Approximate Reasoning using Fuzzy Set Theory
Prof. Balasubramaniam Jayaram
Department of Mathematics
Indian Institute of Technology, Hyderabad

Lecture - 40
Interpolativity of FRI - Single SISO Rule

Hello and welcome to the 2 of the lectures. This week 8 of the course titled, Approximate Reasoning using Fuzzy Set Theory, a course offered over the NPTEL platform.

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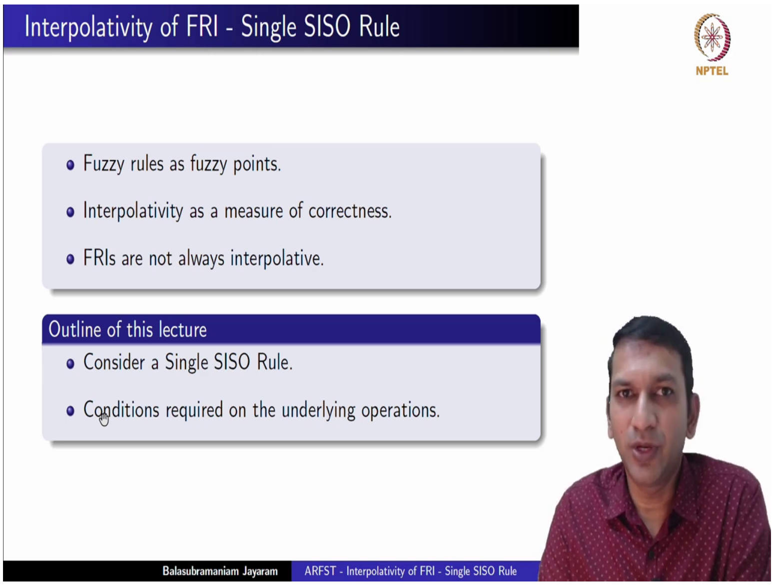

In the previous lecture, we have seen that fuzzy rules can be viewed as pairs of fuzzy points. And under this context we could look at the approximation offered by a fuzzy inference system, a measure of correctness of it could be thought of in terms of its interpolativity property.

We have seen at least in the case of FRIs, that they are not automatically interpolative. In this lecture we will begin our quest towards exploring when FRIs can be interpolative. In this quest we will start with the simplest of them all, a single SISO rule that is we consider that the fuzzy rule base has only one rule and its single input single output rule.

We will see the conditions required on the underlying fuzzy logic connectives to ensure that this inference is interpolative.

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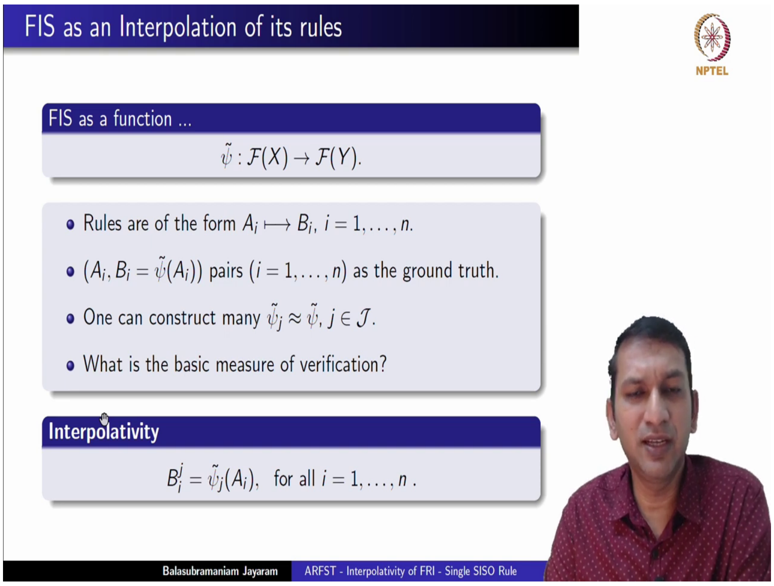

Note that we are looking at a fuzzy inference system as an interpolation of its rules, because we know that a fuzzy inference system it approximates any given function by overlapping patches of its rules. So, you can look at FIS itself as a function from F(X) to F(Y), we are looking at rules of the form A i implies B i as in fact, pairs of data points pairs of fuzzy sets A i, B i. Such that B i is actually the output obtained from the fuzzy inference system psi tilde, when A i is given as the input.

Rules are considered as the ground truth for us in the inference system, we are only looking at them as pairs of fuzzy sets. Clearly one can construct many such fuzzy inference systems, many such functions from F(X) to F(Y). The question now then is what is the basic measure of verification? Whether this really approximates the system function that we are trying to approximate?

We have seen that one good measure is that of interpolativity. So, pick among all these families, all these functions psi j tilde, that psi j tilde or those psi j tildes at which they are interpolative; that means, at A i they actually give B i.

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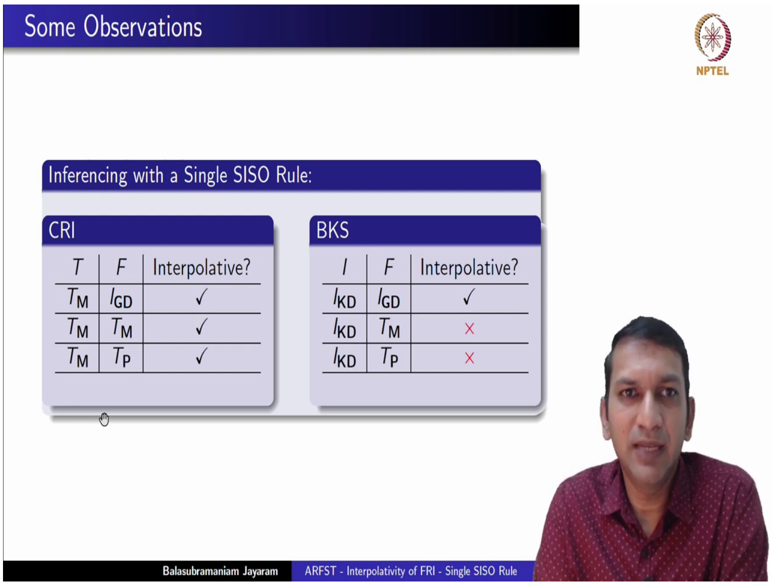

We have made some observations, in the case of single SISO rule if we are using the CRI inference, the composition rule of inference mechanism we have seen that some pairs of operations do make it interpolative, at least for the fuzzy sets that we have considered.

For instance, if the T-norm that is used in the inference in the composition is the minimum T-norm and if we use the Godel implication to obtain the relation between the antecedents and consequence, in this case a single rule; that means, capturing the rule into relation using a Godel implication.

