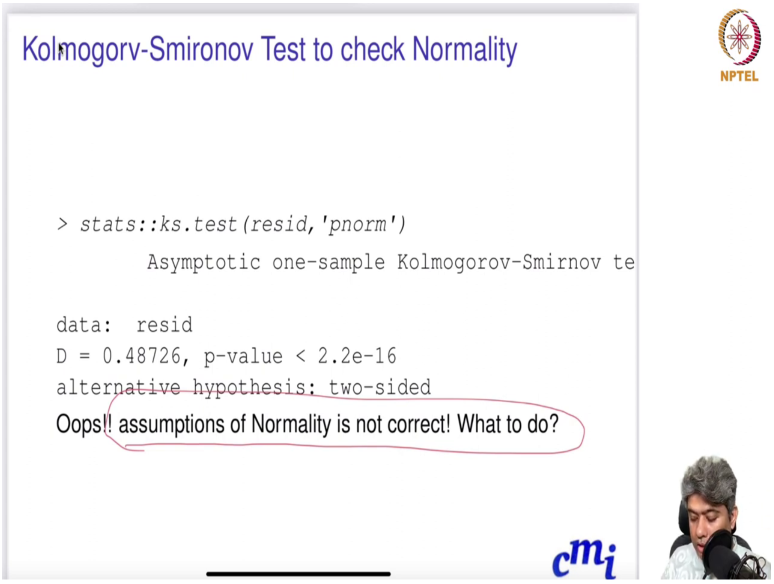

So, we do not know. So, we just ran a Kolmogorov-Smirnov of test and oops assumption of normality is not correct. If the assumption of the normality is not correct then we cannot do the test for the whether the it is correctly priced or fairly priced or under priced or over priced that test is not valid anymore.

So, simple linear regression though its a application capital asset pricing model is a simple application of simple linear regression model, but we cannot do the test for alpha because underlying assumption of normality does not hold good here.

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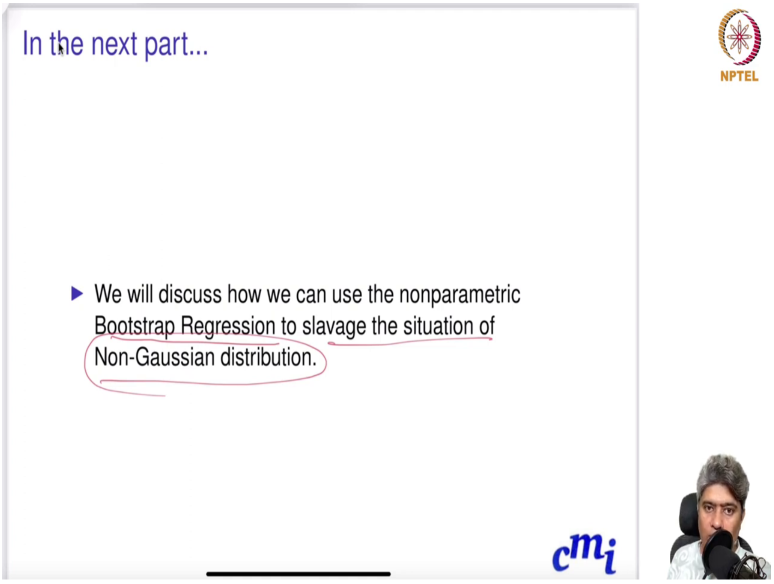

So, how we solve this issue? In the next video, in the next lecture we will discuss how can we use non-parametric bootstrap regression to salvage the situation of non-Gaussian distribution. And next video we will do hands-on on CAPM and after that we will talk about bootstrap regression to salvage the situation of non-Gaussian distribution.

CAPM is one of the celebrated model in the economics and finance and this is the one of the most beautiful application of linear regression in economics. It was done by Eugene Farmer and because of that he won Nobel Prize. So, with this beautiful model, I will stop this video. I hope you enjoyed capital asset pricing model.

Thank you very much. See you soon.