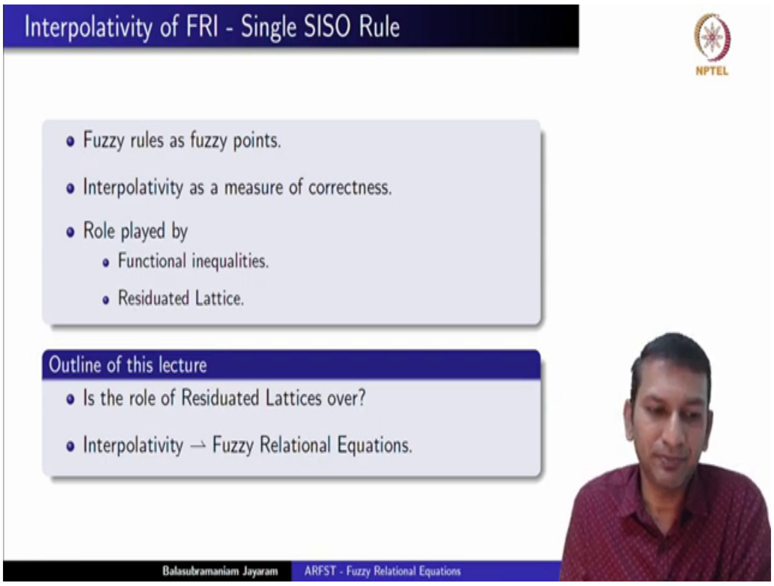
Approximate Reasoning using Fuzzy Set Theory
Prof. Balasubramaniam Jayaram
Department of Mathematics
Indian Institute of Technology, Hyderabad

Lecture - 41
Fuzzy Relational Equations

(Refer Slide Time: 00:16)

Hello and welcome to the third of the lectures, this week 8 of the course titled Approximate Reasoning using Fuzzy Set Theory, a course offered over the NPTEL platform.

(Refer Slide Time: 00:31)

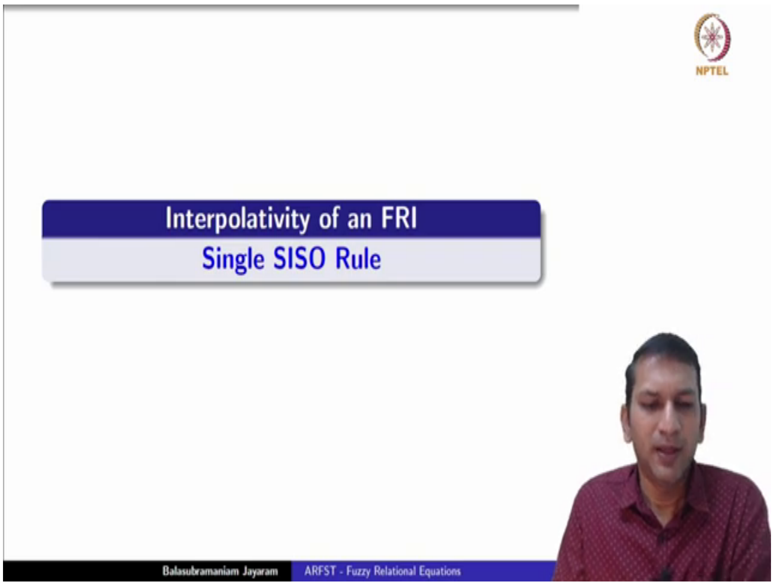


In the last two lectures, we have looked at fuzzy rules as fuzzy points and hence, the interpolativity of the obtained function as a mapping from the spaces of fuzzy sets on X to fuzzy sets on Y appeared to be a good measure of correctness. We have seen while trying to explore interpolativity in the case of a single SISO rule, we have seen that a couple of functional inequalities play a role and also, the algebraic structure of residuated lattice comes into picture.

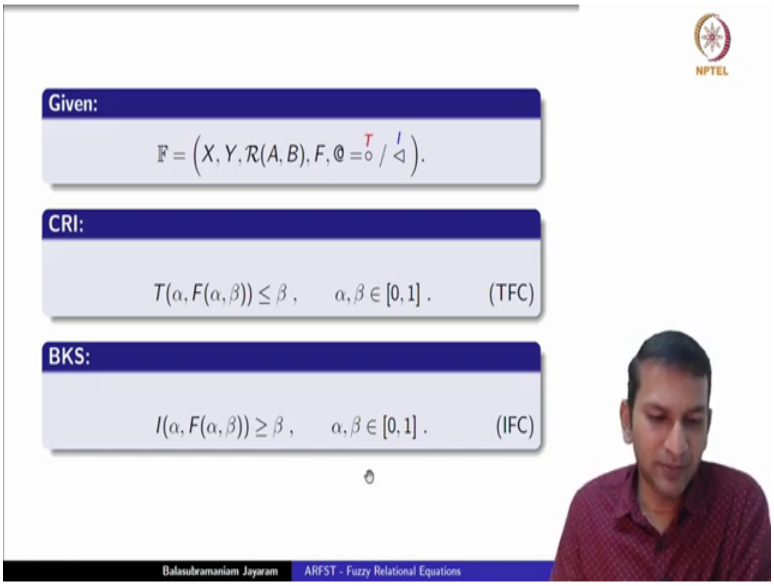
In this lecture, we will see the role of residuated lattice is not over in fact if anything, it has just begun to appear and we will make use of the richness of the structure quite a lot improving a lot of results related to desirable properties of a fuzzy inference system. And in this lecture, we will concentrate on fuzzy relational equations, especially the solvability of fuzzy relational equation with respect to the compositions that we have seen. Namely, that of the sup T composition and the inf I composition.

Of course, we also need to understand how interpolativity is related to the solvability of fuzzy relation equations. That also we will see in this lecture.

(Refer Slide Time: 01:57)

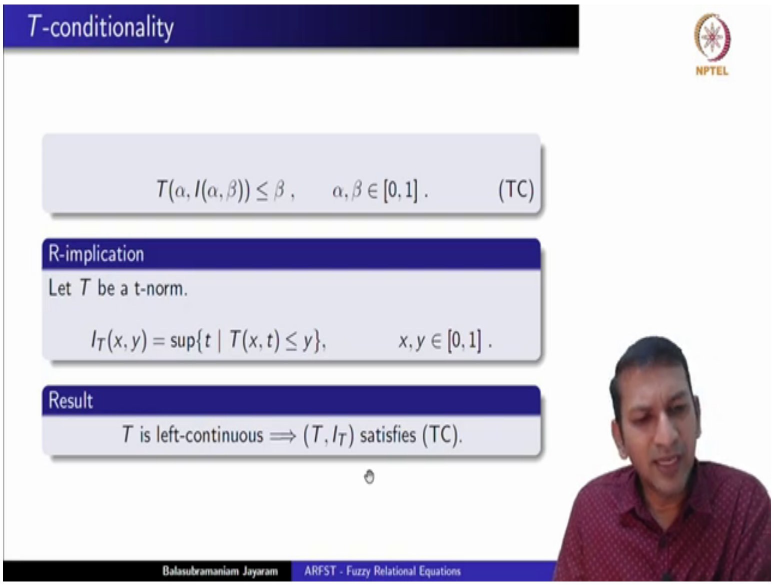


(Refer Slide Time: 02:02)



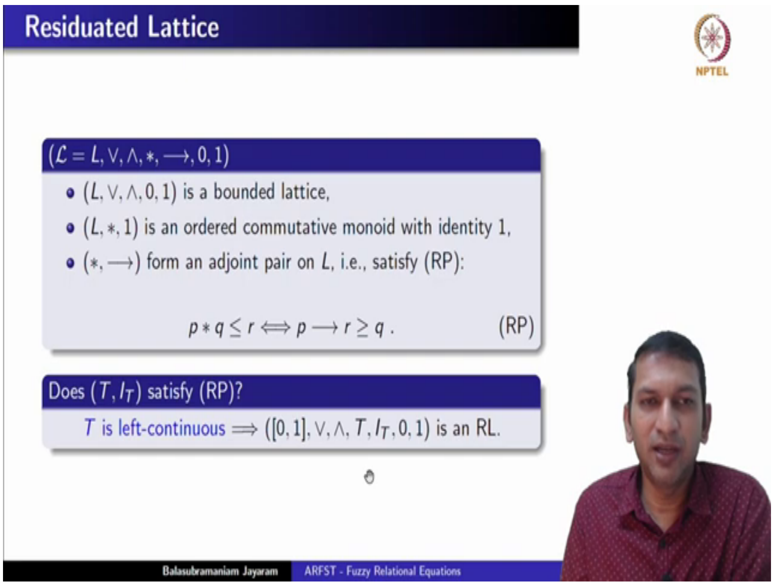
Well, a quick recap of what we have done in the previous lecture. So, this is the framework in which we are dealing with it is an FRI. If you were considering a single SISO rule and for the composition, we either consider sup T or inf I composition. We have seen what we call the T F conditionality played a role in obtaining interpolativity and in the case of the BKS inference scheme, what we call the I F conditionality, a functional equality of this form seems to play a role in ensuring interpolativity.

(Refer Slide Time: 02:38)



We have also seen, this is in some sense a special case or a generalized case of what is known in the literature as T conditionality. Now, we have seen that one solution for this equation inequality comes from the family of R implications; especially, when the T that we consider, the T norm that we consider is left continuous.