We saw that for this pair of operations it is interpolative for the fuzzy sets a and B that we have considered. Similarly, if we consider both of them to be the T-norm, minimum T-norm then the it is interpolative or if we consider min T norm for the composition, the product T-norm for capturing the relation and the rule, we saw that it is again interpolative.

In the case of BKS, we have seen that if I were interpreted as Kleene-Dienes and F were interpreted as the Godel implication it is interpolative at least for the fuzzy sets a and B that we considered. However, for another pair of combinations where we fix the implication the with the Kleene-Dienes but varied the function, F which gives us the relation between the rule between the antecedent and the consequent.

If you vary them as minimum T-norm or the product T-norm even for the fuzzy sets a and B that we have considered to form the antecedent and consequent in the single SISO rule that we have considered, it is not interpolative. This is what we would like to investigate in this lecture.

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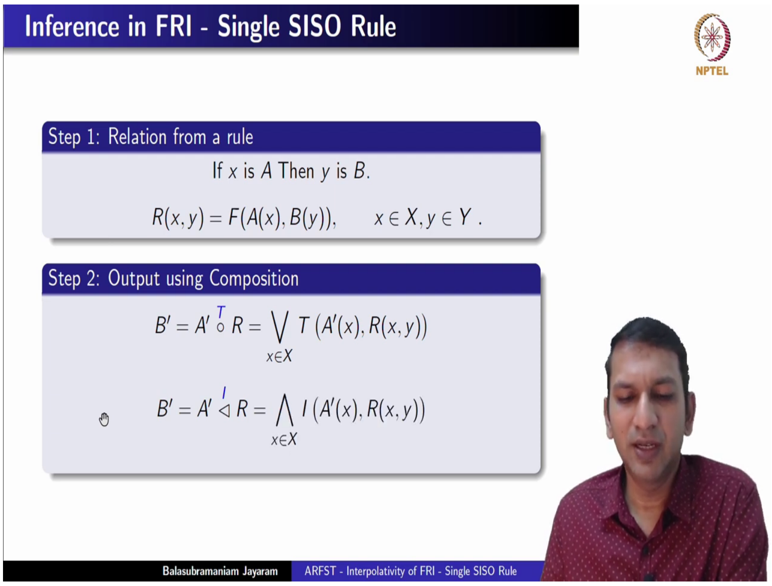

Note that, in the case of an FRI with the single SISO rule we have just this rule that if x is A, then y is B. We use an operation F to obtain the relation R from x cross y to 0, 1 as F of A x comma B y and the output of for a given input A dash is obtained by composing it with this R.

So, you could use the CRA, the compositional rule of inference where we use the sup T composition, this is how it looks like the final formula or we could also use the BKS inference, which is essentially using the BKS product the Bandler Kohout of product composition, where I is an implication and this is how it looks like.

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Well, let us look at a single SISO rule and a singleton input now.

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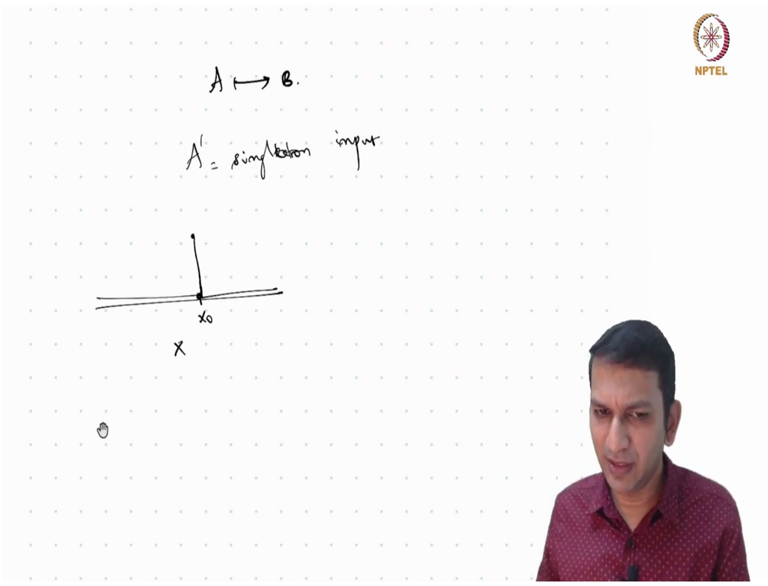

So, let us assume that A implies B is what we are given and A dash is in fact, a singleton input.

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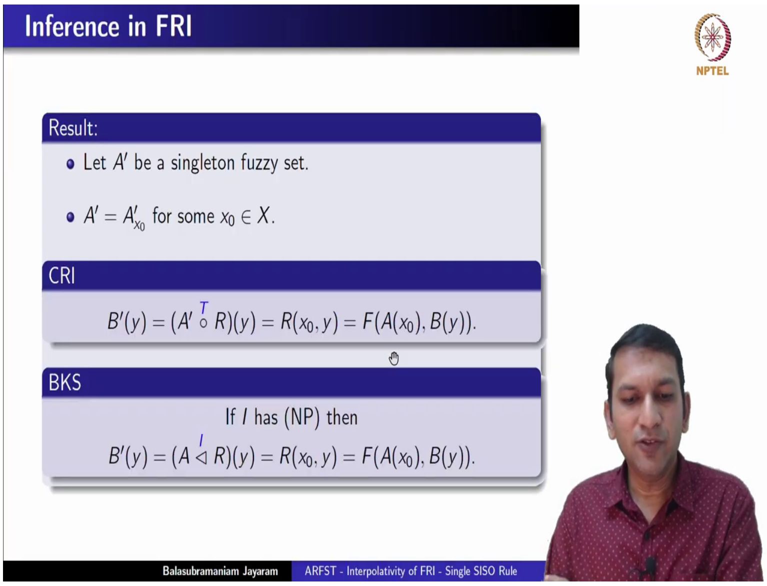

That means, what we have is A dash is fuzzyfied at some point x naught element of X.

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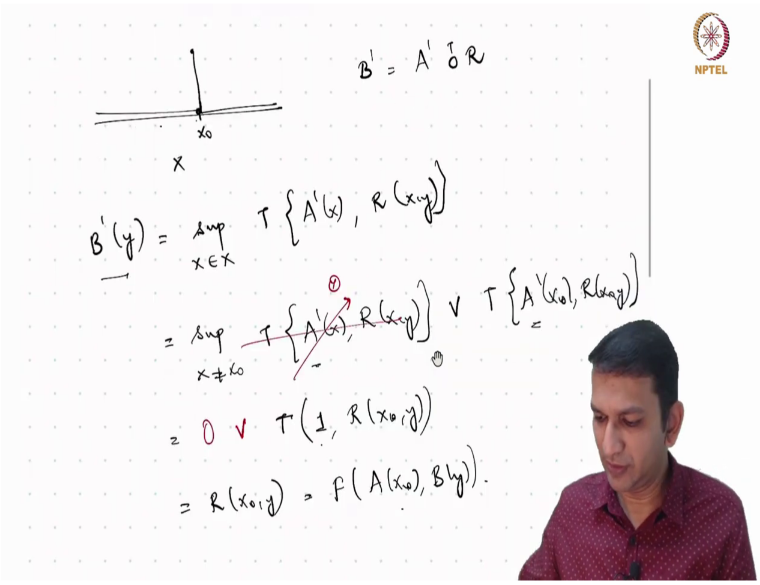

That means, if you look at it this is your X, this is your x naught, it is 0 everywhere else except at this point where it is 1. So, this is the fuzzy set that we are consider. Now, let us look at what happens at this point if we do. So, we are looking at CRI, B dash is A dash circle T R. So, B dash is a fuzzy set on y.

