Approximate Reasoning using Fuzzy Set Theory
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Lecture - 44
FRI ~ SBR : FITA ~ FATI :Some Connections

Hello and welcome to the first of the lectures in this Week 9 of the course titled Approximate Reasoning using Fuzzy Set Theory, a course offered over the NPTEL platform.

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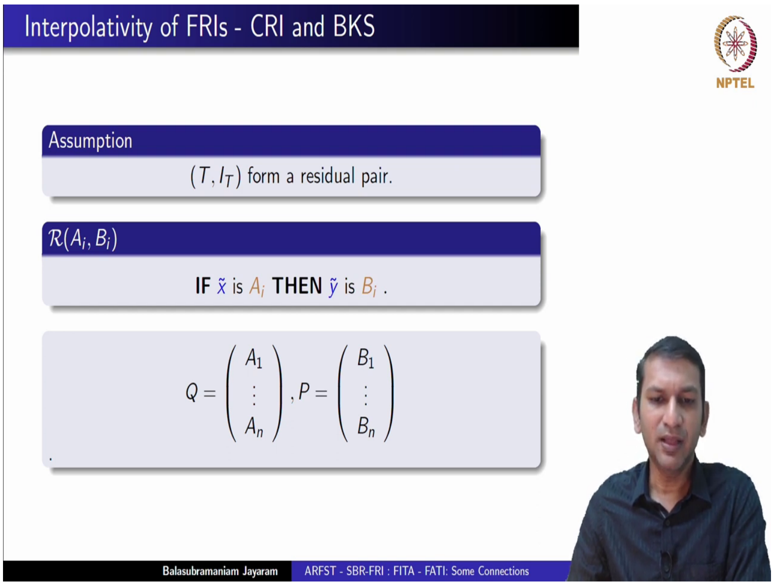

In the last week, we discussed the interpolativity of fuzzy inference mechanisms specifically that of FRIs predominantly wherein we looked at the compositional rule of inference and the Bandler–Kohout subproduct. Note that when we discuss this we were assuming the operations to come from residuated lattice given a set of rules, single input single output rules the Q matrix was formed from the antecedents and the matrix P was formed from the consequents.

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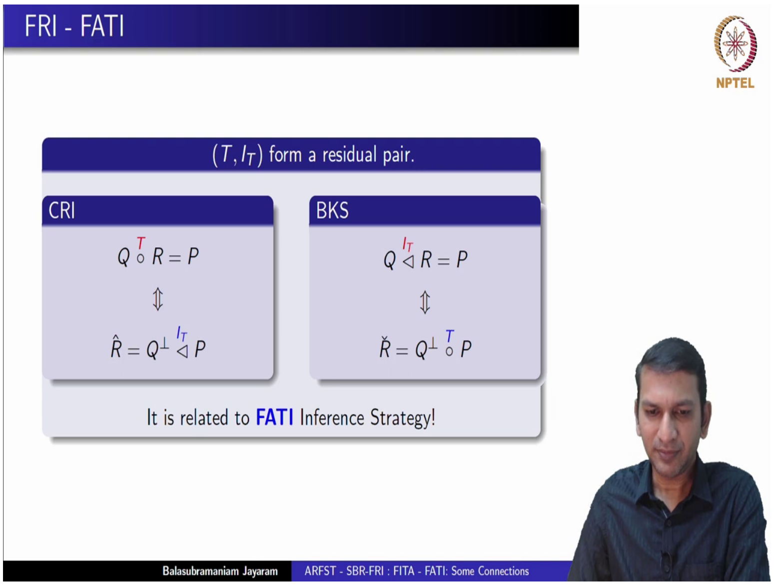

This led us to study the solvability of fuzzy relation equations and through that we came up with conditions on interpolativity of the underlying FRI mechanisms. For instance, when the T and I T form a residual pair in the case of CRI we are looking at Q sup T composed with R is equal to P we saw that this equation has a solution for R when Q and P are fixed Q is given in terms of the antecedents and P is formed from the consequents.

We saw that this system has a solution if and only if this R cap which is the Q transpose in I T composed with P is a solution it is a maximal solution. Similarly, in the case of Bandler–Kohout subproduct we have seen that Q in I T composed with R is equal to P when Q and P are fixed it has a solution for R, if and only if R check which is the Q transpose with sup T composed with P if this is a solution and if it is a minimal solution.

Now, the question that comes to mind is from the fuzzy relational equations we have obtained the interpolativity of the underlying FRIs. But, when we have multiple rules we have two inference strategies that are FATI and FITA. Now, it is clear that in the case under which interpolativity is ensured we are looking at a single relation which means it is in fact, the FATI inference strategy first aggregate, then infer. Then, the natural question that comes up is what happens to FITA in this case are these two same?

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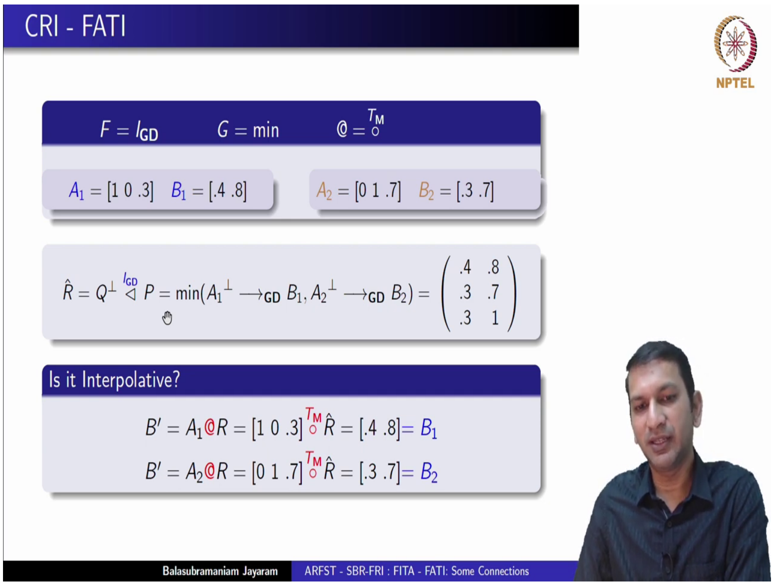

Let us take a couple of examples and check it this was the first example that we have seen note that here the composition is sup T. So, we are looking at CRI and we are using the Gödel implication which is the residual of the minimum T norm. Now, we have seen that R cap is given as this and when we ask the question is it interpolative all we did was we took A 1 and composed it with R cap and we found that yes we were getting B 1.

Similarly, when we took A 2 and composed with R cap we were getting B 2. Note that this is a single relation which we are using which is composed of the two relations obtained from these two rules using the Gödel implication and we are taking the minimum over. So, this is clearly falling into the FATI region. What happens if you use FITA?

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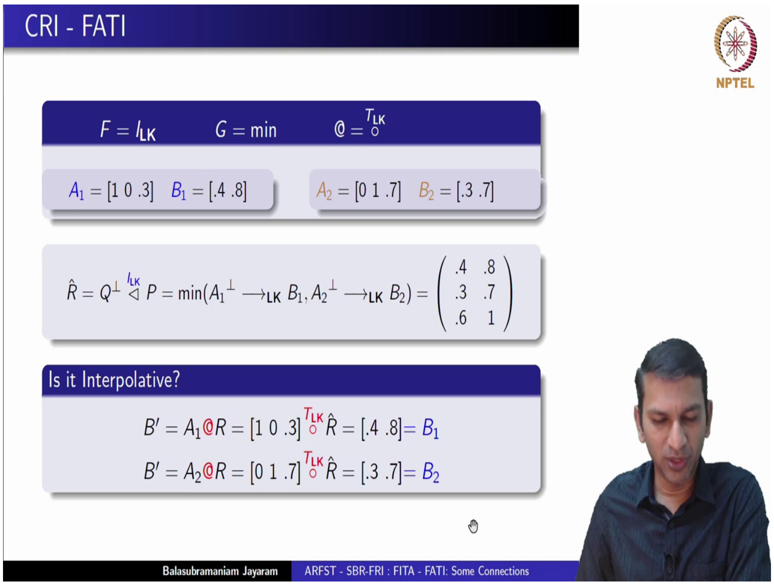

Note also the other examples that we have considered where we changed it to sup T LK. So, it is still CRI, but with the Lukasiewicz T norm and the corresponding Lukasiewicz implication which is its residual has been used, and we saw that once again we obtain interpolativity.

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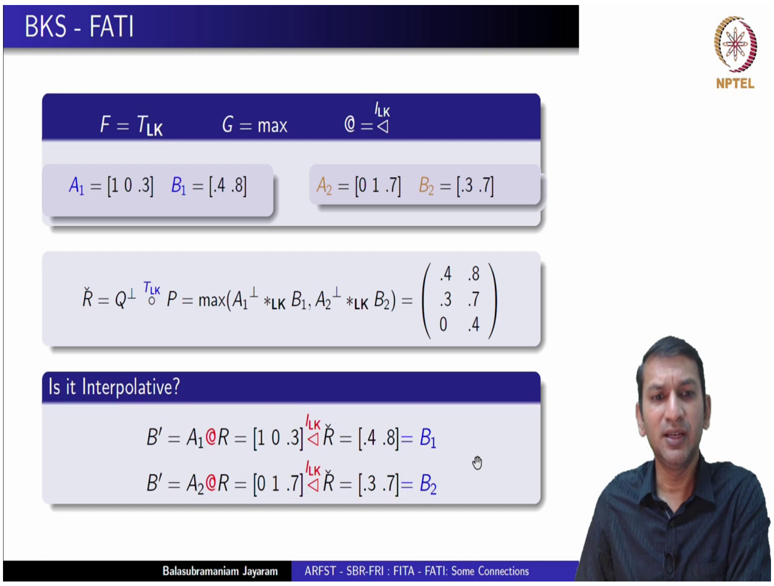

We have seen also an example for the BKS inference mechanism where we took inf I LK; that means, the implication was Lukasiewicz implication. So, clearly the corresponding T residuum is the Lukasiewicz T norm and that is what we have consider. You can also see that this is the case here where we are obtaining the relation using the T norm there, A 1 transpose B 1 using the T norm here and then we are taking a max over.