(Refer Slide Time: 03:07)



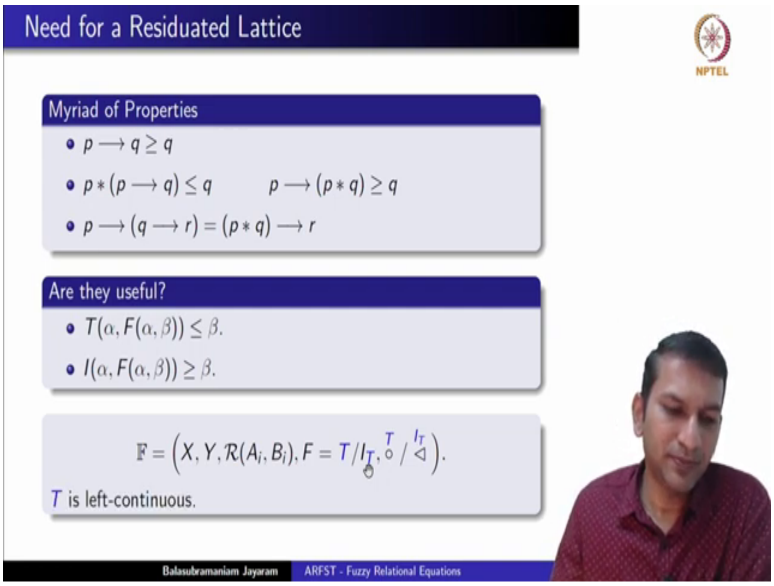
And this led us to revisit the residuated lattice structure, what is a residuated lattice? It is a set L endured with a few operations and constants, such that L with respect to the meet and join at the bound 0 and 1 is a bounded lattice. With respect to the star operation, it is an ordered commutative monoid with identity 1 and finally, the star operation on the arrow operation, they form an adjoint pair. What does that mean? They satisfy this restitution property.

We have seen that if we consider a T norm and the corresponding R implication, they actually satisfy the residuation property, if T is left continuous. And hence, this entire structure becomes a residuated lattice structure. When we looked at residuated lattice structure in one of the earlier lectures, the early part of this course at that time, we were interested in making sure that with respect to the set of fuzzy sets, we were able to obtain as richer structure as possible mirroring what happens in the case of classical set theory.

Now, you will see here, in the rest of the lectures throughout the course, the power of residuated lattices in ensuring many of the desirable properties that we expect on a fuzzy inference system. So, you we have seen the importance or richness of residuated lattices in the context of theory in coming up with good structures with some properties.

But now, we will also see how these properties, this myriad of properties that residuated lattices have, how they are going to impact in applications by ensuring that the inference systems that we have considered have some nice and desirable properties.

(Refer Slide Time: 05:00)

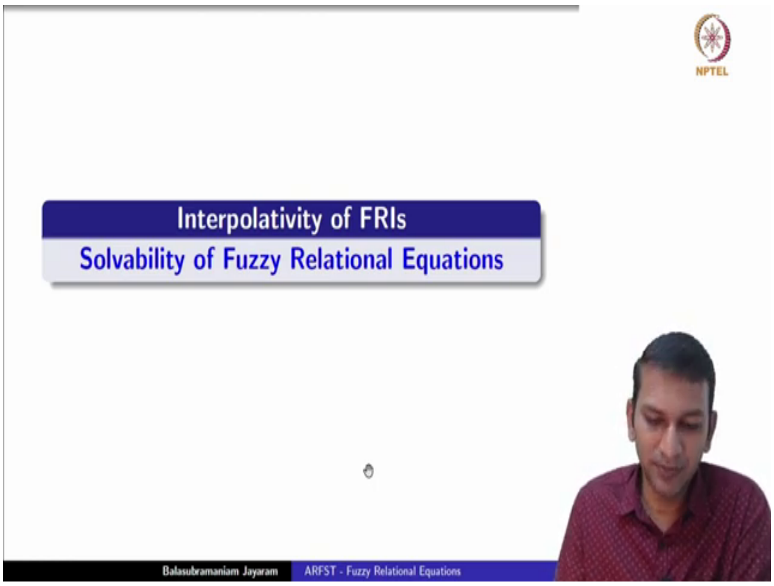


Well, even when we discuss the need for residuated lattice, we saw that it has a lot of properties. A couple of them, we are listed out. You might recall these are some of the properties that we have listed out. If you look at the properties on the second line, in fact, they are effectively some special cases of what we consider or what we have introduced as T F conditionality and I F conditionality.

So, if we consider this as a T norm, then this arrow is a corresponding residuated operator and if you consider the arrow to be the corresponding residuated operator which is the implication, then star could be the operation from which you had obtained the arrow.

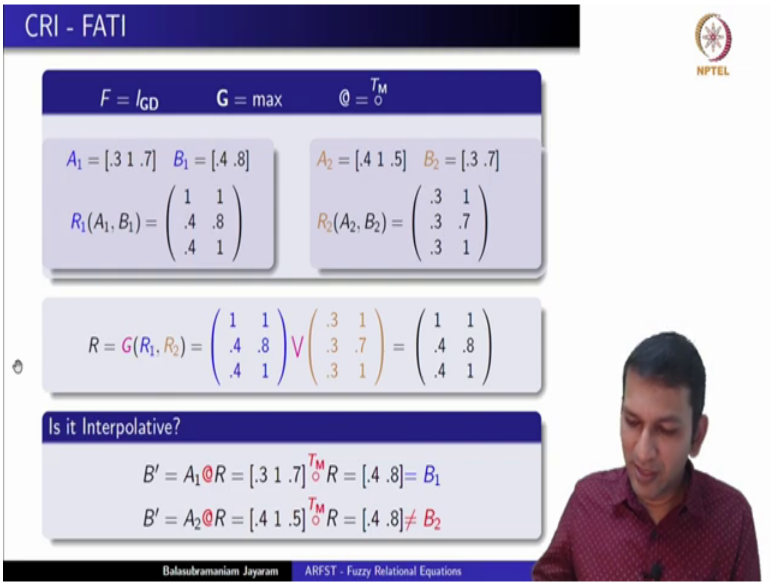
So, this actually once again shows that perhaps it is good to stay within the framework of residuated lattice structure. So, in that sense, we will explore FRIs for multiple SISO rule case in this context, where the operations T and I T are actually coming from a residuated lattice structures, which means of course T is left continuous.

(Refer Slide Time: 06:14)



Now, how is studying interpolativity of FRIs in some sense equivalent to studying the Solvability of Fuzzy Relational Equations.

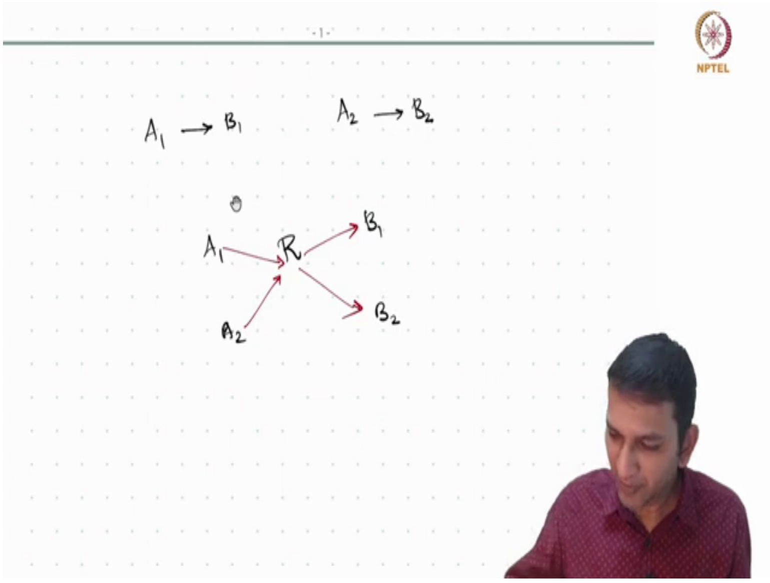
(Refer Slide Time: 06:24)



Let us revisit one of the examples that we have seen in an earlier lecture. So, now, we have a couple of rules, single input single output rules and we have seen that if you use a girdle implication for obtaining the relation and max for aggregation and the sup T composition, we see that given these two rules; A 1 implies B 1 and A 2 implies B 2 these are the relations that you obtain using this operation F.

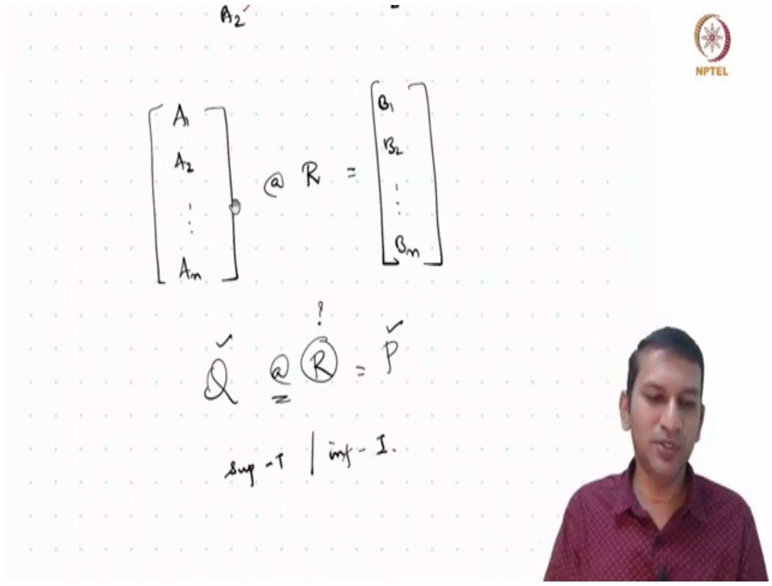
Now, if you aggregate them, this is the relation R you get and we wondered about its interpolativity properties. We saw that if we gave A 1 as the input, then A 1 composed with R, we actually obtain B 1. However, when we gave A 2, this was not the case because we could not obtain B 2.