So, at a particular point it look like this, supremum x element of X, T of A dash of x comma R of x, y. Now, what we can do is we can write it like this the supremum x not equal to 0 T of A dash of x comma R of x comma y joint max with, when x is equal to x naught.

So, when x is equal to x naught is A dash of x naught comma R of x naught comma y. Note that A dash is singleton fuzzyfied; that means, outside of x naught, when X is not equal to x naught this is in fact, 0. And for a T-norm when one of the argument is 0, the entire thing become 0. So, this is 0 joined with T of when A dash at x naught is in fact, 1, 1Rr of x naught comma y.

Clearly this is equal to because, one is the left neutral element both left and right neutral elements for the T-norm T what we have obtained is R of x naught comma y which is nothing but F of a at x naught comma B of y. So, you see that B dash of y is nothing but the membership value that x naught takes in a suitably modified on the corresponding membership value that y takes in B that is your B dash of y.

So, if you are looking at this CRI B dash of y is A dash composed with R at y which is R of x naught y which is equal to F of a x naught comma B of y this is in the case of CRI what happens in the case of BKS, let us check that out.

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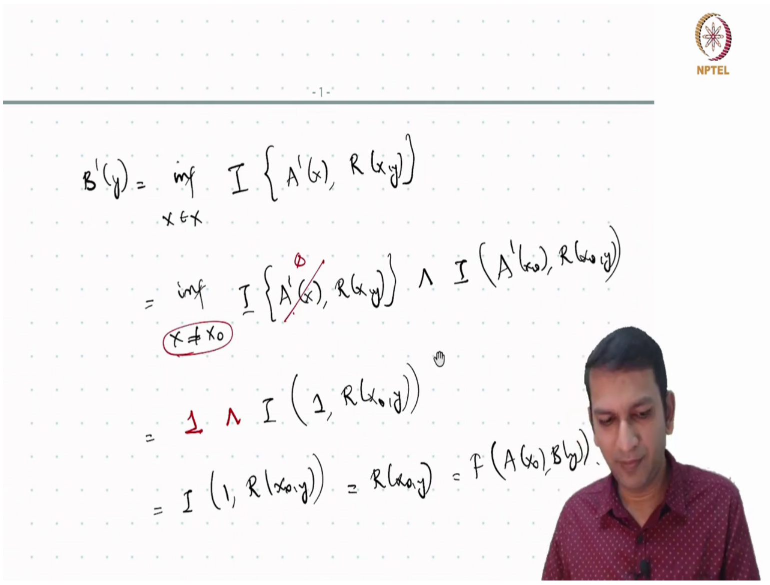

So, in the case of BKS, B dash of y is given as infimum x element of X, I of A dash of x comma R of x, y. Once again we can split it like this, infimum x not equal to x naught I of A dash of x comma R of x, y meet, ok anyway, I of A dash at x naught comma R of x naught comma y.

Now, note that once again at this point when X is not equal to x naught A dash of x naught is 0 R of x, y definitely will can have some non-zero values, but what we do know is I is an implication I of 0 comma x is 1, which means this entire thing becomes 1 for every x not equal to x naught and what we are left with is this I of A dash of x naught, which is essentially at this point it is 1, 1 comma R of x naught comma y, which is essentially I of 1 comma R of x naught comma y.

Now, if I has the neutrality property; that means, 1 is the left neutral element of I, then essentially this also becomes R of x naught comma y, which is F of A at x naught comma B of y. So, in the case of BKS, again if the considered I in the composition, if it has the neutrality property, then B dash of y is nothing but F of A x naught comma B of y, essentially it means the membership value of x naught in A, the antecedent of the rule is used as the similarity value to modify the consequent B.

So, you see here the moment we have a singleton input, we are essentially making FRI look like a similarity based reasoning scheme ok.

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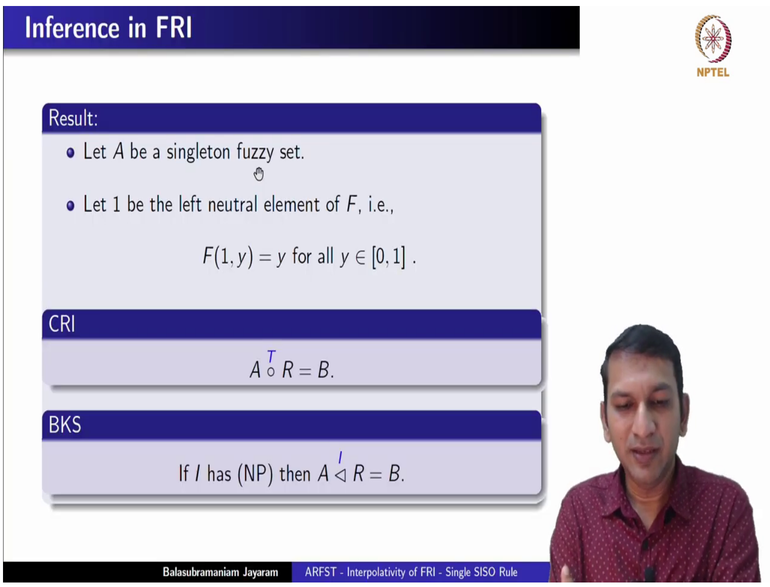

Now, with this we can have a pretty simple trivial result, that if the antecedent that we considered original if it were a singleton fuzzy set. So, the antecedent is a singleton fuzzy set and if 1 is a left neutral element of F that is F of 1 y is equal to y for all y, then what we obtain is this result in the case of CRI. That is A composed with R is equal to B, which means we do obtain interpolative. How do we obtain this?