Once again we have seen that we obtain interpolativity. Let us consider a couple of examples from this and see what happens when we actually apply the FITA inference strategy that is first infer, then aggregate.

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The question is are they equal because interpolativity seems to be available only when we have this FATI inference strategy.

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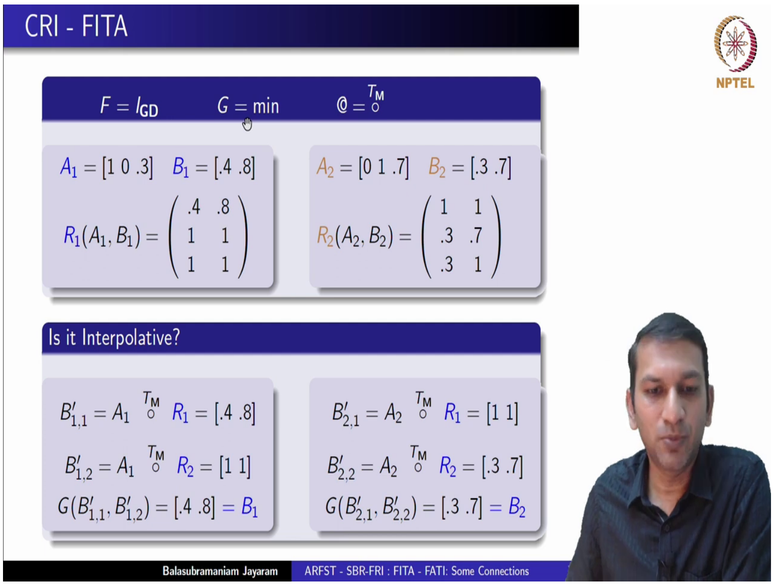

This is the first example we have seen sup min composition with a Gödel implication giving the relation between the rules. These are the As and Bs and these are the individual conditions. Let us ask whether it is interpolative; that means, applying FITA and seeing whether we get the same value that we got in the case of FATI. In the case of FATI we knew that it is interpolative that is if you give A 1, you will get B 1 and A 2 you would get B 2. So, now, in the case of FITA we are taking A 1 and composing with R 1, let us write this.

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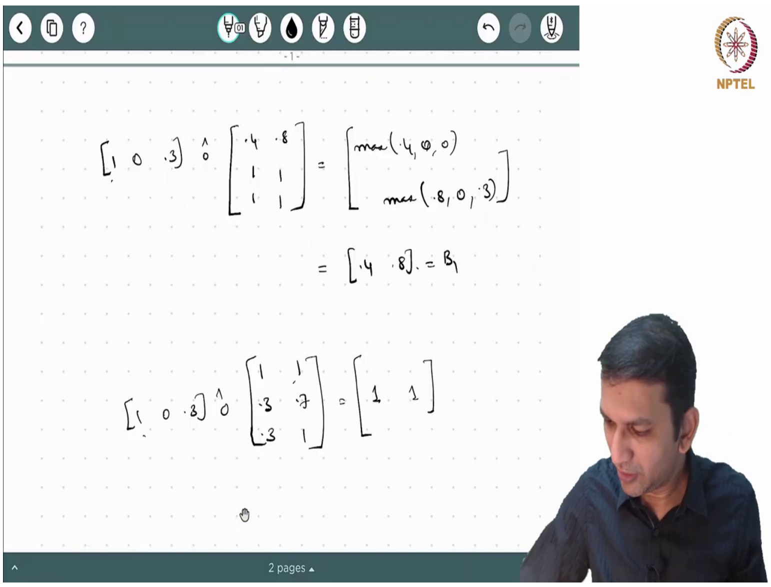

So, we are looking at 1, 0, 0.3 composed with R 1; R 1 is given as 0.4, 0.8, 1, 1, 1, 1. Note that we are using sup min composition. So, when we apply this is max of 1 and 0.4 is 0.4, 0 and 0 and 1 is 0, 0.3 and 1 is 0. Similarly, second component max of 1 and 0.8 is 0.8, 0 and 1 is 0 and 0.3 and 1 is in fact, 0.3. So, now from these two what we get is 0.4, 0.8 now, which really is your B 1.

So, we see here A 1 we compose with R 1 is B 1. However, note we are looking at FITA inference; that means, A 1 has to be composed with R 1 to get B 1 1 dash then A 1 has to be composed with R 2 also. So, let us take the R 2 relation which is 1 1 0.3 0.7 0.3 1. So, now, we are going to compose 1 0 0.3 sup min. Allow me to write it directly 1 1 is 1 max. So, then essentially it is 1. Once again 1 and 1 it is 1 max of this cannot be greater than 1. So, it is 1 1. So, what we get is 1 1.

Now, these are individual inferences B a dashes. So, we need to aggregate them using g which is min and what we find is in fact, we get B 1 itself. Let us look at A 2 whether we infer using the FITA strategy whether it gives B 2.

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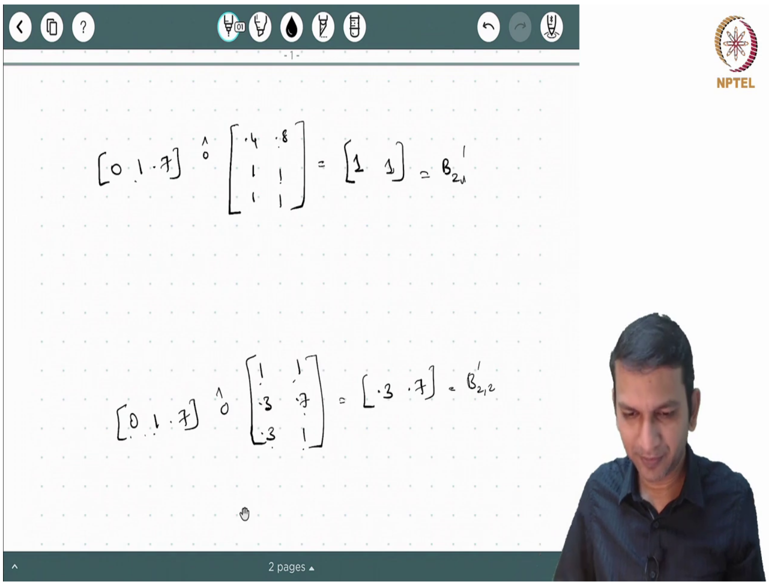

So, once again allow me to write it here. So, A 2 is 0 1.7 remember A 1 and A 2 they are normal and they form a corresponding partition. So, when you apply sup min composition 0 1 0.7 which is 1, 0, 0.8 is 0, 1 and 1 is 1. So, that is a maximum. You do not get any better than that this is what you get. So, this is what we get as B 2 1 dash 1 1.

So, what is B 2 2 dash? That is A 2 composed with R 2. So, once again let us erase this A 2 is 0 1 0.7 when composite 0 on 1 is 0 1 on 0.3 is 0.3 7 on 0.3 is 0.3. So, 0.3 0 on 1 is 0 1 on 0.7 is 0.7 0.7. So, the max is 0.7. So, this is our B 2 2 dash. So, it is 0.3 0.7. Now, once again applying the aggregation which is min operation here G of this is in fact, 0.3 0.7 which is B 2. So, in this case we see that when we give the antecedents as the inputs both FATI and FITA seem to be interpolative.

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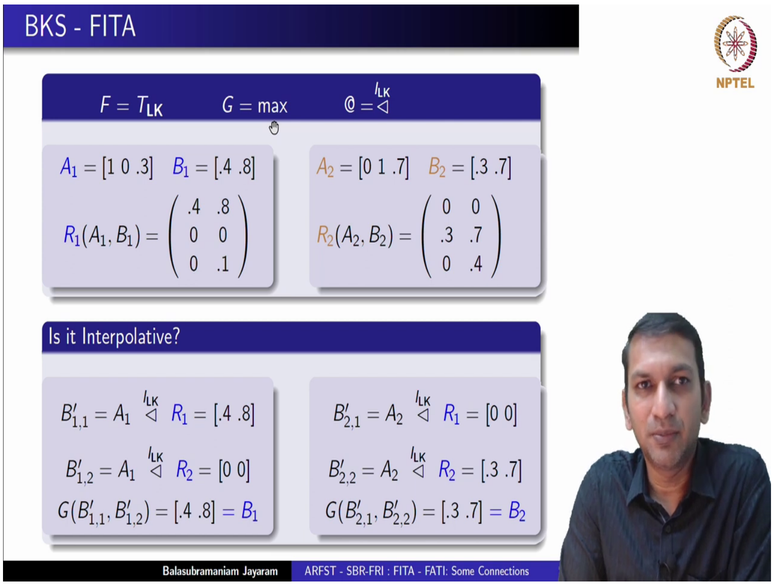

Now, what about the case of BKS it can be easily verified these are the relations that we get and if you give A 1 to R 1 we get this 0.4, 0.8; if you give A 1 to R 2 we actually get 0 0 and now note that the maximum is the aggregation operator. So, when you aggregate these two we get 0.4 0.8 which is B 1. Similarly, if you take A 2 and do the FITA inference A 2 as the input and do the FITA inference we obtain the overall output to actually be B 2.

So, from these two examples it appears that FATI is equal to FITA in the case it is actually interpolated, it appears from the numbers that we have taken. But, is it really true?