(Refer Slide Time: 07:19)



Now, what does it mean? We have A 1 implies B 1, A 2 implies B 2. What we want is a single R such that when we gave A 1 to R, then we obtain B 1 and when we give A 2 to R, we obtain B 2.

(Refer Slide Time: 07:55)

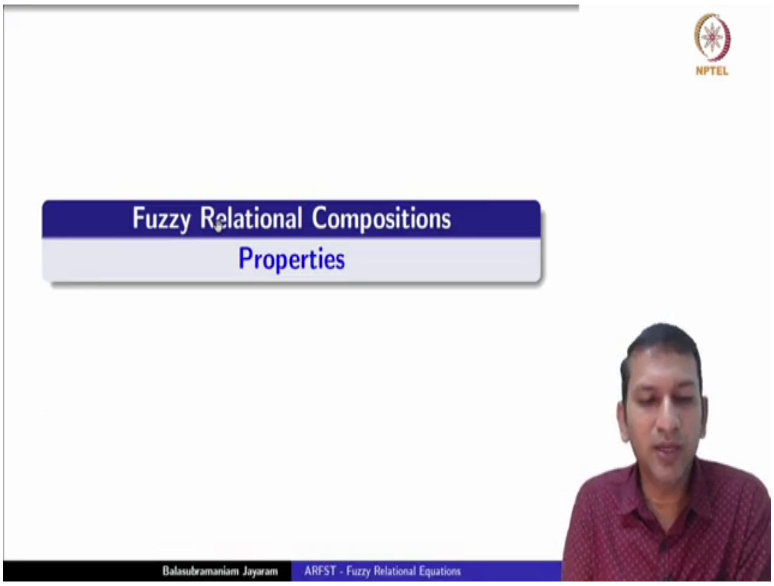


We want a single R and we see here this R does not seem to make the cut which essentially implies that if we put the set of antecedents on one side, note that we are considering these as row vectors and perform the composition, then what we are expecting is the corresponding rows the corresponding consequence.

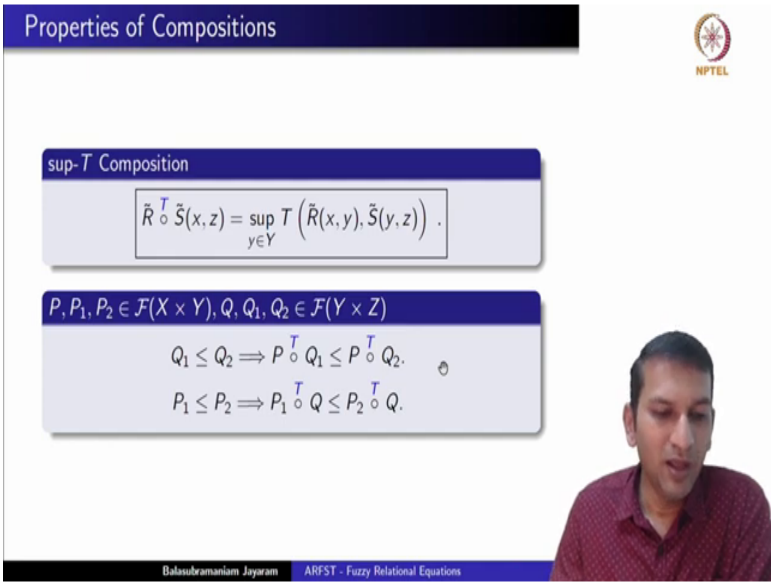
So, now, this entire thing can be looked at as studying this fuzzy relational equation, where Q is known, P is known; these are coming from the antecedents; one may to look at it is that way and what we are interested is in obtaining this R. So, how should we obtain the R? That is the question we are asking.

So, this is much like the matrix equations given A and B, we want to find the next such that A x is equal to B. So, here we are given Q and P and we are interested in finding an R. And this composition of course, we are in the statement for us our interest is from the point of view of fuzzy relational inference, which means we are going to talk about CRI and VKS inference which essentially means we are either talking about sup T composition or inf I composition.

(Refer Slide Time: 09:05)



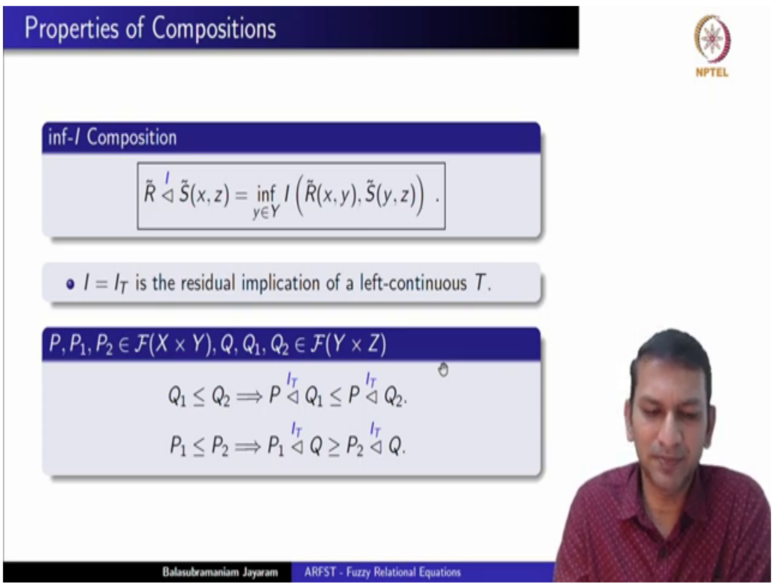
(Refer Slide Time: 09:12)



Well, let us look at some properties of these fuzzy relational compositions. This is the sup T composition, we have seen in one of the earlier lectures, we will discuss fuzzy relational relations that if Q 1 is less than or equal to Q 2, then if you pre multiply pre compose with another P, then what we see is this is the inequality we have every time. It essentially says that it is monotonic in the second variable, of course because of symmetry of T norm of the T norm underlying T norm that we use.

It can also be shown it is monotony in the first variable.

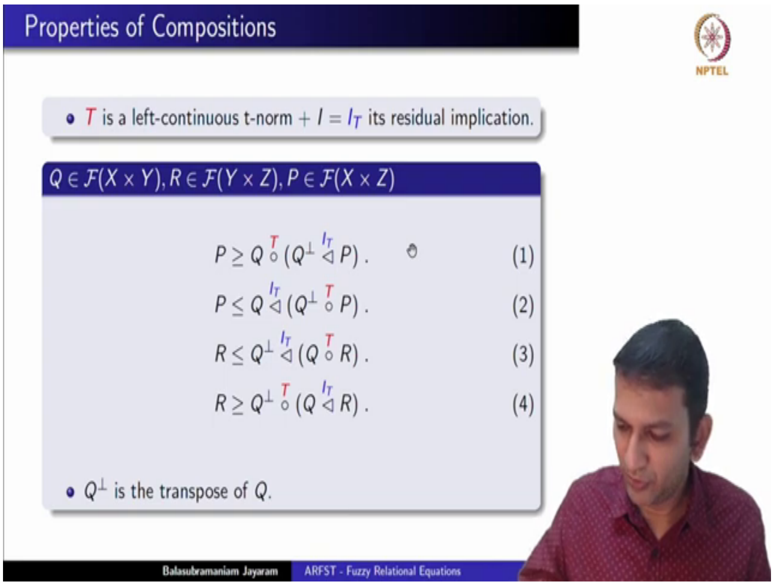
(Refer Slide Time: 09:51)



However, if you look at the inf I composition, if we take I T to be the residual implication of left continuous T. Since we want to discuss in the framework of residuated lattice structure, the rest of the properties we will discuss when T comes T and I T are related to each other as a residual path, which means T is the left continuous is a left continuous T norm and I T is the corresponding residuated implication R implication obtained from I T.

So, let us fix this and what we can see is in the second variable, the inf I T composition is monotonic. However, in the first variable, it is anti-monotonic. Of course, once again this stems from the properties of the implication, the R implication which is a fuzzy implication just mixed monotonic in nature; decreasing in the first variable and increasing in the second variable. Of course, not in a strict sense.

(Refer Slide Time: 10:50)

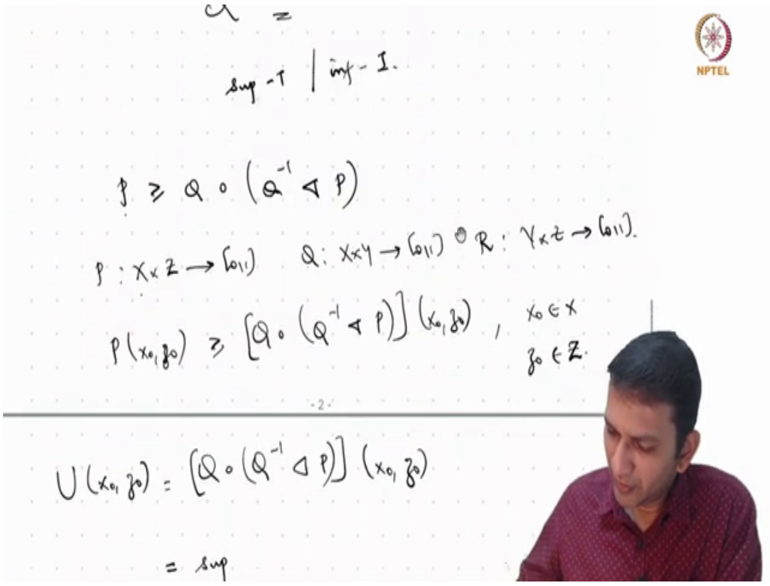


Now, what is interesting is in this setting, if Q is a fuzzy relation on X cross Y; R is a fuzzy relation on Y cross Z and P is a fuzzy relation on X cross Z, then there are these are some properties that we can prove. Note that here it is a nice mix of sup T with inf I composition; of course, we should always remember we are discussing in the context where T and I T are coming from the residuated structure, which means T is a left continuous T norm and I T as its corresponding R implication.