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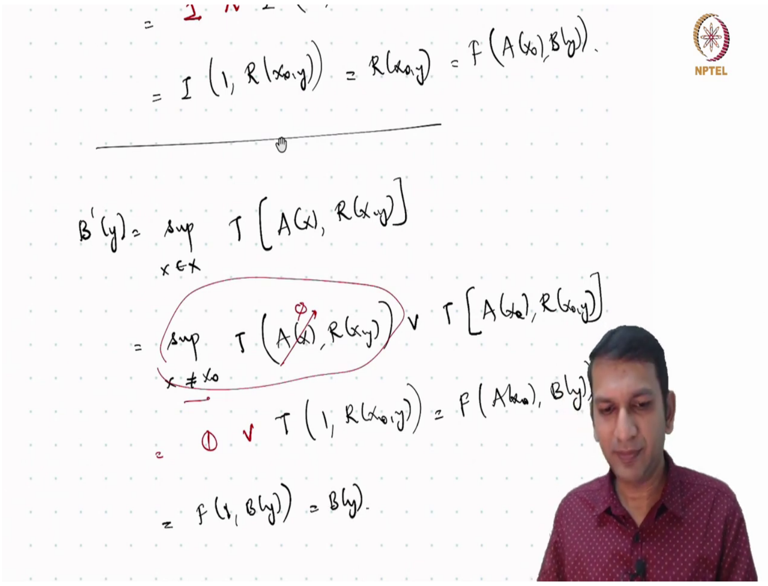

Let us look at this, we are looking at B dash of y, once again this supremum x element of X, T of now A dash is A of x comma R of x, y. Once again we split this as X not equal to x naught, remember A dash is a singleton fuzzy set without loss of generality we can take that domain, the point x naught at which it attains normality 1 and rest all places it is taking the 0 membership value. T of A of x comma R of x, y.

So, T of A at x naught comma R of x naught comma y, once again at this point it is 0 for every such x equal to x naught. So, this entire thing becomes 0, joined with T of 1 comma R of x naught comma y. We have seen that this is nothing but since one is the neutral element of T is nothing but F of A x naught comma y and A x naught it is 1. So, we have F of 1 comma y here 1 B of y. So, if F has the neutral (Refer Time: 12:54) property, that if 1 is the left neutral element of F, then we actually obtain this B of F.

Similarly, we can show just following the same procedure step by step, we see that if I the implication used in the BKS inference does have the neutrality property. That means, one is the left neutral element, then we see when the antecedent A is a singleton fuzzy set, then we do have interpolativity. Means, if you give A dash as A, the singleton fuzzy set which is the antecedent, then we obtain the B could be any fuzzy set not necessarily singleton, ok. So, this is the case with singleton fuzzy set.

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Let us consider the case where we have a non singleton input.

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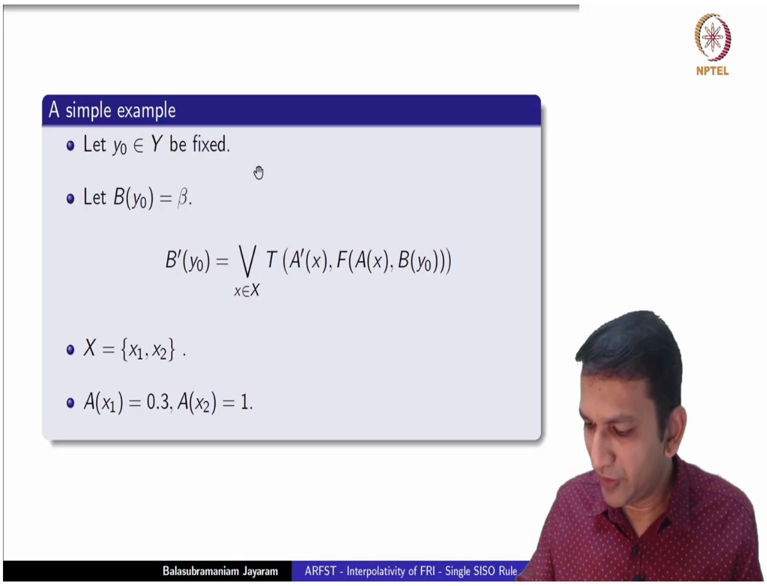

Towards helping us in this, let us look at a simple example. Let us fix a y naught element of Y and we are still in the single SISO rule case, let us look at the membership value of y naught in B, the consequent of the rule that we are considering. Let us assume this to be beta.

Now, so B dash of y naught is in fact given as follows. So, we fix y naught, it is supremum over x, the running variable is x, it does not have anything to do with y. So, we can fix this Y to be y naught. Let us for the moment also assume that X has only 2 elements and the antecedent A takes these as the membership values there and let us try to calculate what is B dash of y naught.

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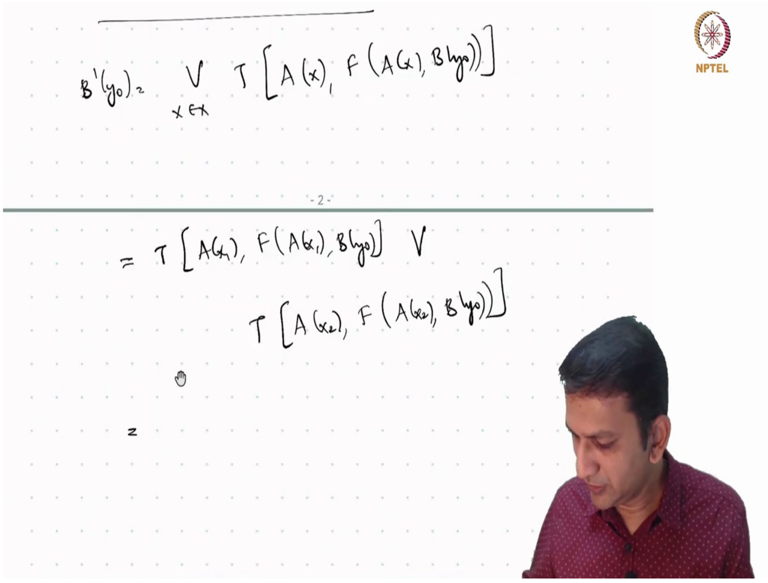

So, B dash of y naught is equal to supremum x element of X, T of A of x. So, note that we are giving A dash, but now we are checking for the interpolativity. So, A dash is a F of A of x comma B of y naught. Now, since x has only two elements, x 1, x 2 we could write this as T of A of x 1 comma F of A of x 1 comma B of y naught supremum T of A of x 2 comma F of A of x 2 comma B of y naught. Now, if you look at this, look at that, look at the second component.