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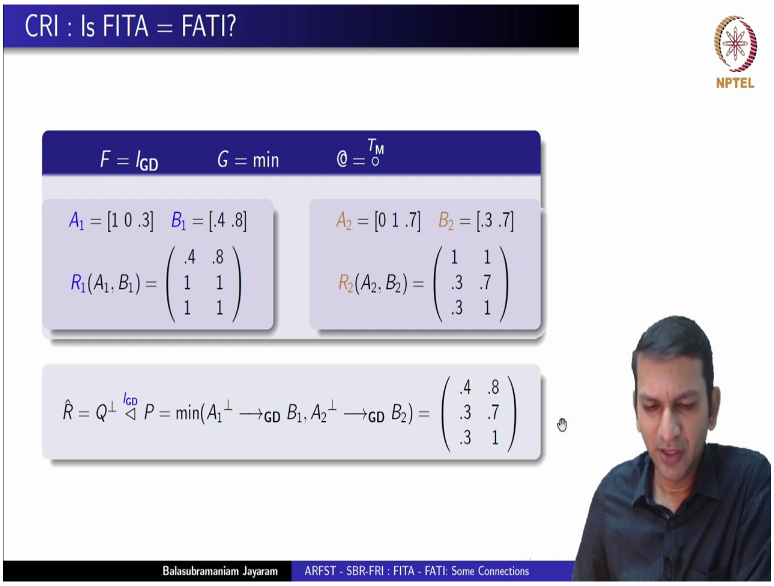

Let us once again take the very first example that we considered. So, now these are the individual relations and this is the composed relation, the aggregated relation.

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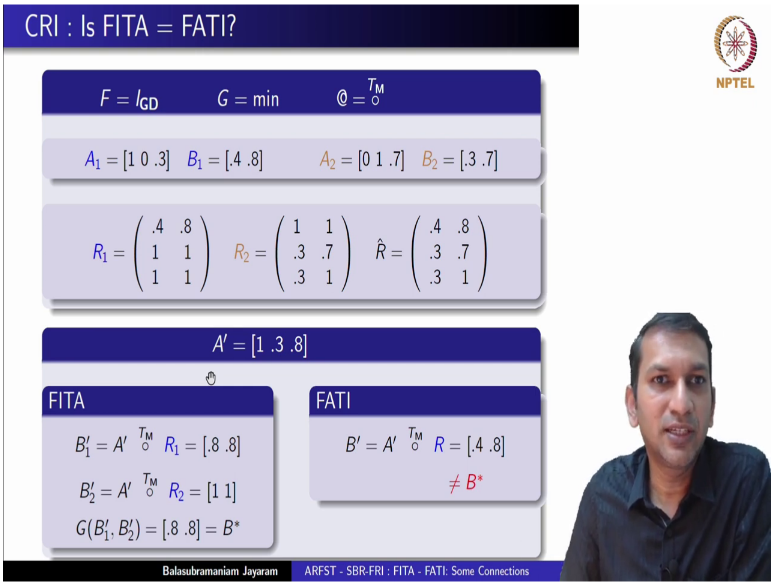

So, let us retain them A 1 B 1 A 2 B 2 this is R 1 R 2 and the combined relation which is in this case R cap because we want it to be interpolative, that is the context in which we are considering this let us give A dash to be 1 0.3 0.8. So, it is something different from either A 1 or A 2.

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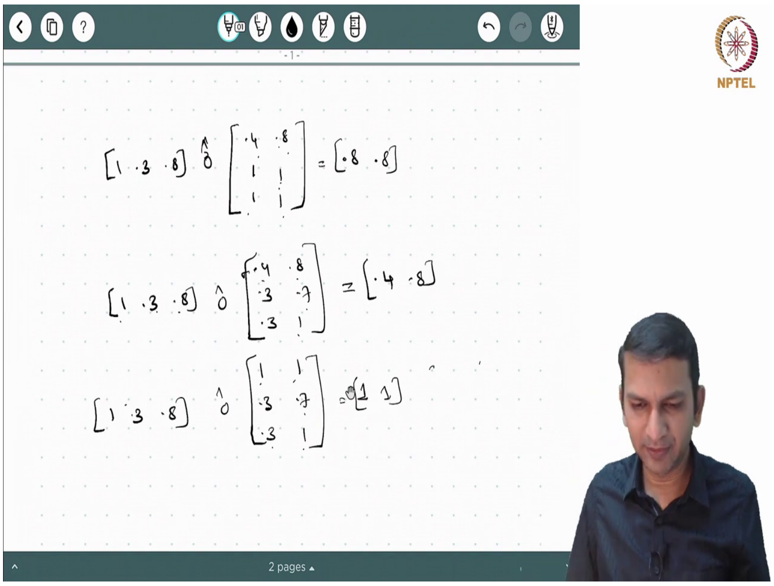

Let us calculate what happens in the case of this equation. So, now, A dash is 1 0.3 0.8 1 0.3 0.8 sup min what we get is 0.4 0.3 0.8 0.8 0.3 0.8. So, we get 0.8 0.8 as the output. Now, we need to apply this A dash also to R 2. R 2 is here 1 0.3 0.8. So, max is 1, again max is 1. So, this is 1 1.

Now, we aggregate these two local outputs and what we see is we get 0.8 comma 0.8 as the B star. Right now we are not discussing interpolativity because A dash is not one of the antecedents. So, let us apply the FITA inference. So, for that what we have is the combined R which is the R cap here. So, we have 1 1 0.3 0.8 sup min R cap is 0.4 0.8 0.3 0.7 0.3 and 1. So, this is the R cap that we have.

If you apply the sup min composition with this 1 0.4 0.3 0.3 is 0.3, 0.8 0.3 is 0.3. So, what we get is 0.4 which is the maximum 1 and 0.8 is 0.8, 0.3 0.7 is 0.3, 0.8 1 is 0.8. So, the maximum is 0.4 0.8, clearly this is not equal to B star. So, if you give a new input an input which is not one of the antecedents in this system at least we have seen A dash the B dash obtained from FITA is not equal to FATI.

So, just because we have interpolativity it does not mean that FATI is equal to FITA in that context when the operations are considered later when the relation is also considered according to the condition satisfying interpolativity.

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Now, the question arises is there some situation where we can ensure FITA is in fact, equal to FATI? Well, this is what we will deal with next.

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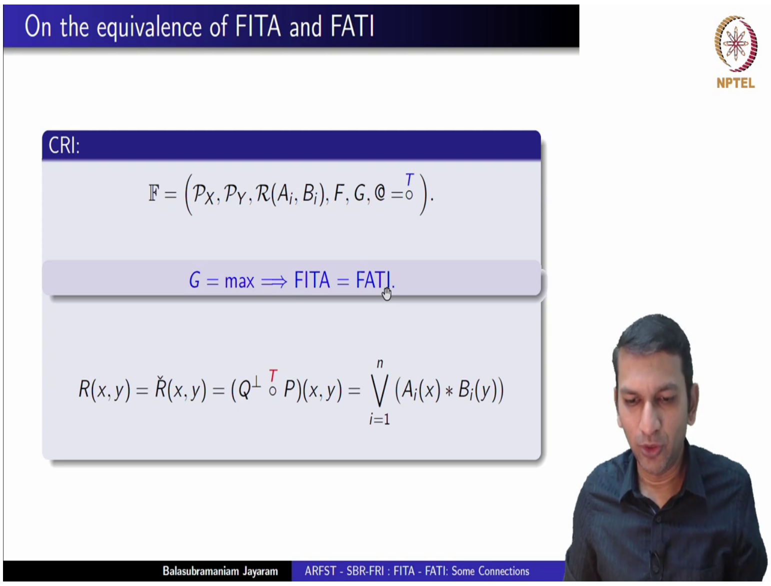

Look at this is the FRI system the CRI system where only the composition is fixed as sup T. If we take G to be the maximum that is if you aggregate it using the maximum aggregate all the relations using maximum, then it can be shown that FITA is in fact, equal to FATI. Now, how do we show this what does it mean?

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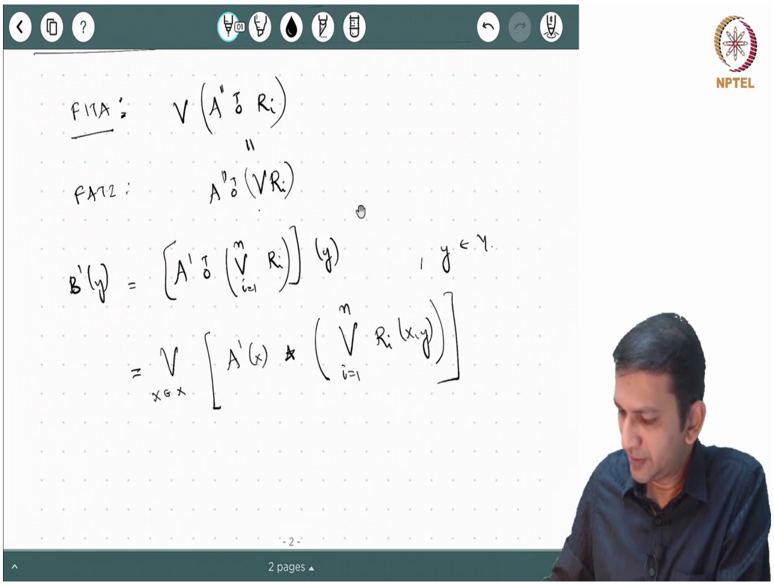

Look at this what is the FITA inference? FITA inference says, first infer then aggregate. So, that means, given an A dash we will compose it with the R i and then aggregate it for the aggregation we are using max this is the FITA inference; FATI is it says first aggregate all the relations and then apply the composition for the given (Refer Time: 14:06). We want to show that these two are actually same. So, what does it mean? So, let us start with a FATI inference.

So, B dash of y according to FATI is A dash circle T supremum i is equal to 1 n R i evaluated at a particular y. Note that y element of y is arbitrary. Now, let us expand this. So, this is supremum x over x A dash of X, let us use the infixed notation star i is equal to 1 to n R i of x comma y.