We have seen similar inequalities as properties for general compositions; but these are some special properties when we consider T and I T to come from the residuated lattice structure and we will see how these inequalities will help us in discussing the solvability of fuzzy relational equations. We will among these four inequalities that we have, we will pick up a couple of them and try to prove them.

We will pick perhaps the first and the third one because the variables do change and so do the inequalities. So, allow me to copy this. So, we need to prove P is greater than or equal to Q inverse.

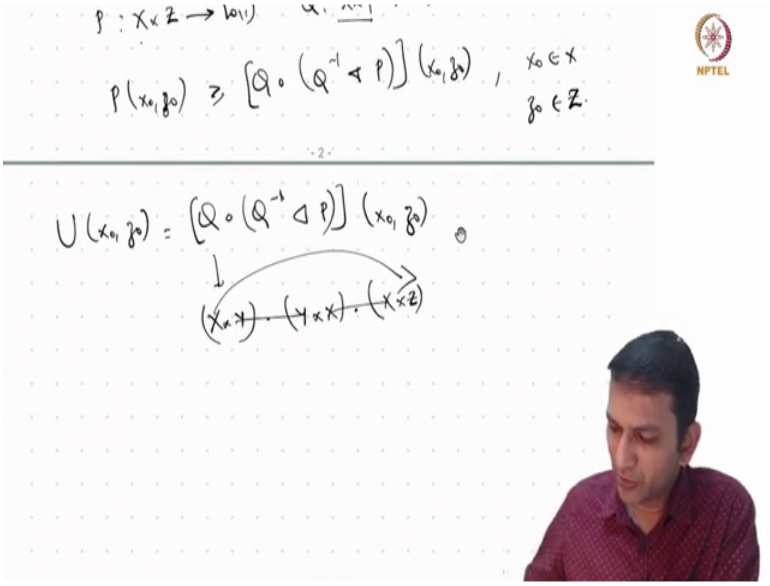
(Refer Slide Time: 12:23)



Since we have fixed the T and I T to be coming from the residuated lattice structure, just to avoid conversion notation, we will not indicate the T or I over the symbols circle in this left triangle. But that will be understood in the context. This is what we need to prove.

Note that P is a function from X cross Z to 0 1; Q is from X cross Y to 0 1 and R if you consider is from Y cross Z to 0 1. So, this is what we need to prove. So, now, let us if you want to prove this note that P acts on two values X and x naught and z naught. So, we need to show this is greater than or equal to Q circle Q inverse for arbitrary z naught from X and z naught from Z. This is what we need to prove.

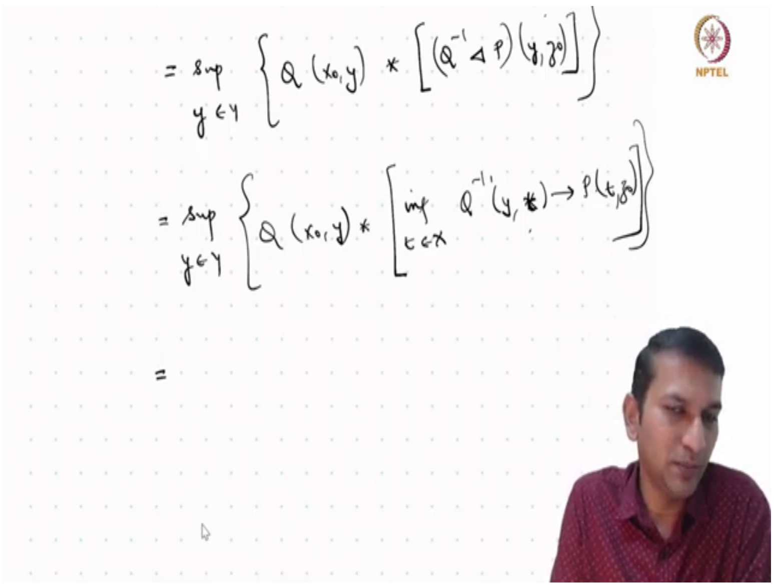
(Refer Slide Time: 13:35)



Now, let us write the right hand side as U of x naught z naught is equal to Q circle Q inverse delta P x naught z naught. Now, let us expand all this. So, this is sup T composition. So, what we have is supremum. Now remember Q is a Q is from X cross Y. So, before that perhaps one thing that we should we can ensure, we should ensure is this remember Q is from X cross Y; Q inverse is the transpose, so it is from Y cross X and P is from X cross Z.

So, now if you see this, this entire U is actually from X cross Z to [0,1] and P is from X cross Z to [0,1]. So, dimensionality wise, we are very much in the table.

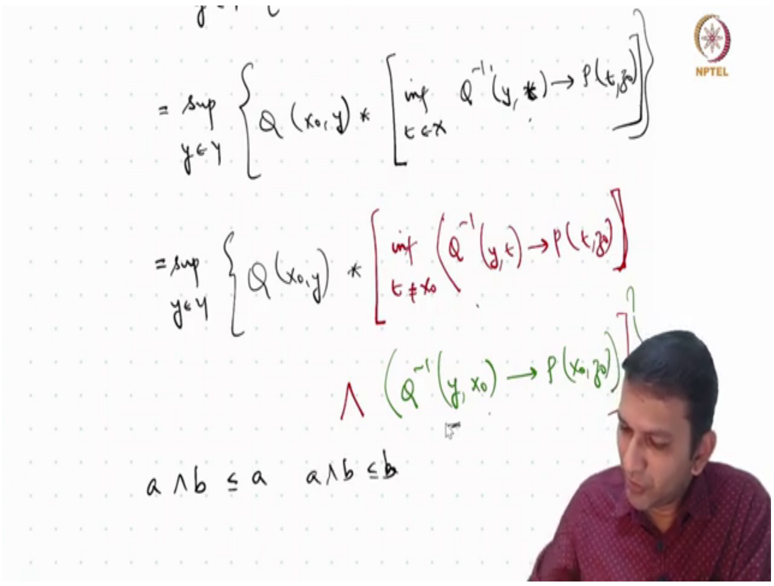
(Refer Slide Time: 14:47)



So, let us expand this is supremum Q is over X cross Y, X and Z are fixed. So, supremum of y coming from Y Q of x naught comma y star Q inverse P of y comma z naught. So, now, once again expanding this Q x naught comma y star. Now, this is the inf I composition because I is the R implication obtained from the corresponding T.

There is not just the infix notation so that notation gets easier to see inf and note that we have to write this as Q inverse implies P of z naught y; Q inverse is from y crosses to 0 [0,1]. So, which means this variable here varies over x. This is what we have. Now, let us do a similar algebraic manipulation as we have done in the previous lecture, where we considered the kernel of A separately; the sub port of the kernel of A separately and the rest of them separately.

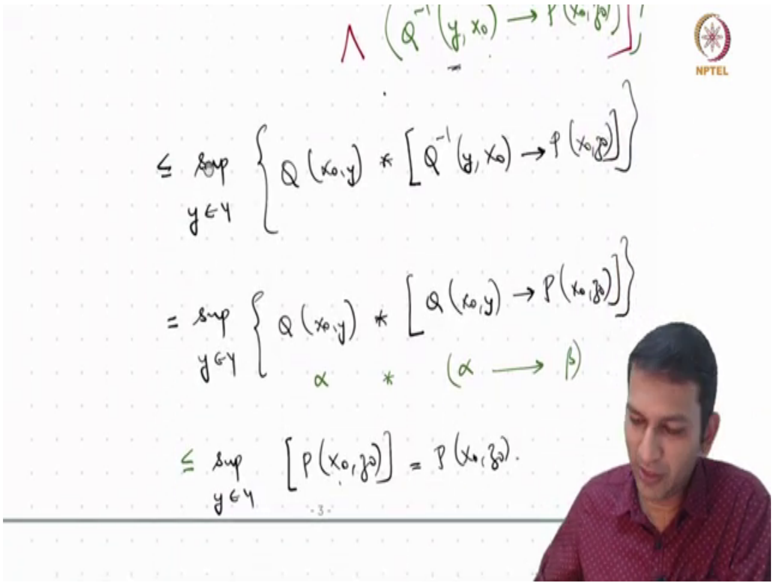
(Refer Slide Time: 16:27)



So, we could write this as supremum over by Q of x naught comma y star, we will divide this into two parts; the part where inf T is not equal to x naught, Q inverse y comma t implies P of t comma z naught (Refer Time: 17:15) the part, where t is equal to x naught which is Q inverse y comma x naught implies P of x naught comma z naught. This is exactly like how we are done in the previous lecture.

So, all we are doing is as t varies over X, we will consider for all the t which are not x naught and when t is equal to x naught, we are considering a separately. (Refer Time: 17:51). Now, if you look at it supremum of star or and here, we know that infimum of these two things is always smaller than this whenever a and b is less than or equal to a; a and b is less than or equal to b. So, using this property here, what we can write this is and supremum is monotonic over this.