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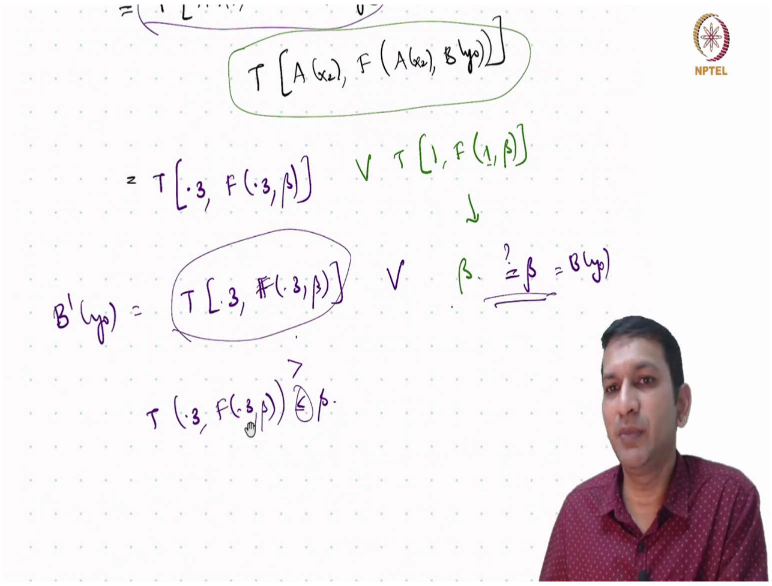

This is nothing but T of A of x 2 is 1. So, because T of T of 1 comma F of 1 comma B of y naught is beta is what we obtained. Now, if you assume. So, one is a neutral element of T, if assume that F also has one as its neutral element, this essentially turns out to be beta.

Now, what is this value? So, what we have here is nothing but T of A of x 1 is 0.3 comma F of 0.3 comma beta. So, on the whole what we have is T of 0.3 comma F of 0.3 comma beta or beta maximum with beta. If you want this to be beta you can ask the question when will this B equal to beta, because we want this B dash of y to be B of y naught. Remember this is essentially B dash of y naught.

So, this is B dash y naught, we want this to be equal to B of 1. So, when will this be equal to beta? Well, clearly we want that this value here should be smaller than beta; that means, we want that T of 0.3 comma F 0.3 comma beta is less than equal to beta, if it is less than or equal to beta then the supremum of these two which is max in this case, is actually beta and we obtain this.

If this were value were to be greater than beta, then it will no more beta at the maximum will be this first component which will be greater than beta. And we will not obtain interpolativity. Taking a cue from this, let us do some algebraic calculations and maybe perhaps a little manipulation.

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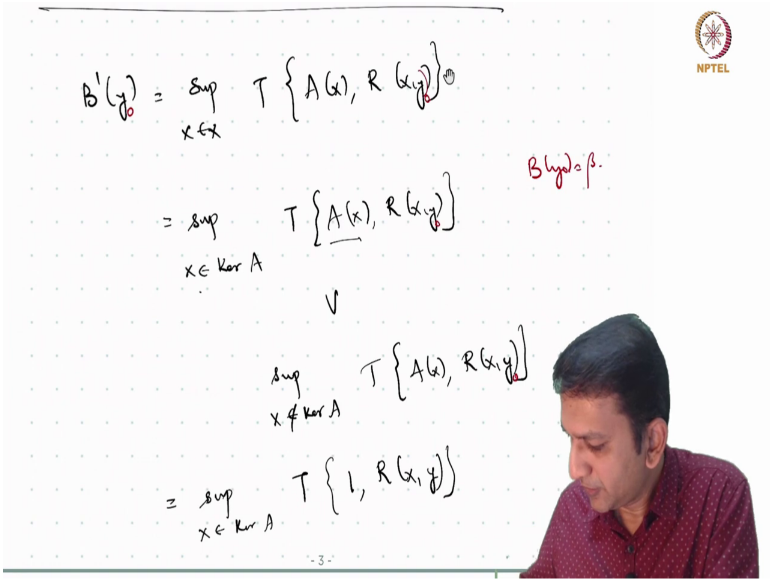

So, what we have is B dash of y is equal to supremum x element of X, T of A of x comma R of x comma y, yeah. Now, let us assume for a moment that A in fact, is normal.

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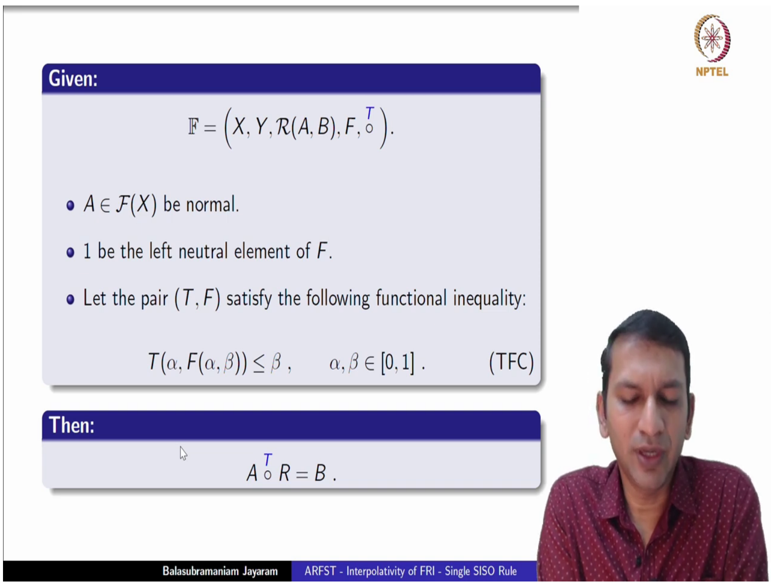

So, we are considering the CRI composition with a single rule A, B, F is the operation used to relate the antecedent A to the consequent B to obtain the relation R and we are considering the CRI inference which means its sup T composition. Let us for the moment assume A is also normal. So that means, we could write it like this, supremum x element of Kernel of A, T of A of x comma R of x, y.

Note, that Kernel of A means the value 6 in the support of A, where the membership value is 1 in A, means where A attains normality this joined with supremum x not in the Kernel of a T of A of x comma R of x, y. Now, let us fix y naught. Once again let us fix B of y naught to be beta. So, now, what we have here is equal to, look at this when x belongs to the Kernel of A, then this value is 1. So, what we will have is supremum over x element of Kernel of A, T of 1 comma R of x comma y.

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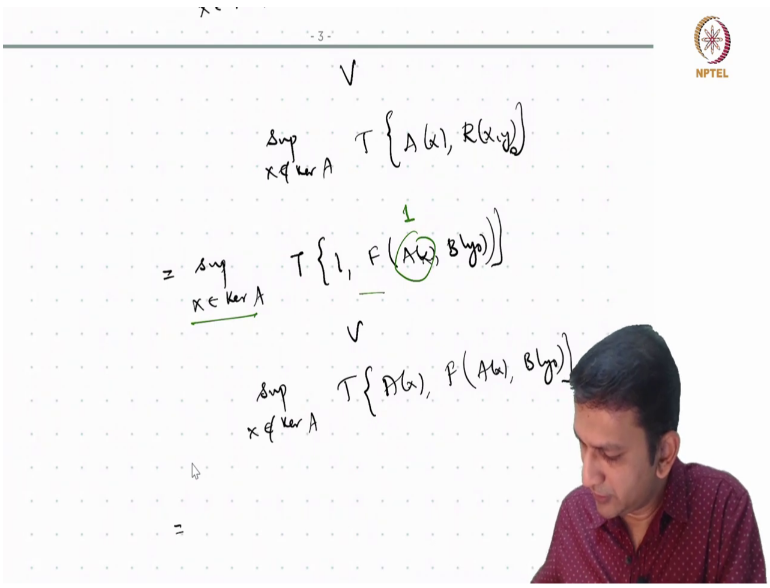

Max with supremum x not in Kernel of A, T of A of x comma R of x, y. Now, writing it in terms of the corresponding F, what we obtain is supremum over x element of Kernel of A, T of 1 comma F of R of x comma y is F of A x comma B y naught by fixing the y naught supremum sup x not in Kernel of A, T of A of x comma F of A of x comma B of y naught.