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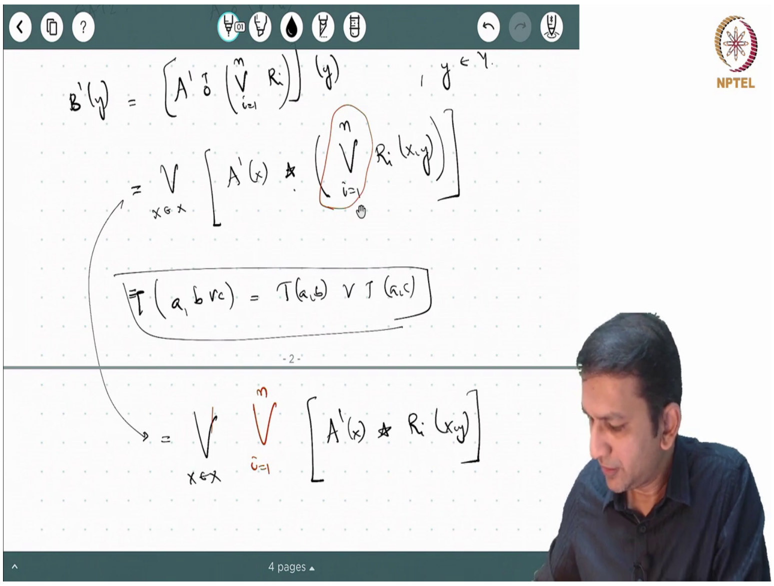

Now, the star is a T norm; we know that T norm is increasing in both the variables. So, by monotonicity and also otherwise it can be shown that any T norm is distributive over max which means if you have T of a b of cth is equal to T a comma b or T of a comma c. So, using this distributive equation what we can do is we can rewrite this like this. We can pull out this quantity.

So, this becomes x element of x i is equal to 1 to n note that this supremum here is of the elements varying over the domain x and this R here is the disjunction of the rules i is equal to 1 to n A dash of x star R i of x y.

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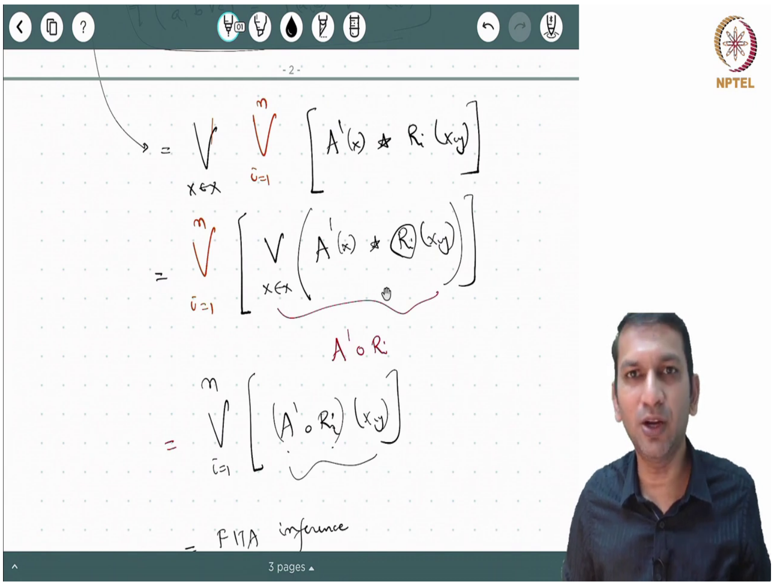

Now, these are maximum or supremum taken over two different domains easily we can switch them you can write this as x element of x A dash of x star R i of x y, but note what is this is essentially this A dash circle R i. So, we can write this as i is equal to 1 to n A dash circle R i at x y this is nothing, but inferring each one of them for a given i and then aggregating. So, clearly this is the FITA inference.

What did we start with? We started with FATI inference. This is FATI for us. First we aggregated the rules and then took a dash and composed it. Now, what we have seen is it is same as composing first and then aggregating the outputs. So, clearly in this case we see that FITA is equal to FATI when G is equal to max and we use the sup T composition. Note that here we are not expecting any conditions on the T norm, any T distributes over max. So, that is the only property that we have made use of here.

What is interesting is if you consider R check which is nothing, but the Q transpose sup T composed with P. If you rewrite this what you see is this is what is your corresponding relation; it is not R cap the corresponding R check. So, now, you see here it is essentially applying the maximum as the aggregation and for f we are using the corresponding T norm star itself. So, now, in this situation when you use R check, it is clear that G is max and hence FITA will be equal to FATI.

Note that in this proof that we have seen we did not specify how R i is going to be. We have not specified how R i has been obtained. So, you could obtain R i from any operation f and in this case the f is the corresponding T norm itself.

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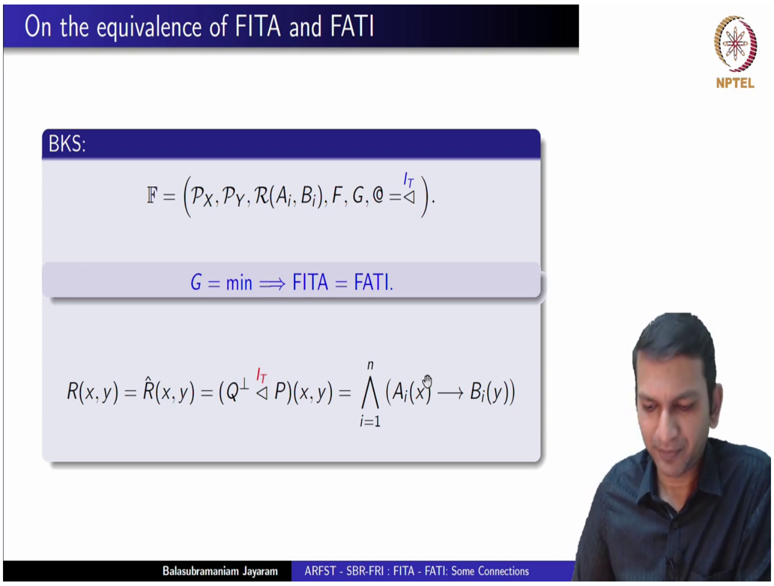

Similarly, we can show that if you are looking at BKS the composition is fixed as delta I T if you take G is equal to min then we can show that FITA is equal to FATI.

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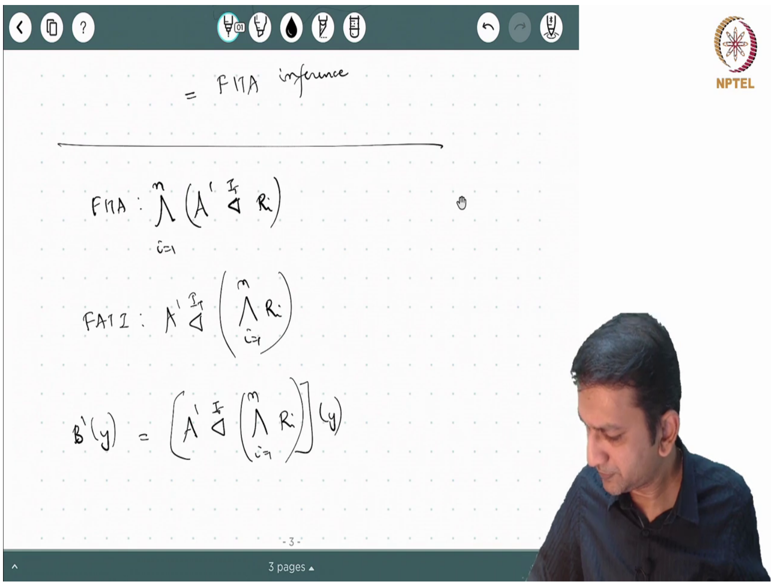

So, once again let us look at what is it that we are dealing with here. The FITA inference is A dash composed with each of the R i's and then we are taking the minimum of them because G is min here that is the aggregation FATI is a mode first composing all the relations using the aggregation and then applying A i's. Well, once again if you start with FATI at a particular y we know that this is nothing, but A dash R i applied by particular y, right.

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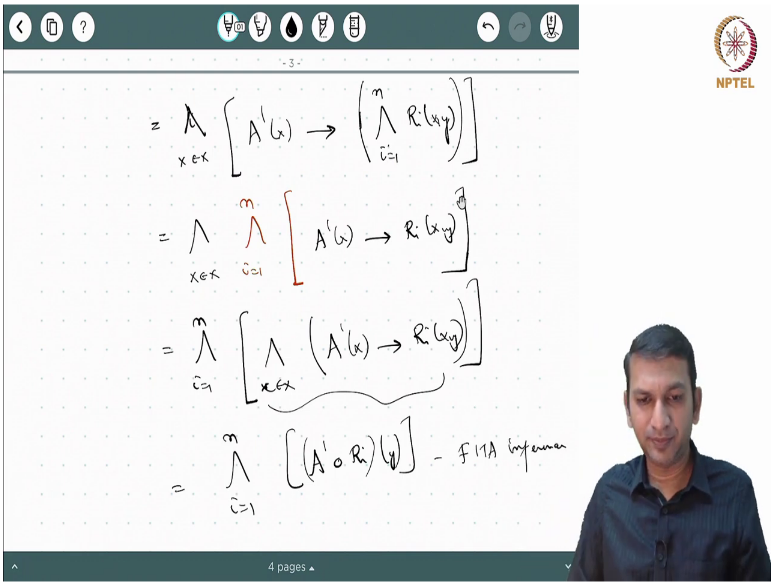

This here this is inf x element of x A dash of x implies R i of x y. Now, implication as we know is increasing in the second variable and this is conjunction of the rules. So, once again we can pull this outside and write it as A dash of x implies R i of x y. Now, once again interchanging the conjunctions what we get is R i of x y. Now, you can readily recognize this is nothing, but A dash composed to R i at a particular y i is equal to i is equal to 1 to n. So, clearly this is the FITA inference.