(Refer Slide Time: 18:28)



So, this is less than or equal to supremum y element of Y; Q of x naught comma y star Q inverse y comma x naught implies P of x naught comma z naught. Now, this is equal to supremum y over Y (Refer Time: 18:58) Q of x naught comma y star. Look at this Q inverse of y comma x naught is nothing but the Q transpose. So, it is essentially Q of x naught comma y implies P of.

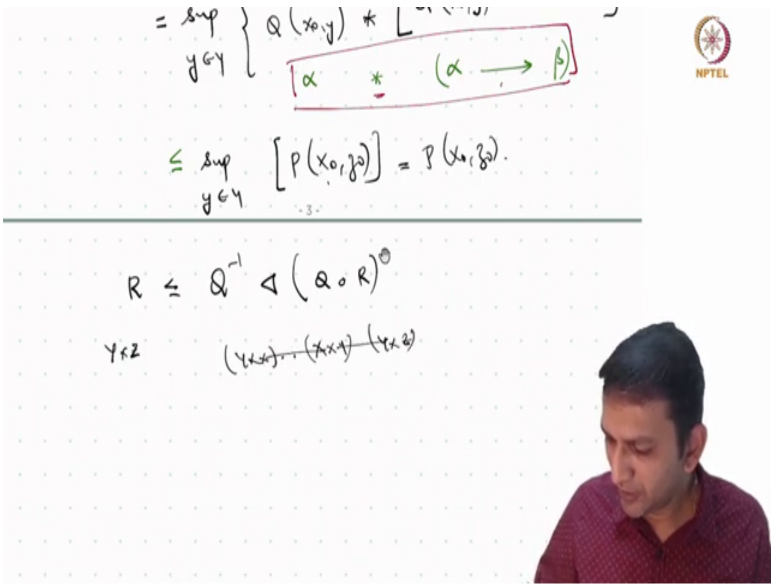
Now, if you take this to be alpha, this to be alpha and this to be beta. So, what we have is alpha star alpha equals beta. Now, we know from the residuated lattice, it is essentially a star a implies now b is less than or equal to b from the residuated lattices which means you could write this entire thing as less than or equal to supremum over y belong to Y P of x naught comma z naught.

But note that the variable that runs is free here is running over the domain y; whereas, here the arguments are actually fixed x naught and z naught which are from different one which means it has no bearing on this value (Refer Time: 20:29) P of x naught comma z naught. So, what we have proven is we started with U of x naught comma z naught which is this value and we have shown that this is actually less than or equal to P of x naught comma z naught.

And since we took x naught and z naught arbitrarily, what we have actually seen is this property plus property which is P is greater than or equal to Q composed Q sup composed sup T composed with Q transpose inf I composed with p. So, we have seen notice one thing this is the equation that we need to understand.

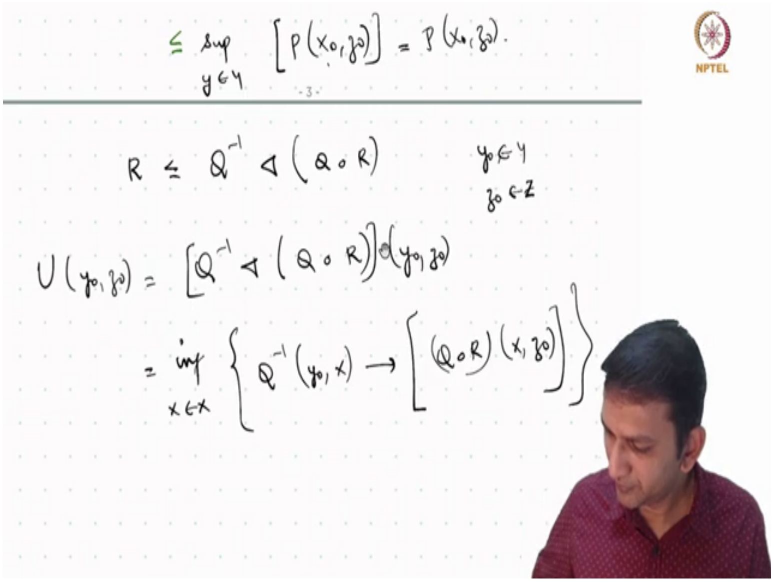
This essentially T F conditionality or T conditionality because if consider this as T norm and this is as an implication, it is the T conditionality in the setting of residuated lattices which has actually helped us in showing this inequality. Of course, we have not seen the use of this inequality which we will see presently. We will take up one more inequality to prove and then, we will go into discussing the solvability of relational equations.

(Refer Slide Time: 21:48)



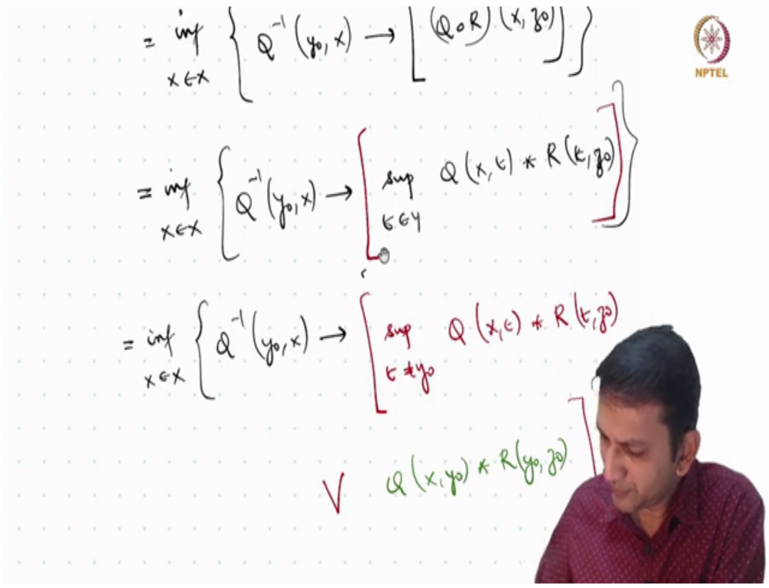
Let us consider the third inequality. Allow me to just writing. This R is less than or equal to Q inverse Q circle R. Now, once again let us look at the dimensionality R we know is from Y cross Z; Q inverse Q is from X cross Y; Q inverse is from Y cross X; R is from Y cross Z. So, now if you look at it this from R to Z which means dimensionality wise we are doing from.

(Refer Slide Time: 22:23)



So, now let us start once again here. So, R acts on some arbitrary y naught belongs to Y and z naught belong to Z. So, now, we have seen that the same thing happens here. So, once again let us look at U of y naught comma z naught is equal to Q inverse Q transpose at the point y naught comma z naught. Let us expand this once more. So, now, what we have is infimum Q inverse, notice it is from Y cross X to [0,1]. So; that means, it varies over X Q inverse of y naught comma x implies Q circle R of x comma z naught.

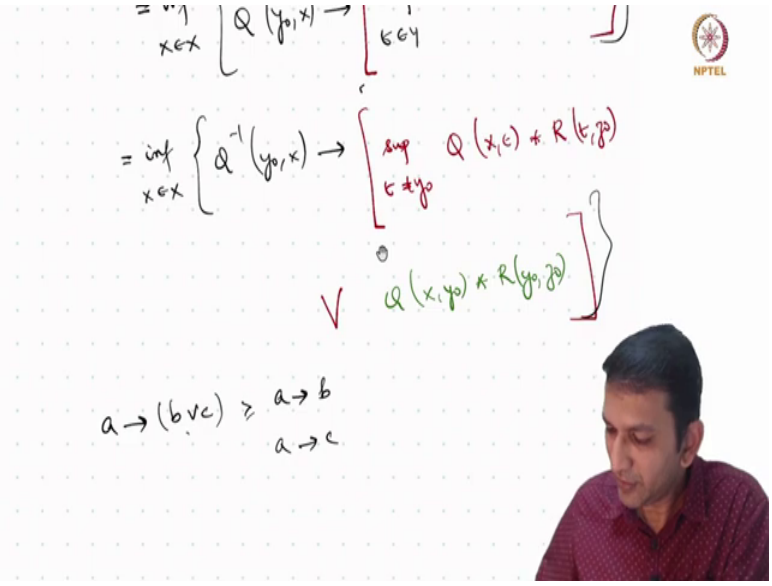
(Refer Slide Time: 23:17)



Now, this is Q inverse of y naught comma x implies (Refer Time: 23:25). Let us once again expand this is supremum over remember Q varies Q is a function relation over X cross Y. So, now, X is fixed, it is the Y that will vary. So, supremum t belong to Y; Q of x comma t star R of t comma z naught.

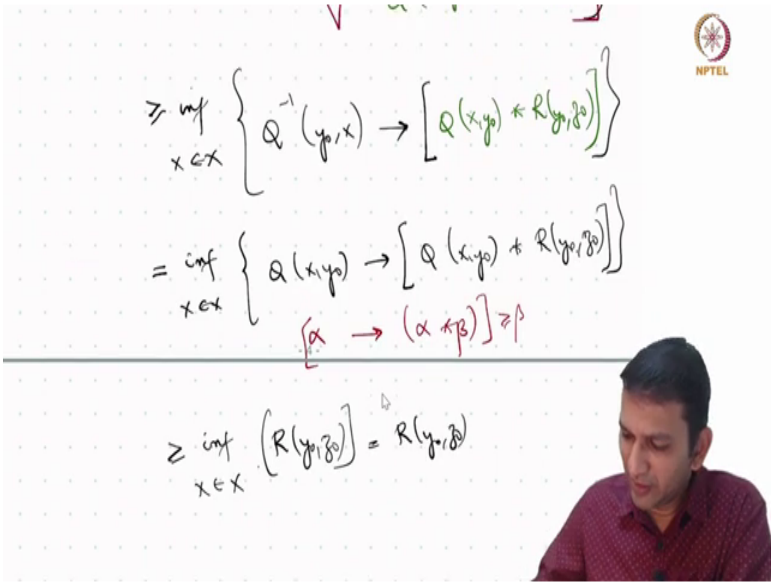
This is what we have here. Once again, we will split this Q inverse y comma x implies supremum over t coming from y t not equal to y naught Q of x comma t star R t comma z naught; join with at that point t is equal to y naught x comma y naught star R of y naught z naught.