Now, let us assume, on this point A of F(X) here since we are picking x from Kernel of A, we know that F of A x is 1, which means rewriting this what we obtain is the first component turns out to be, for every x there the supremum can be removed now. Because it does not affect B of y naught if 1 turns out to be the neutral element of F. So, let us add this assumption, that 1 is the left neutral element of F.

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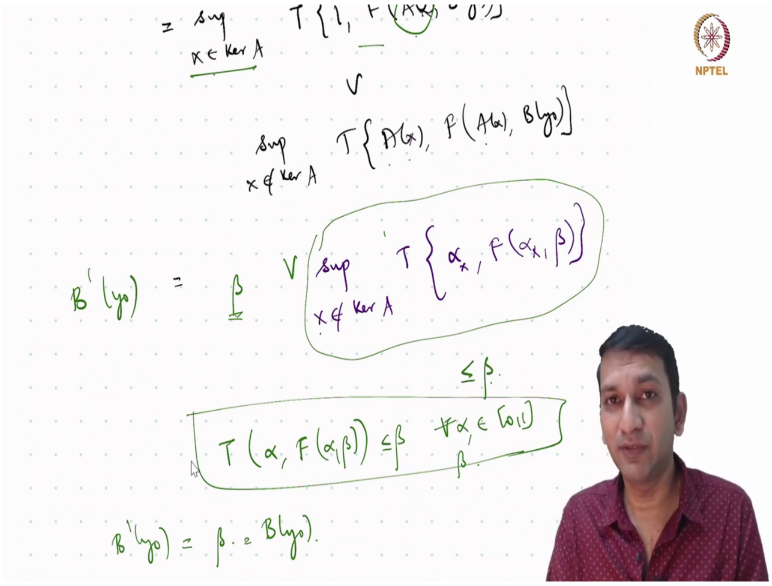

Then we obtain this to be B of y naught or supremum x not in Kernel of A, T of for the moment let us assume that for each of this x it is in fact, some alpha x, F of alpha x comma beta. So, we have assumed that beta of y naught is beta B of y naught is beta this is the equation that we get.

Now, what we want is we know that this is beta B dash of y naught, we want this to be equal to in fact beta. Which means this entire quantity should be less than or equal to beta and for every alpha x remember x is varying over the non-Kernel part of A; that means, alpha x which is nothing but A of x is actually a value less than 1, but it could be any value less than 1. So, now, we want that this entire supremum is less than or equal to beta.

Remember this beta is fixed quantity. So, in a sense what we want is for any alpha T alpha F alpha beta should be less than or equal to beta for all alpha and 0 1, perhaps including one its fine and note that once again this beta is an element that we have arbitrarily fixed B dash of y naught is arbitrarily fixed as some value beta which means it should happen not only for alpha, but also for beta.

So, this is the inequality that we want T and F to satisfy. So, if we add this also as the condition, let the pair T comma F satisfy the following functional inequality, T of alpha comma F of alpha beta is less than or equal to beta for all alpha beta coming from the 0, 1 interval. If we have these three conditions we see that in fact, B dash of y naught is equal to B, which is B of y naught.

Note that y naught was originally an arbitrary point that we took, which means B is in fact equal to, B dash is in fact equal to B. So, now this is the condition that we need on the underlying operations T and F for CRI to be interpolative, given that the antecedent A is normal fuzzy set and F has one as the left neutral element.

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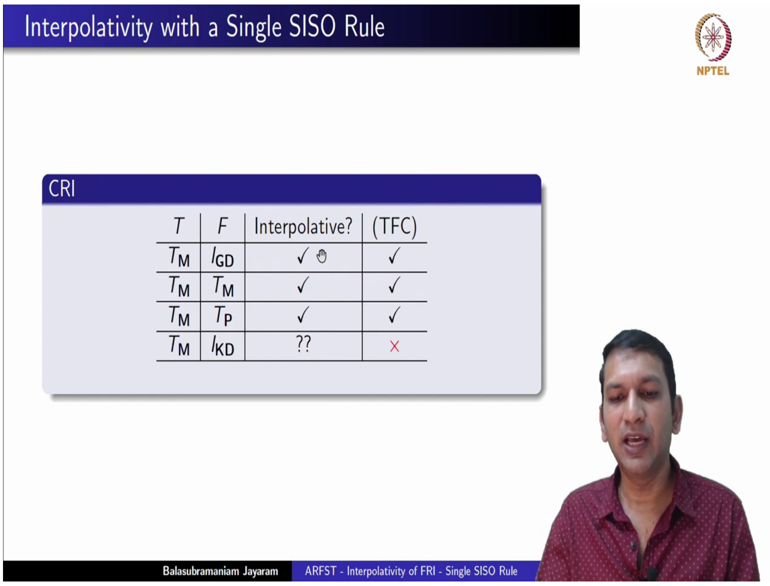

Let us quickly check the observations we have made earlier, the examples we had. We saw that if we take the minimum T-norm for the T and the Godel implication for F it turned out to be interpolative. In fact, it can be shown that it does satisfy this functional inequality right. Let us verify this.

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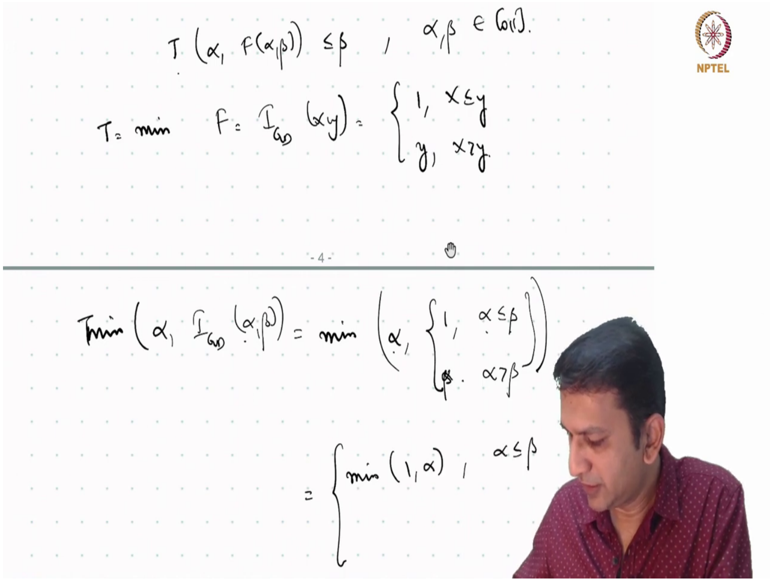

So, the function inequality that we are considering is this T of alpha comma F of alpha beta should be less than or equal beta for all alpha element of 0, 1. Now, let us construct T to min and F to be the product implication.