So, in the case of BKS no matter what the relation R i is, what operation F you have used to obtain the R i, if G is min you would get FITA and FATI to give the same output for any input A dash. Now, notice that if you take the R cap relation it is given like this which essentially means it is infimum of A x implies B i y.

So, note that what we are interested in is the aggregation that we have used in this case it is i is equal to it is actually infimum which is the minimum as was mentioned the operation F is in some sense inconsequential; of course, in the case of R cap that gets fixed to be the corresponding implication the I T that we use.

So, in that sense note that for interpolativity in BKS case we want that R check should be a solution and if it is a solution it should be the minimal solution; whereas, here we are asking for R cap in the presence of R cap R cap is one of those relations which will lead to the FITA and FATI inference being equal.

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Well, we have seen the relation between FITA and FATI when will they be equal. Let us see whether FRI and SBR two of the major fuzzy inference mechanisms whether they themselves are related or not.

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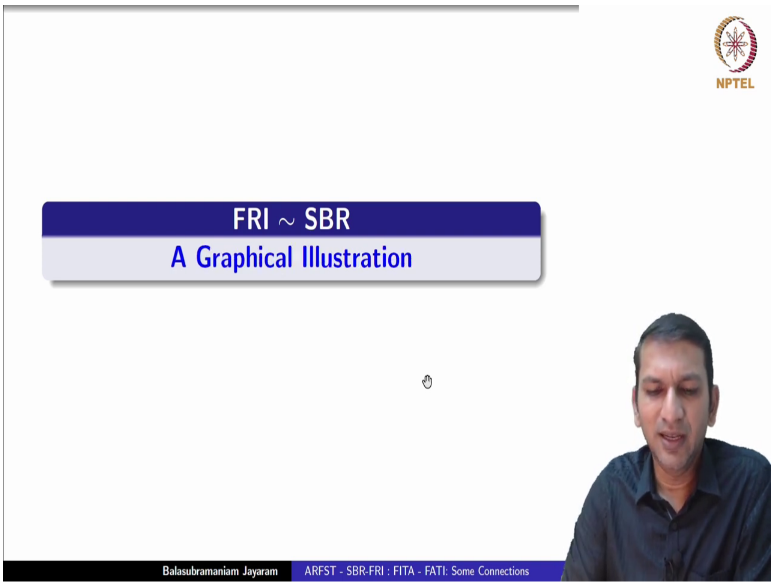

Towards this end, let us look at recall the graphical illustration that we have seen earlier about how to look at these inferences themselves.

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Let us look at the FATI inference strategy in an FRI.

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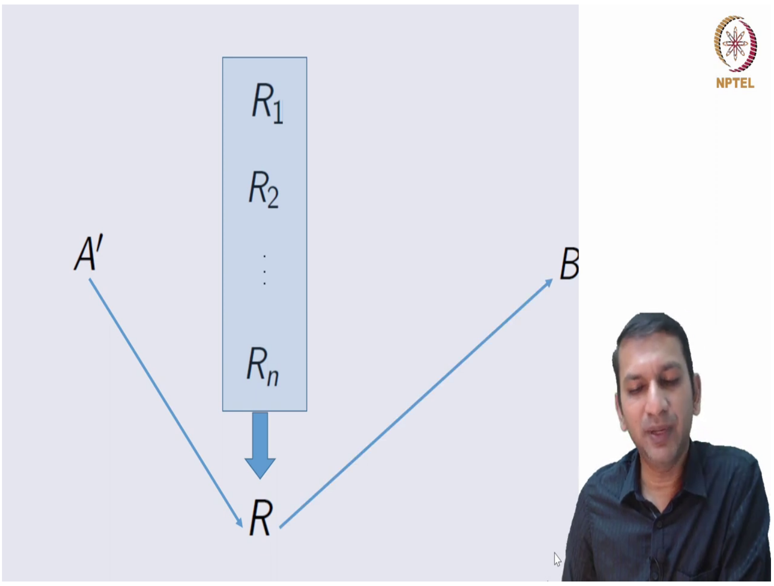

We are given an A dash which is the input we need an output B dash to obtain this we are given the ground truth in the form of rules. Now, from these rules we abstract the relations R i. So, we have n such rules. So, we have n such relation. In first aggregate then infer first we aggregate these relations into a single relation R and we compose A dash with R to obtain R B dash.

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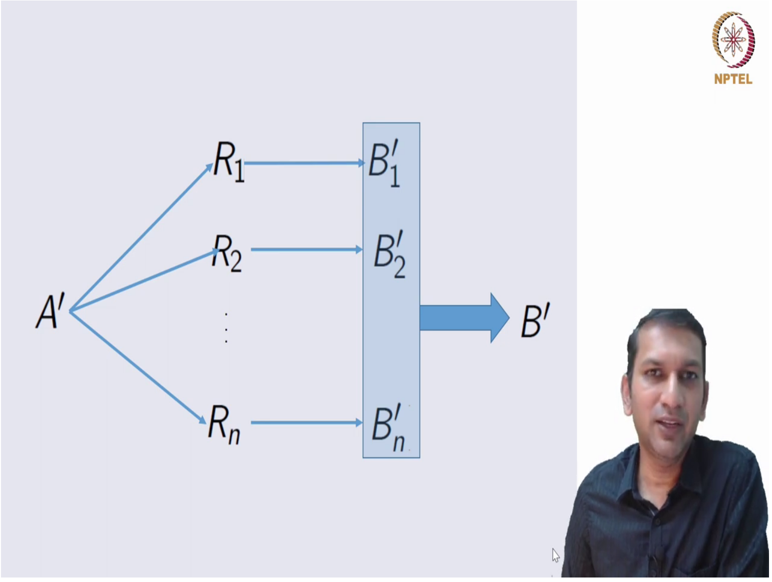

In the case of first infer then aggregate strategy, we still have these R relations, but what we are going to do is instead of aggregating these relations? We are going to compose A dash with R 1 to obtain B 1 dash A dash with R 2 to obtain B 2 dash so on till obtaining B n dash. So, locally we are going to compose and obtain local outputs B 1 dash B 2 dash so on till B n dash.

Then after inferring them we are going to aggregate these outputs to obtain the B dash. So, this is the first infer then aggregate strategy. These are the two inference strategies that we have in an FRI.

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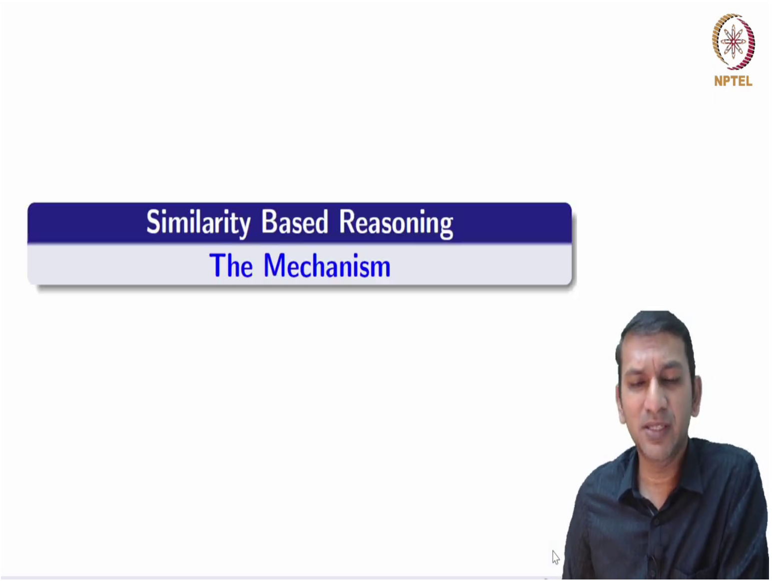

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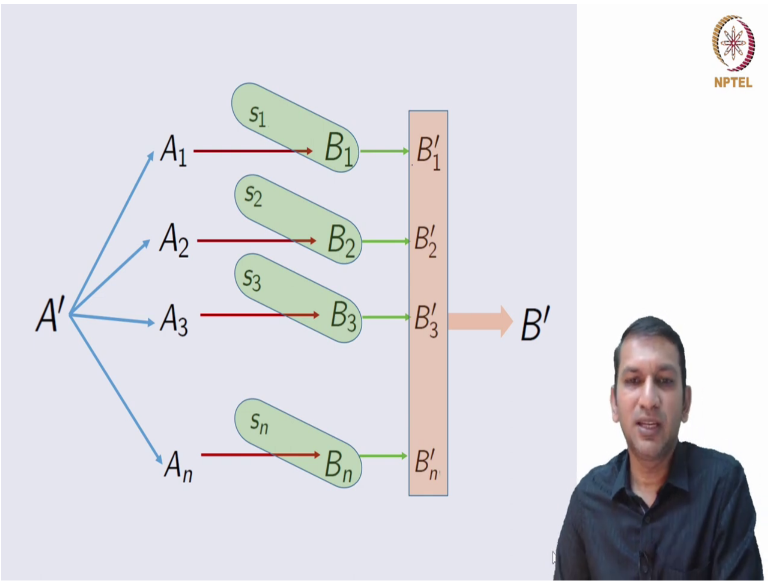

If you look at the similarity based reasoning scheme what we have is we are given an input A dash and we still need an output B dash, but we are retaining the rules as such. So, we have n rules of the form A 1 implies B 1; A 2 implies B 2; A 3 implies B 3 so on till A n implies B n.

Now, what we do here is first given this A dash, we are going to match it against each of the antecedents A 1, A 2, A 3 so on to obtain a similarity value S 1. So, A dash matched with A 1 gives us a similarity value S 1, with A 2 it gives S 2, with A 3 S 3 so on till S n. Now, we take this similarity value S I and modify the corresponding B i. So, S 1 modifies B 1 to give B 1 dash S 2 modifies B 2 to give B 2 dash.

Similarly, we get B 3 dash so on till B n dash. So, now, this is finally, aggregated to obtain a B dash. So, it appears that the FITA strategy and the SBR mechanism they seem to have some kind of a correspondence or some kind of a similarity. Let us explore this little further.