(Refer Slide Time: 25:00)



So, with this, close this bracket with. We notice that for an implication a implies b of c, it is increasing in the second variable. So, this is always greater than or equal to a implies b and also, a implies c.

(Refer Slide Time: 25:25)

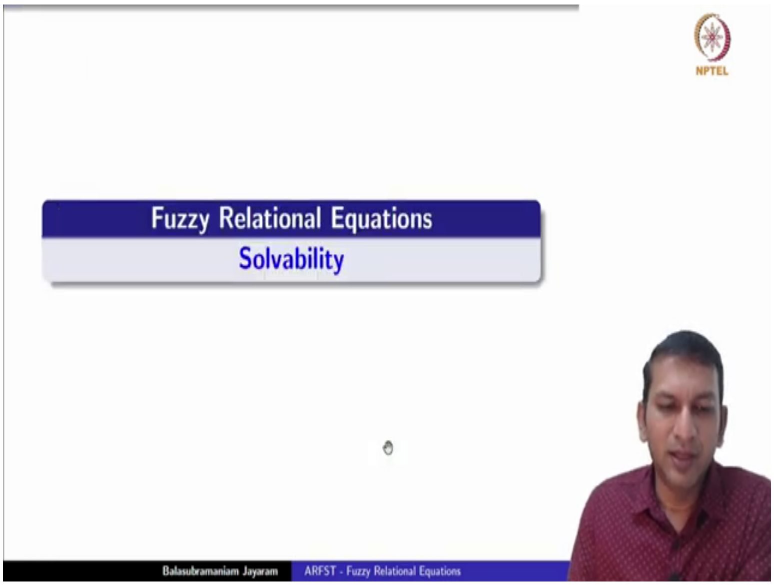


So, using this, we can say this is greater than or equal to infimum x element of X Q inversion of y naught comma x implies the quantity in bracket. Well, we do a similar thing as before if you look at this, this is Q inverse which is Q transpose. You could also write this as Q of x comma y naught. This implies Q of x comma y naught star R of y naught comma z naught.

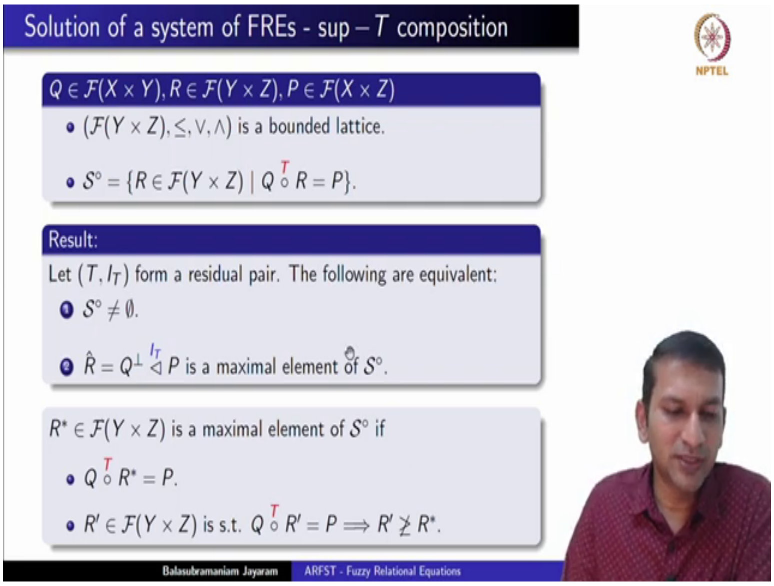
Once again, if you were to look at this as an alpha; this alpha implies alpha star beta, you might readily recall this looks like the I F conditionality which we know is greater than or equal to beta in the case of residuated lattices. So, what we obtain here is that this is actually greater than or equal to inf x belong to X R of y naught comma z naught. Once again, the running variable is X, it has no bearing on the arguments that R acts upon which means this is equal to R upon y naught comma z naught.

To note that y y naught and z naught they are actually arbitrarily chosen elements from Y and Z. So, that means, what we have shown is this quantity is bigger than R of y naught z naught, essentially we have shown that this inequality is well. We have seen the inequalities 1 and 3; the inequalities 2 and 4 can be proven similarly. Now, let us look at how to put them to good use.

(Refer Slide Time: 27:24)



(Refer Slide Time: 27:29)



Let us discuss the solvability of fuzzy relational equations. Now, once again Q is a fuzzy relation on X cross Y; R is a fuzzy relation on Y cross Z and P is a fuzzy relation on X cross Z and let us look at a system of fuzzy relation equations, where we use the sup T composition. Now, if we consider a set of all fuzzy relations over Y cross Z, then with respect to the usual component wise ordering and the max and min operation, we see it is a bounded lattice.

The matrix with all ones is the top element; matrix with all zeros is the bottom element. Now, let us indicate by this S circle, the set of all relations are coming from fuzzy relations over Y cross Z such that for the fixed Q and P, Q sup composed sup T composed with R is equal to P. So, that means, given a Q and P, we are picking all those R's such that Q circle R is equal to P.

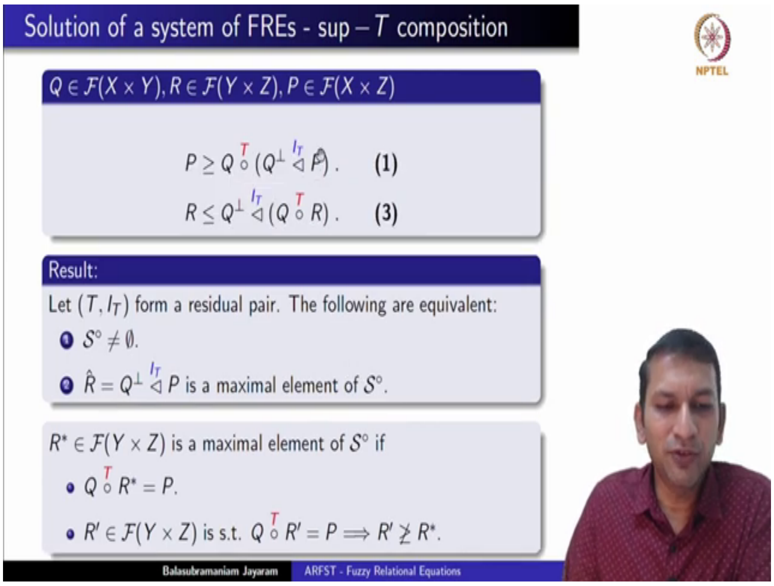
This is the interesting result that we have if your T and I T, they form a residual pair; that means, the T norm that you use in the sup T composition, if it is a left continuous T norm, if you consider the corresponding residuation of it, the following are equivalent. What does it say?

This space which is essentially the solution space means given Q and P, all those fuzzy relations R and Y cross Z which satisfy this equation that is non-empty, if and only if, this particular relation R cap which is given as Q inverse inf I T composed with P is a maximal element of this set. So, it is a very interesting result. It says that this solution space is non empty if and only if, this relation that you obtain from Q and P itself much like in the case of invertible matrices how we obtain X as a inverse b.

So, it says that in such a case this relation that you obtain, of course by using sup I inf I T composition between this the transpose of Q and P, this relation not only should be an element of S circle which means not only should it solve this equation; but it should also be a maximal element of this space. What we understand by maximal element of this space?

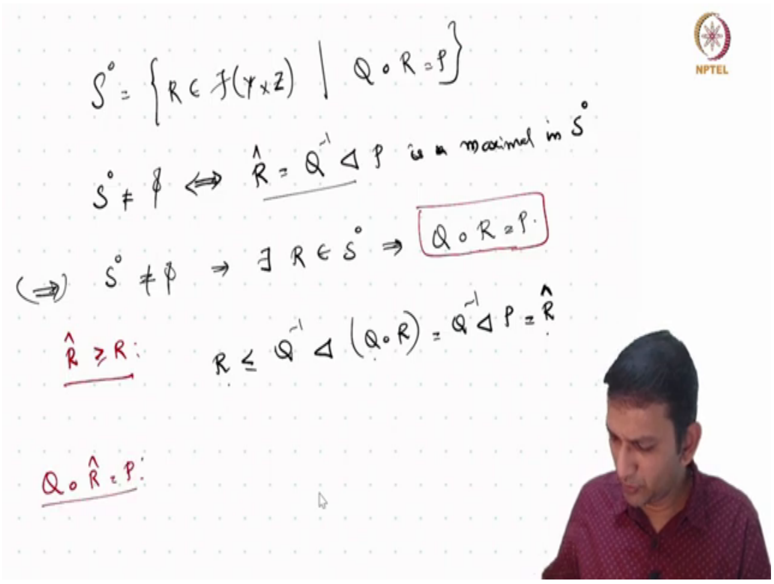
So, if we consider the solution space and R star coming from the set of all possible relations on Y cross Z, it is a maximal element of S circle if first of all it should be a solution to this equation. And if there exist some other solution R dash, then this R star is not smaller than that. Note that it is a maximal element; it is not a maximum element. So, if there can exist other solution. But however, no solution can be greater than this. So, this is the result that we have.