Student: Ok.

This is nothing but 1, if x is less than or equal to 1 and y if x is less than y. So now, consider this you know here; so, substituting minimum for T, minimum alpha comma I Godel, alpha comma beta. Now, you can write it like this is equal to minimum of alpha comma, note that if alpha is less than or equal to beta, it is 1 and if alpha is greater than beta it is alpha it is beta.

So, now this is essentially, if alpha is less than or equal to beta it is minimum of 1 from alpha, alpha is less than or equal to beta.

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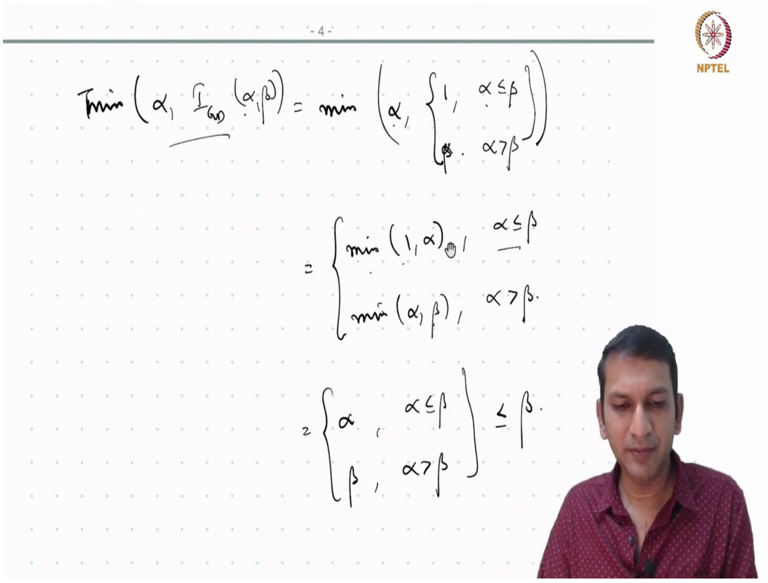

And it is minimum of alpha comma beta, if alpha is greater than beta. Clearly if alpha is less than or equal to beta, it is alpha and alpha is greater than beta, the minimum of alpha beta is beta and we see that the entire thing is always less than or equal to beta. So, the pair T comma Godel implication this pair T comma F does satisfy this equation in functional inequality TFC and hence we also see that it is interpolative.

It can be easily verified that if you consider the min T-norm for both T and F once again it is it does satisfy TFC.

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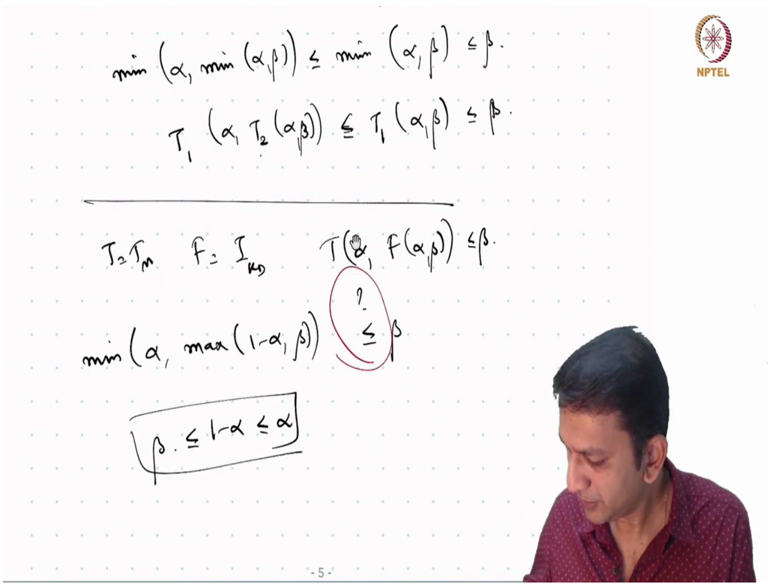

Essentially the property of a T-norm what you know it is min of alpha comma min of alpha comma beta. We know that min of alpha comma beta is always less than or equal to beta and this is less than or equal to beta. In fact, any key T 1, T 2 we consider by the properties of T norm, we know that T 2 of alpha beta is less than or equal to beta.

So, it is T 1 of alpha beta less than or equal to beta. So, we see that even for the other pair where we consider min for the T-norm and product for the function F, which gives us the relation with the rule we see that they do satisfy this inequality. If we consider this pair T M and the Kleene-Dienes implication for the F, we have not seen an example, but we can ask the question is it interpolative.

Let us verify whether it satisfies, whether it satisfy this is the equation TFC. Note that we have here T is T m F is I KD and we are looking at this singleton for every alpha beta. So, this translates into minimum of alpha comma I KD is maximum of 1 minus alpha comma beta. So, the question is it always less than or equal to beta?

This is the question that we are asking. Well, easily it can be shown that if let us assume that the alpha given to us is such that 1 minus alpha is less than or equal to alpha and we pick a beta from here.

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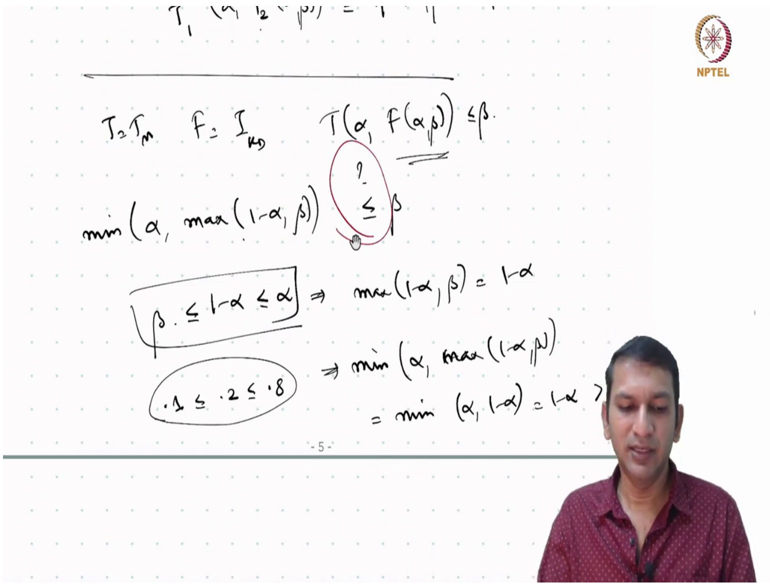

For this pair of beta comma alpha, you see here max of 1 minus alpha comma beta is. In fact, 1 minus alpha, then this implies min of alpha comma 1 minus max of 1 minus alpha comma beta is in fact, equal to min of alpha comma 1 minus alpha it could. From this we see that it is in fact, 1 minus alpha which is greater than beta and not less than or equal to minima.