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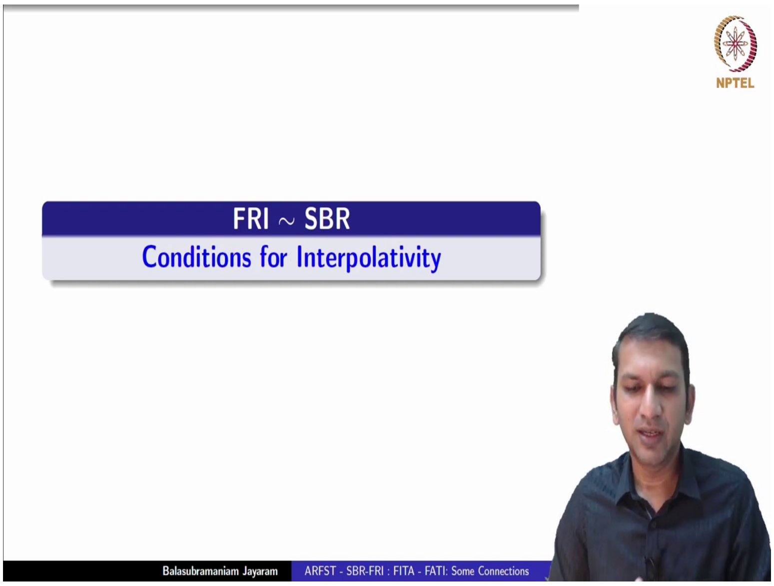

Not just this, if you look at the conditions for interpolating interpolativity that we have seen in the last week of lectures they also show some kind of similarities.

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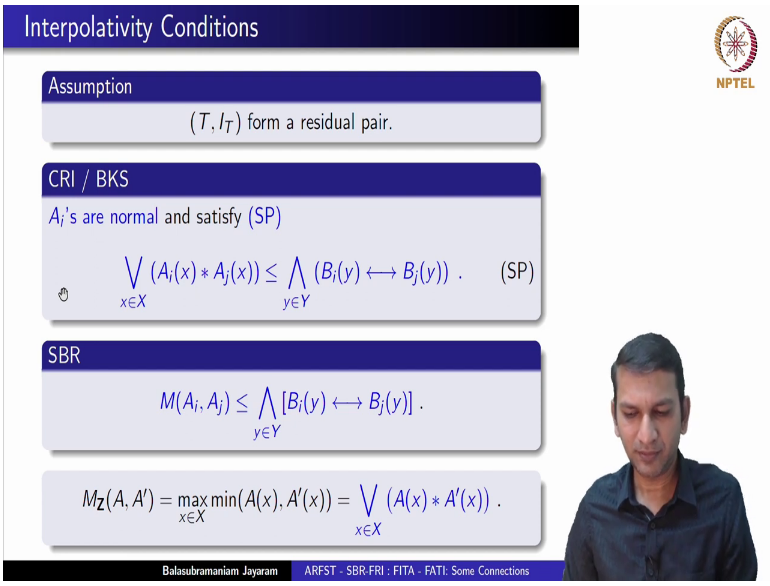

For instance, look at the interpolativity conditions for an FRI. Of course, we have the assumption that T and I T form a residual pair. So, the operations are coming from a residuated lattice. We have seen that one condition to ensure interpolativity either for CRI or BKS that all the antecedents are normal and they actually satisfy the antecedents and consequence together they satisfy this inequality which we call it the semi partition inequality.

Now, under the same context when the operations are coming from a residuated lattice, we have seen in the case of SBR such an inequality. Now, the right hand sides are on exactly the same, only the left hand side seems to differ, but does it really for instance M is only a matching function; consider the Z as matching function. So, either we could write it like this or in terms of infixed notation this is how it appears and this is exactly the same as what you have in the SP equation. So, this gives us an idea as to perhaps SBR and FRI are indeed related. Let us look at this.

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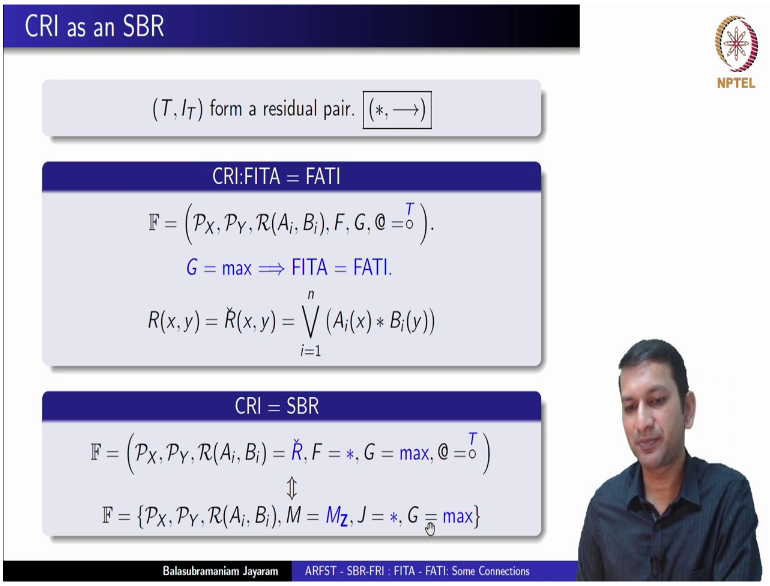

Let us try to look at CRI as an SBR that is an FRI a fuzzy relational inference like this compositional of inference can we look at it as a similarity based reasoning scheme? Once again, we will fix the residual pair; that means, the operations are coming from residuated lattice. We have seen that when we consider a CRI if G were max, we got FITA is equal to FATI and if we consider for the relation R, the R check where the aggregation is max clearly FITA will be equal to FATI. The inferences from both the inference strategies will be same.

Now, we want to show the conditions under which CRI can be looked at as an SBR. Now, look at this. This is the form of this CRI. If we take for the rule the relation R check; that means, F is star and G is max, the sup T composition of course, these are operations are coming from residual residuated lattices then we can show this equivalent to an SBR inference mechanism.

In the SBR inference mechanism what we have is we have a matching function a modification function and aggregation function, note that we are in the context of looking at them as fuzzy inputs and fuzzy outputs. So, H and G that fuzzifier and defuzzier do not come into the picture.

Now, given the CRI where the relation is given as R check; that means, F is star and G is max it can be shown that is in fact, it is equivalent to an SBR where the matching function is a Z s matching function; modification is this is given by the T norm star and the aggregation is given by max.

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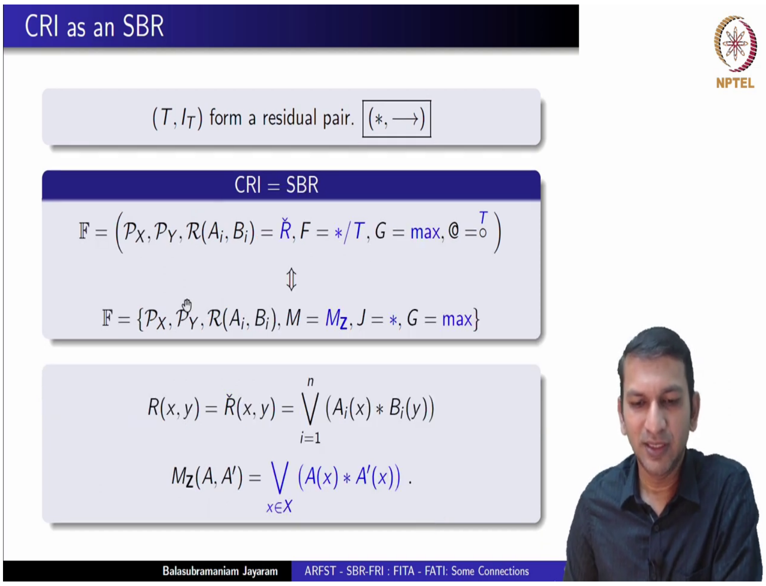

Let us prove this. This is what we want to prove.

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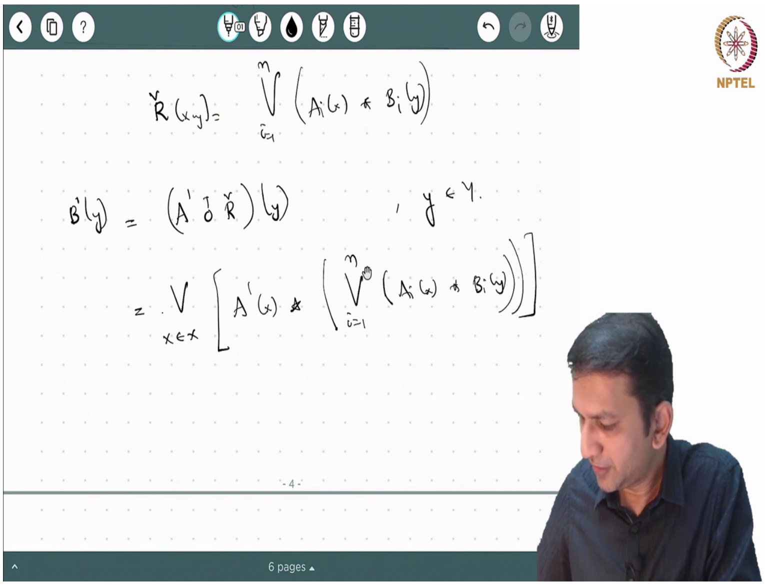

We want to show that in the presence of R check consider R check is nothing, but A of x star B i of y we want to show that such a CRI is in fact, an SBR. Now, what is the output that we obtain from here for a given B dash? Is nothing, but A dash sup T R check of x y. Once again y element of Y is arbitrary is equal to sup of x element of x A dash of x star i is equal to 1; R i of x y which is in this case A i of x star B of y.