(Refer Slide Time: 30:44)



Let us prove this result. Towards proving this result, we will make use of these two inequalities which we have actually proven 1 and 3.

(Refer Slide Time: 30:54)



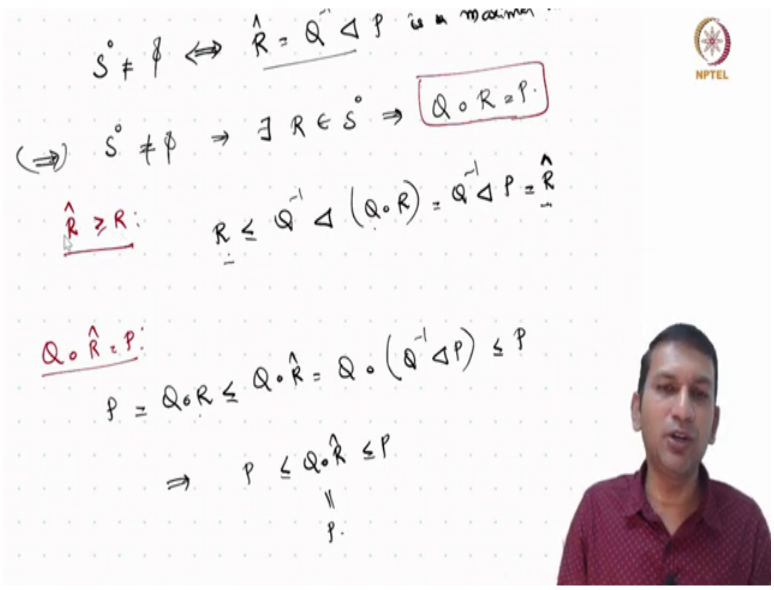
So, now, what do we have? We have S naught which is set of all R from f of Y cross Z such that Q circle R is equal to P. Now, the reason says that if S naught is not equal to phi, if and only if R cap given like this is a maximum or we just say it is maximum in S naught. So, we need to prove two things that it is a solution, this R cap is a solution and it is also the maximal solution. One way is very clear.

If R cap is a maximal element of S naught, then it is also an element of s naught which means it is a solution for this equation. It is essentially means S naught is not empty. So, let us prove 1 implies 2. So, let us assume that the forward direction S naught is not phi, this implies there exists an R and S naught which implies Q circle R is actually equal to P. So, this is what we have to prove.

First let us prove that R cap is greater than or equal to this R cap. Now, look at this third equation. We know that for any R, this is true; R is less than or equal to Q inverse Q circle. But now, what is Q circle R? Because R is the solution. It is P, this is Q inverse; but what is Q inverse P? Q inverse delta P? This is nothing but R cap.

So, what we see is for any R which is a solution R is less than or equal to R cap; that means, if R is a solution R cap, R does not definitely dominate R cap. But however, we do not know whether R cap itself is a solution R naught. So, let us prove that R cap itself is a solution. That means, Q circle R cap is equal to P this is what we need to prove. Now, look at the equation number 1.

(Refer Slide Time: 33:38)

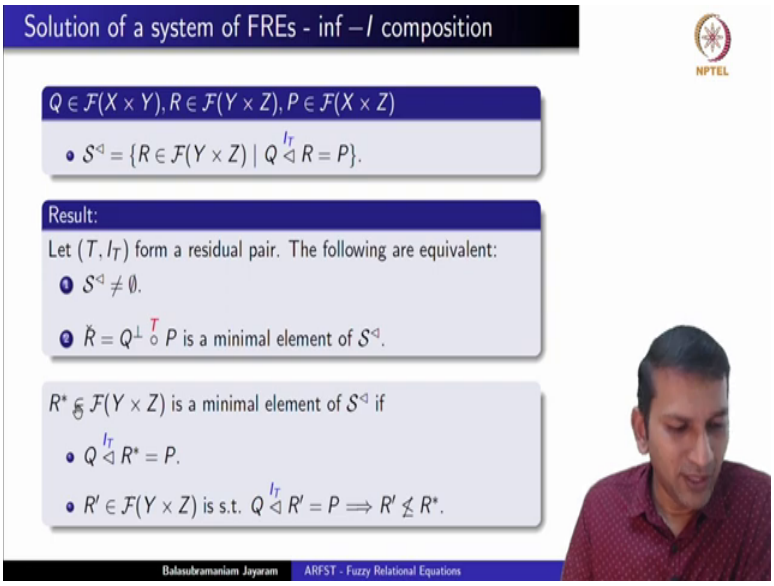


So, now Q circle R cap is equal to Q circle Q inverse delta P. But now if you look at the equation the inequality 1, we see that this is actually less than or equal to P. But we also know that any R given R is smaller than R cap and we have seen by the monotonicity of sup T composition Q circle, R will be smaller than Q circle R cap. But what is Q circle R?

It is nothing but P because R is a solution of this equation the relational equation. So, from here, what we get is Q circle R cap is both less than or equal to P and greater than or equal to P which means this is actually equal to P. So, what we have shown is that R cap is actually a solution and also, that it if there exists some other solution, it definitely does not dominate R cap.

So, essentially, we have proven this result that the solution space for sup T composition, where the Q and P are fixed with R unknown does have a solution, if and only if this R cap which is Q inverse delta P is a maximal element of the solution space.

(Refer Slide Time: 35:06)

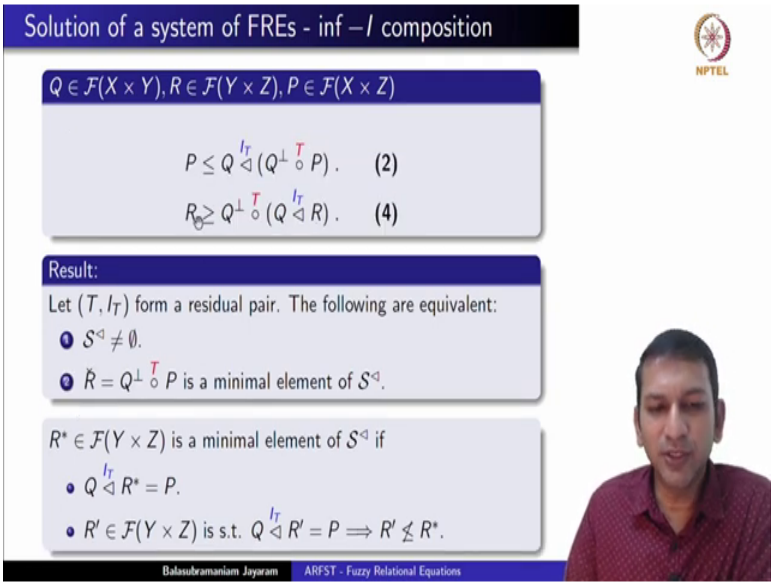


Considering the same set of relations moving from X cross Y to [0,1]; Y cross Z to [0,1] and X cross Z to [0,1]; let us now discuss the solvability of equations involving in inf I composition. Once again, we are fixing Q and P and finding out all those R which are fuzzy relations of Y cross Z such that Q delta R is actually equal to P. Once again, we are in the setting of residuated lattices; the T and I T form a residuated pair, a similar kind of result as earlier can be shown. .

So, now, this solution space is non empty, if and only if you see here we are introducing the sub T composition, Q transpose with sup T composition P which we denoted as R check is an element of this space and it is also a minimal element of this space. What do we mean by this? A minimal element of this space means a fuzzy relation on Y cross Z, such that it also satisfies the equation.

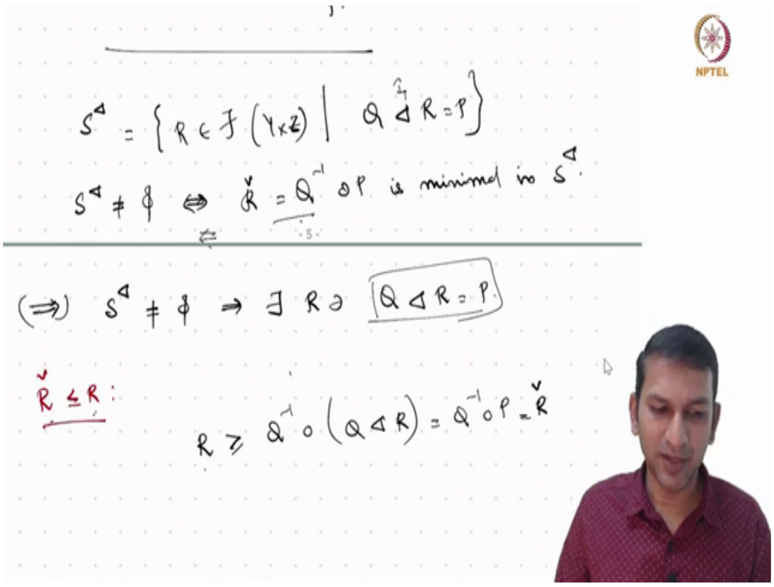
First of all, it should satisfy this fuzzy relational equation and if there exist some other solution to this equation, then that is definitely not bigger than R dash. So, that that solution is not smaller than R star that we are considering here. So, this is the dual of the maximum element ok.

(Refer Slide Time: 36:44)



Once again, we will prove this. To prove this, we will make use of the other two inequalities that we have considered.