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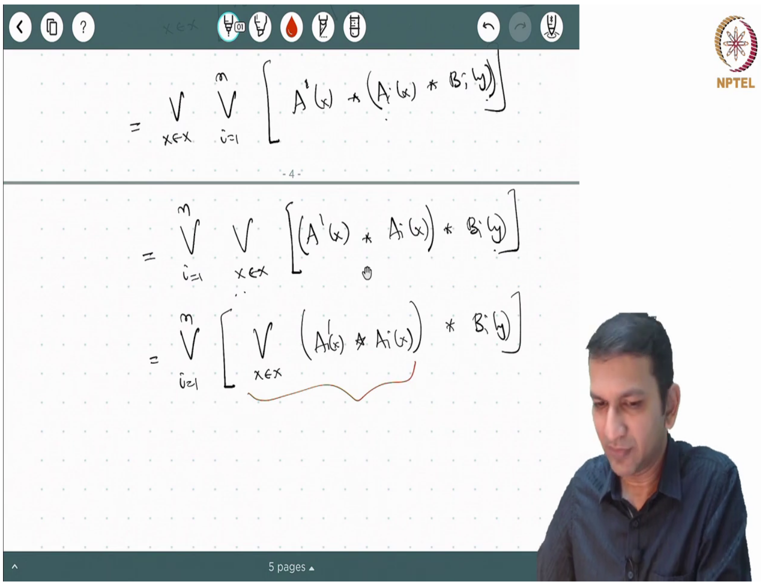

Now, once again we pull out the distinction over the rules, but you have is A dash x star A i x star B i y. Now, note that we can interchange these disjunctions even though we should put brackets because of associativity we do not have to really worry. However, now look at this here we have this bracket here by associativity we can put the bracket like this also.

Now, if you look at it this is a disjunction maximum a supremum running over x it has nothing to do with B i because its argument is y. So, we could also write this as i is equal to 1 sup over x A dash of x star a i of x star B i of y. Now, look at this particular quantity.

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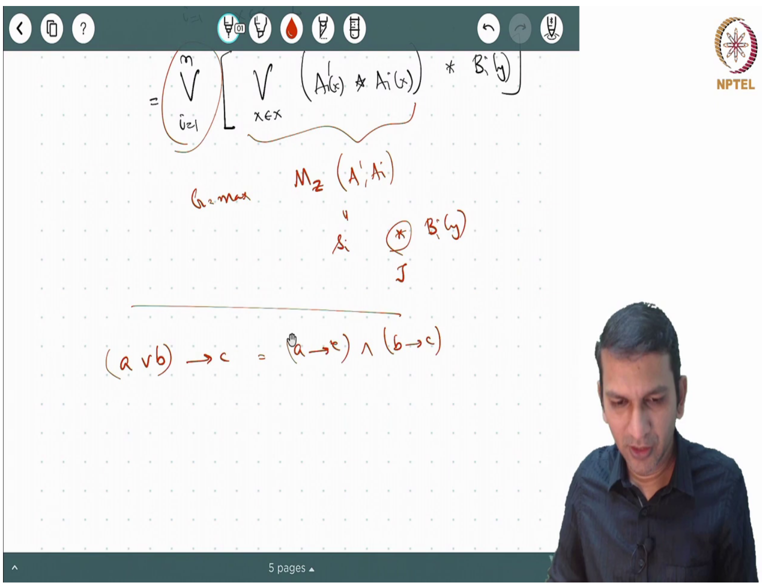

This is in fact, exactly M z of A dash A i. So, now, this will give us an s i. Now, this is s i star B i of y. So, this is our J star and for aggregation we have G is equal to max. So, we see that CRI when we use the relation R check we see that it is in fact, equivalent to an SBR where the matching function is the Z s function, the modification is given by the same T norm and the aggregation is the maximum aggregation.

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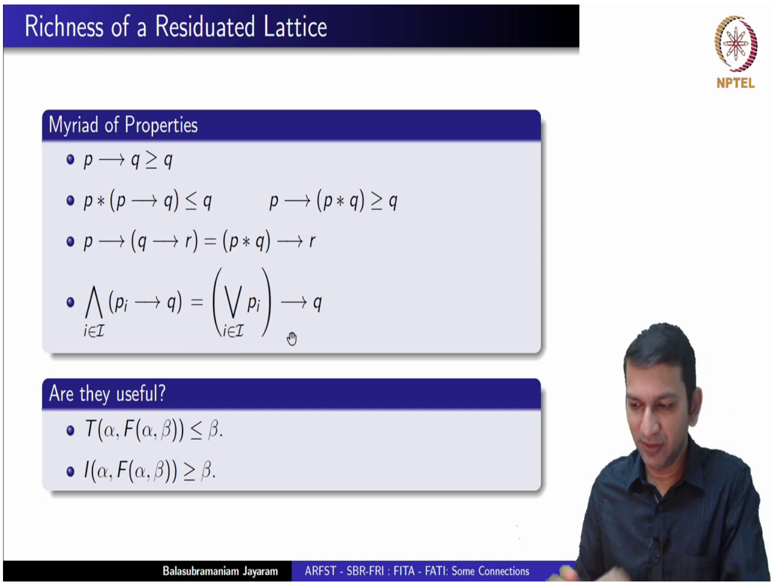

Now, can we look at BKS also as an SBR? Yes of course, we can, but before that let us revisit some of the properties that are available for us from the residuated lattice structure. Let us look at some of these properties. A few of them we have already known to be useful. For instance these two properties on the second row are what we have seen as the T F conditionality and the I F conditionality which were instrumental in showing interpolativity of an FRI in the case we had a single seizure.

Now, the third equality the property is also called the law of importation, we will have reasons to look into this function as a functional equation and in much more detail in one of the upcoming lectures. The last of the properties is essentially the distributive of conjunction and implication. For instance this comes from the classical logic tautology which says that if it is A or B implies C, then this is A implies C and B implies C.

So, this is how we understand it p 1 or p 2 implies q is p 1 implies q and p 2 implies q of course, in this case i is an infinite possibly infinite index set.

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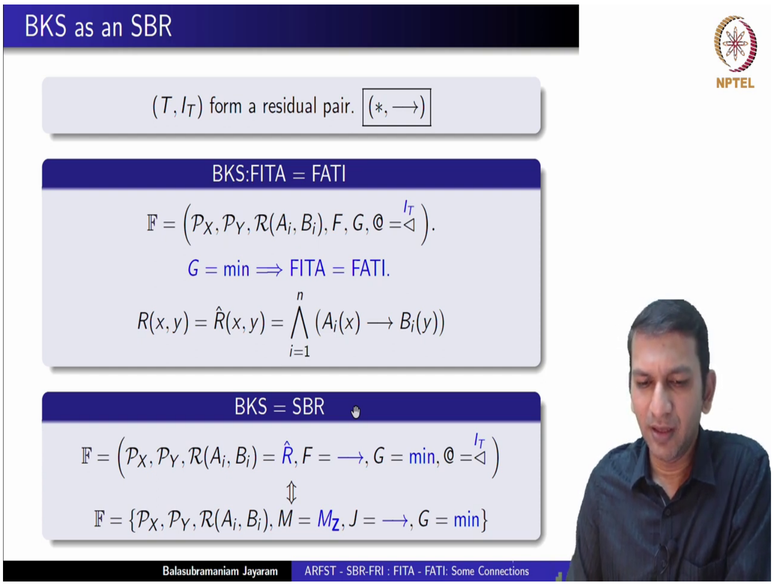

Well, we will make use of the third and fourth properties in showing that BKS can also be seen as an SBR. Once again we are in the realms of residuated lattices. We have seen that for a BKS case when G is min that if you aggregate all the relations using the min operation, then FITA is equal to FATI.

Now, we have seen immediately that R cap relation essentially aggregates these relations using the minimum operation. Note that for F it is using the implication itself. We will see that if we use R cap in the presence of infimum I T composition; that means, in BKS if you use R cap essentially it means G is min and F is the corresponding implication.

We can show that it is in fact, equivalent to an SBR inference mechanism where the matching function once again is the Zs matching function, the modification is given by the corresponding R implication that we are considering and G the aggregation is the minimum aggregation.

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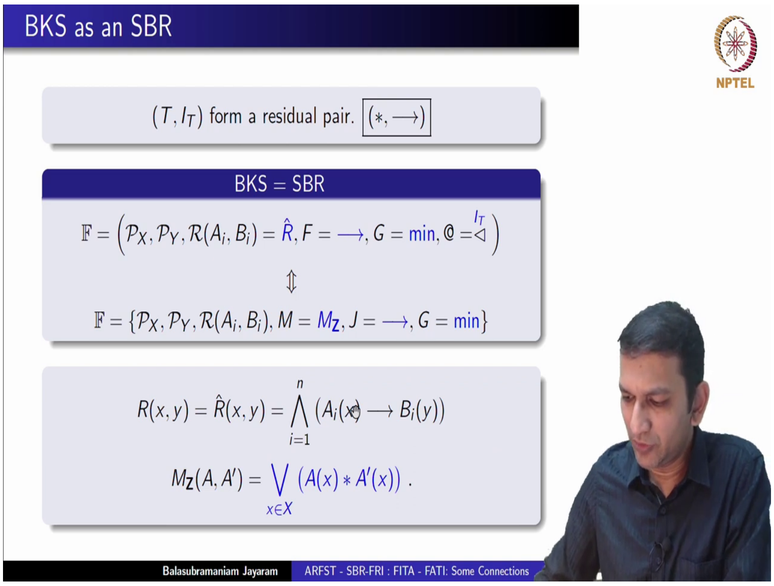

Let us see how to prove this.

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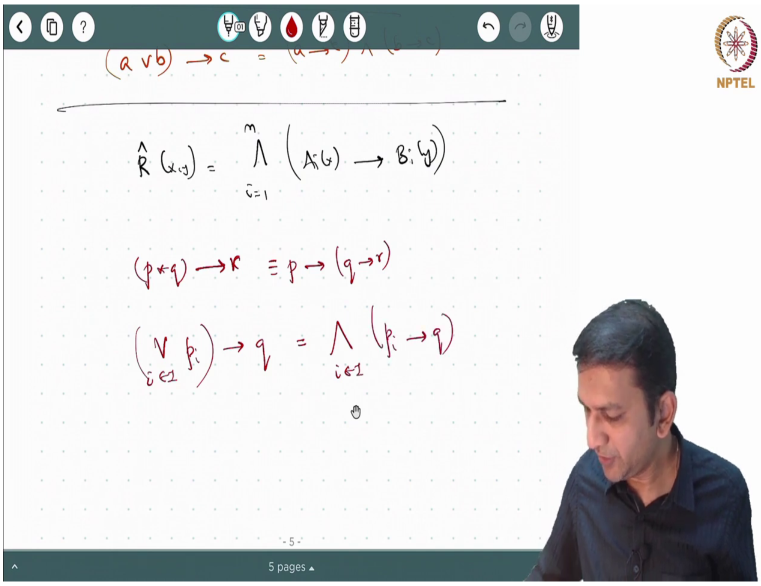

So, we need to show that FRI is also an SBR and what we are given is R cap x, y nothing. But, i equal to 1 to n A i of x implies B i of y and we already know there are two properties that we are going to make use of and that is p star q implies R is equal to p implies q implies r and also that (Refer Time: 35:53) have this equal to we will make use of both these properties now.

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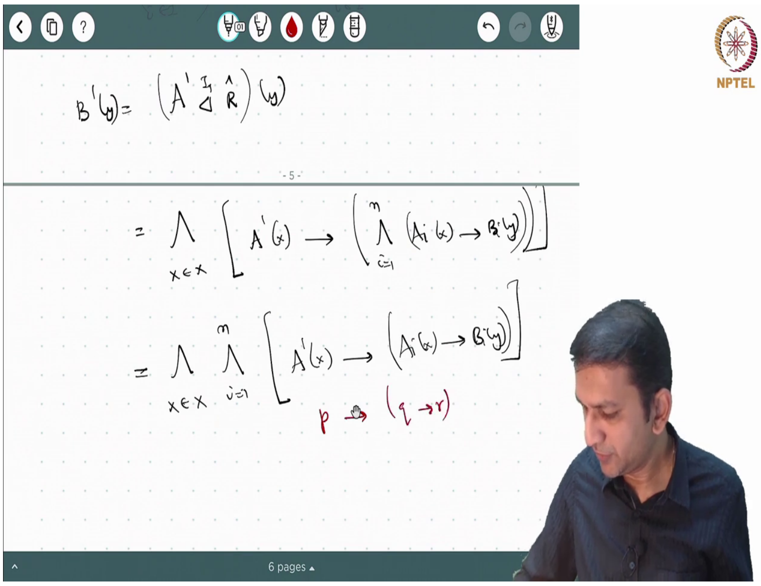

So, let us look at inferencing using the BKS with A dash I R cap. So, B dash of y is this if we were to expand it, it would look like this infer x element of x A dash of x implies i is equal to 1 to n; R i now is A i of x implying B i of y. Once again as we did earlier we can pull out the conjunction over the rules outside and what we are left with is A dash of x implies A of x implies B of y.

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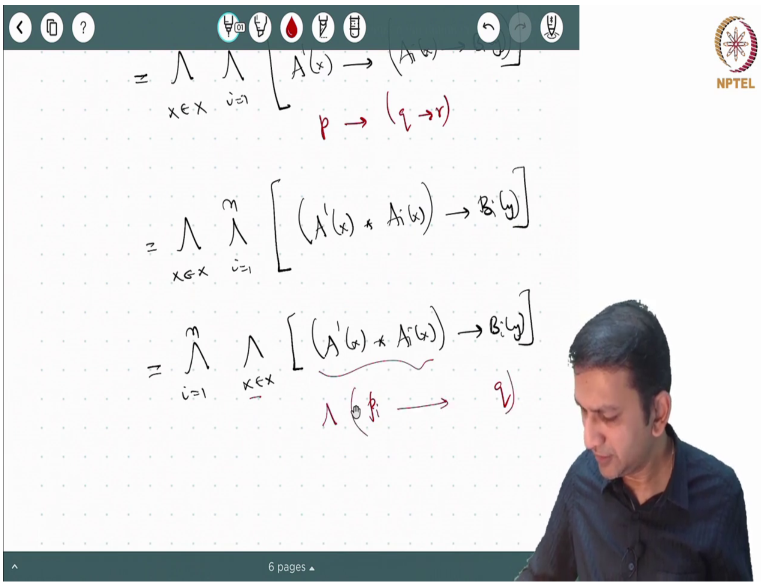

So, now we can look at this as p implies q implies r which means we can use this property here and then write this as p star q which is A dash of x star a of x implies B i of y. Once again by swapping the infimum both the infima we have to get the this an i is equal 1 to n x element of x A dash of x star A i of x implies B i of y. Now, note that if we consider this quantity the x here affects only this.

So, you could look at this as the corresponding p i's and this are the q an so, what we have is something like p i implies q and this we know with respect to this property can be modified like this.

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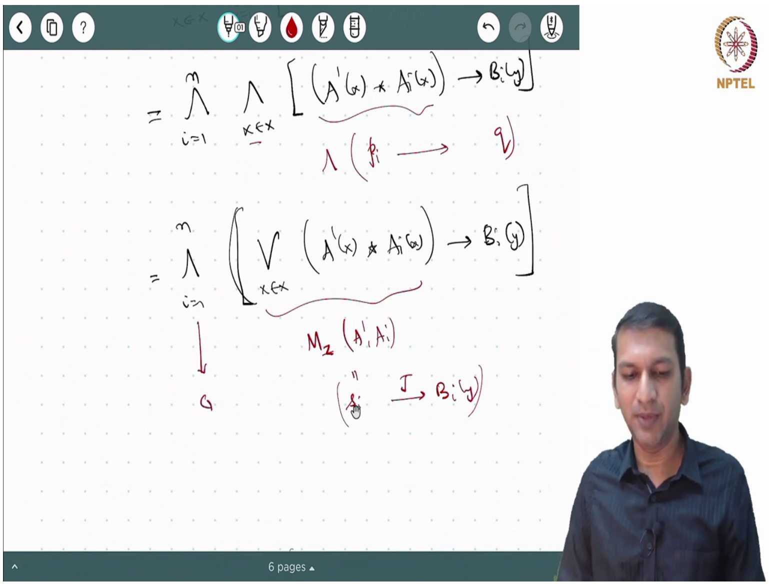

When you pull it push it inside then it actually becomes supremum over x element of x A dash of x star A i of x implies B i of y. Note once again now is essentially M z of A dash comma A i which we know is s i. So, this is a similarity value. Now, s i is modifying B i using this j which is the implication and this is our G which is the minimum operation.

So, we see once again if we use R cap as the relation in a BKS then it can be seen as an SBR where the matching function is given as a Zs matching function and we take implication for the modification and minimum for the aggregation. Of course, note that these were possible because we assume the operations to come from residuated lattice structure which gave us a lot of properties such rich properties helped us in obtaining this equivalence.

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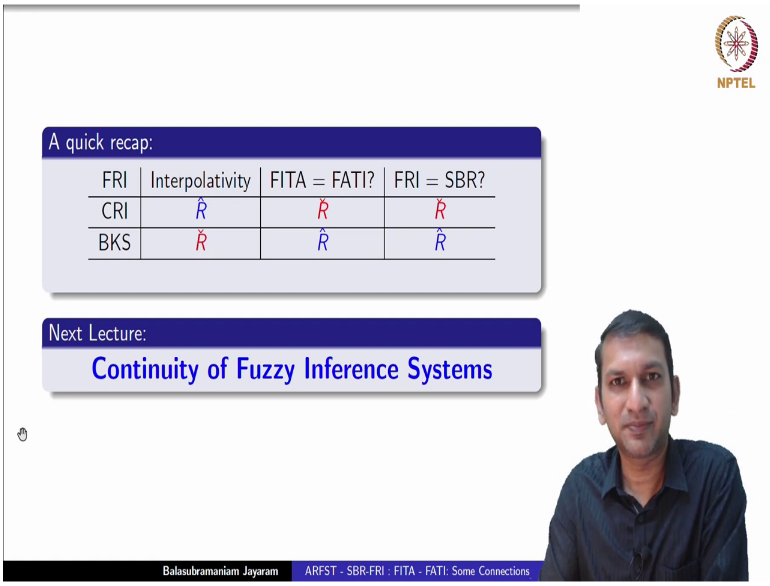

Let us have a quick recap of what we have seen in this lecture. In this lecture given an FRI we ask two questions will the FITA and FATI inference strategies, will they be equal? Will they give the same output and the second question that we discussed was if an FRI can also be seen as a similarity based reasoning scheme.

In the case of CRI while it was R cap which ensured interpolativity that is if R cap is a solution of the appropriate or the corresponding fuzzy relational equation, we saw that it leads to interpolativity. When you wanted to answer the question of equivalence of FITA and FATI or whether FRI this CRI can be looked at as SBR it was R check that came to our rescue that is if you use R check employ it to capture the rules.

Then we know the output from FITA or FATI will remain same and such a CRI can also be looked at as a similarity based reasoning scheme where the mod matching function is essentially the Zs matching function. In the case of BKS, the roles are reversed. It is R check which ensures interpolativity if it were a solution of the corresponding fuzzy relational equation.

However, it is R cap which ensures that FITA is equal to FATI and allows us to look at BKS itself as a similarity based reasoning scheme. In the next lectures, we will start to deal with continuity of fuzzy inference systems both that of fuzzy relational inference and also the similarity based reasoning schemes.

Glad you could join us today for this lecture. Hope to see you soon in the next lecture.

Thank you again.