(Refer Slide Time: 36:52)

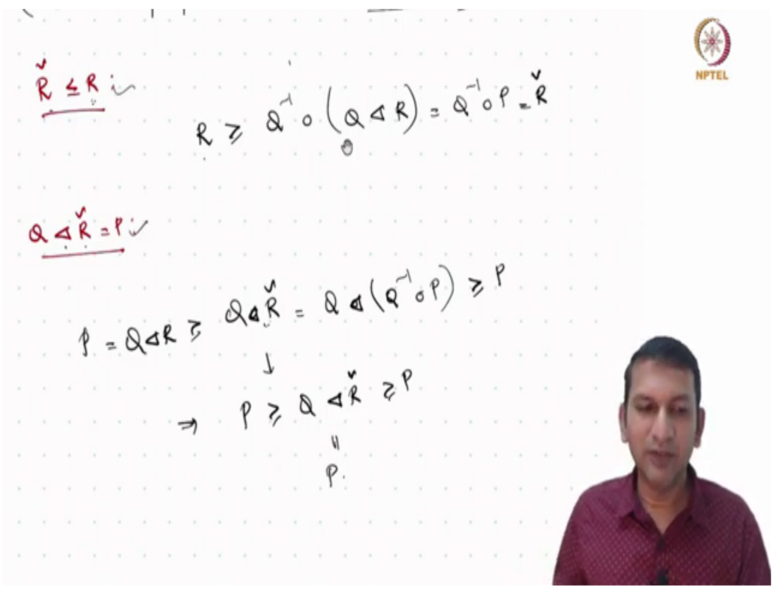


So, what we have is S delta which is set of all R belonging to f of Y cross Z such that Q delta R is equal to P. Of course, the I here is I P. Now, the reason says that S delta is not equal to phi; if and only if R check is given as Q inverse circle P is minimal on S delta. Once again, the reverse implication is clear because if R check is minimal in S delta, it is also an element in S delta which means it will satisfy the fuzzy relational equation which means S delta is non empty.

So, let us prove the forward equation. Once again, we need to prove that if there exist that for given norm is S delta is not equal to phi, this implies there exists some R such that Q delta R is equal to P. What we need to prove here is R check is smaller than any such R that we can considered. To the same, consider the second inequality. Well, what does the second inequality say?

It says that R is greater than or equal to Q inverse circle Q delta R. But note that Q delta R is actually equal to P which means this is Q inverse circle P; but Q inverse will be is essentially R check. So, what we have shown here is for any solution R, R check will be smaller than that or at least R check, R does not go below R check.

(Refer Slide Time: 39:05)

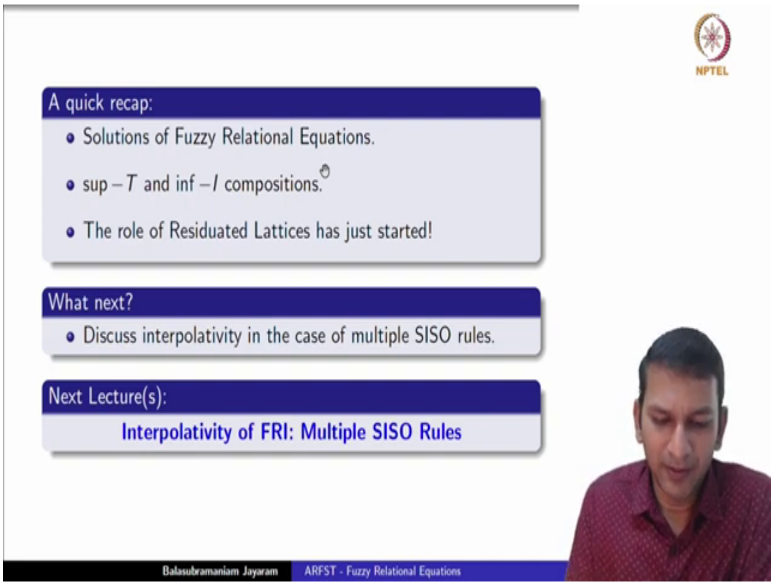


So, what we need to show is second thing that R check is in fact a solution; that means, Q delta R check is equal to P. So, let us look at what is Q delta R check. This is nothing but Q circle Q delta Q inverse circle Q; but now, if you look at the first inequality, it is exactly the same Q delta Q inverse circle P. We know that this is greater than or equal to P.

But we know once again that the inf I composition, where I is I T, residual implication. It is also increasing in the second variable, since R is bigger than this, what we have is Q delta R is greater than or equal to Q delta R check. But what is Q delta R? Because R is a solution to this; this greater than is equal to P.

So, now what we have here is P is greater than equal to Q delta R check greater than or equal to P from which we get that is in fact equal to P. So, that means, R check is a solution and also if there exists some other solution, then it does not go below R check. So, essentially, we have proven that given Q and P, the conditions on R or the solution space so that Q you can find the relation R such that Q circle R is equal to P or Q delta R is equal to P.

(Refer Slide Time: 40:39)



Well, we have looked at solutions of fuzzy relational equations so far and we have seen specifically for the sup T and inf I composition because our the context that we have set is in the is coming from fuzzy relation inference and we are interested in looking at interpolativity there.

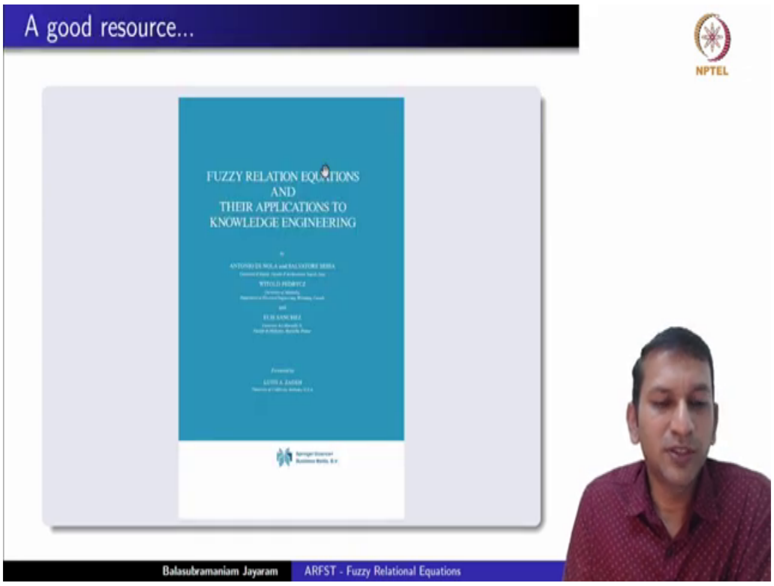
We have seen that residuated lattices play an important role. At the beginning, we asked is the role of residuated lattice over. We have seen that in fact residuated lattices with the powerful, the rich properties that they have, the rich structure that they possess they are able to supply us with all the properties that we require to be able to prove some key results. Oh, what next? We would like to discuss interpolativity in the case of multiple SISO rules.

This is the link part that we have actually discussed. As was shown in one of the slides earlier in this lecture, if you want to discuss about interpolativity, we have the set of rules the antecedent and consequents AIS and BIS, if you take them to be matrices Q and P, what we are interested is in finding R such that when you keep these rows essentially row wise composition should give a corresponding rule that is what we are interested in.

And in the context of FRIs we are interested only in sup T and inf I composition and that is what we have discussed in this lecture. The solvability of fuzzy relational compositions was a relational equation when the compositions are essentially sup T and inf I.

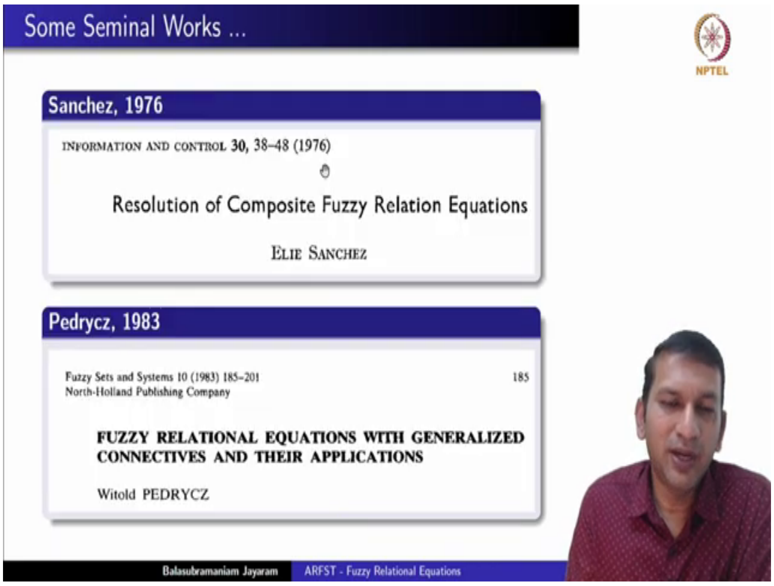
However, the T and I, they actually are related to each other as a residuated pair. So, they come from a residuated lattice structure. In the next lecture, we will make use of this results discussing the solvability of fuzzy relation equation and see how we can relate them to the interpolativity of the corresponding FRI whether it is a compositional rule of inference or the BKS inference. This will take up in the next lecture.

(Refer Slide Time: 42:56)



A good resource for that deals with fuzzy relational equations itself specifically what we have seen in this lecture is the book by Antonio Di Nola Salvatore Sessa, Pedrycz and Sanchez; it is a edited volume.

(Refer Slide Time: 43:15)



We would also like to highlight some similar works which have which are related to fuzzy relational equations; the one by Sanchez, way back in 1976 and also, the work by Pedrycz which is back in 1983. See thus, these are two similar works which dealt with fuzzy relational equations and as you can see they also give a inkling as to where they can be useful. So, glad that you could join us for this lecture. Hope to meet you soon in the next lecture.

Thank you again.