Of course, if we take alpha to be say 0.8, then 1 minus alpha is 0.2, if we pick beta to be 0.1, this couple alpha and beta satisfy this inequality, this triple of inequalities. And we see that for that pair alpha beta T M and I KD do not satisfy this inequality. Thus they are not interpolative.

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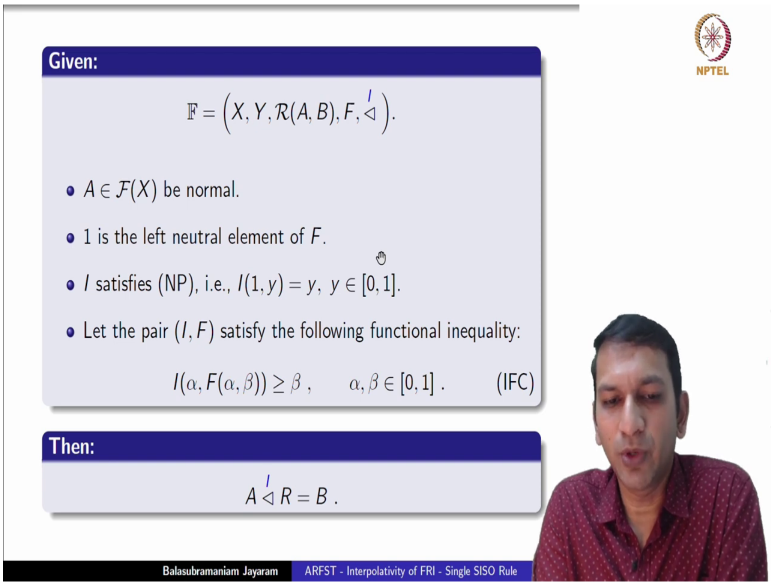

If we consider the BKS of product, we can pretty much get the same result except that we need another condition on I. If we consider the antecedent A to be a normal fuzzy set and 1 as the left neutral element of F and I satisfies, NP. That means, 1 is also the left neutral element of I, note that in the case of a T-norm this was already given, because 1 is a neutral element of T norm. It can be shown that another a dual kind of an inequality functional inequality plays a role in obtaining interpolativity.

What is that equation this equation I of alpha comma F of alpha beta should be greater than or equal to B beta for every alpha beta and 0, 1. If this condition is satisfied, then we obtain that the BKS inference with this I where the antecedent a is normal fuzzy set and F has 1, as it is neutral left neutral element we see that it is also interpolative.

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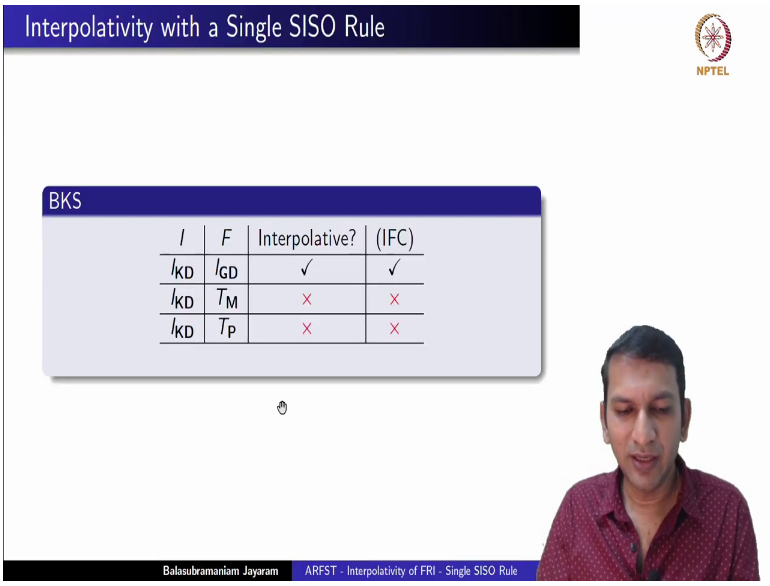

Let us revisit the examples that we had. In the case of BKS, we have seen that if we consider Kleene-Dienes for I and Godel for F, we obtained that the for the considered A and B it was interpolative. We will show it does in fact, satisfy this inequality IFC, it is quite easy to see.

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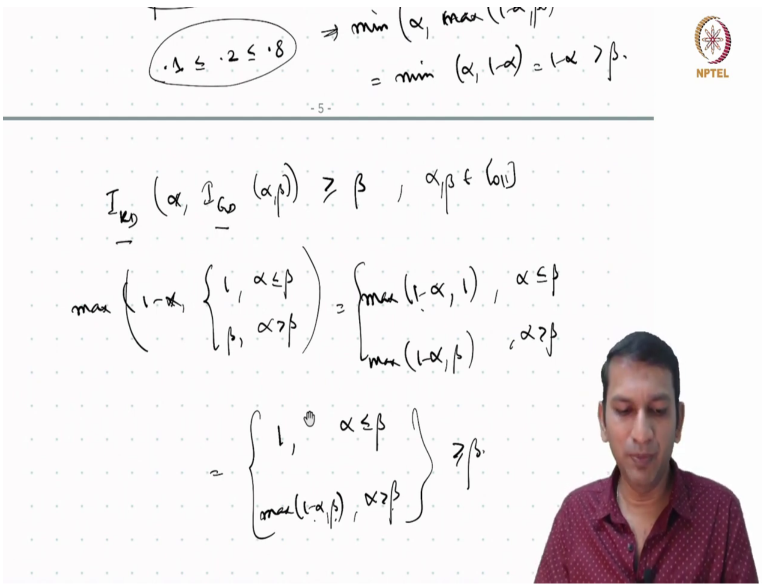

So, what we have is we need to show I KD of alpha comma I Godel of alpha comma beta is in fact greater than or equal to beta for every alpha beta element of 0, 1. Now, the that is the equation that inequality that we have here, greater than or equal to beta. What is this? Left-hand side is maximum of 1 minus x comma Godel is 1, if alpha is less than or equal to beta, beta if alpha greater than beta and this will turn out to be maximum of 1 minus x, 1 minus alpha comma 1.

If alpha is less than or equal to beta and alpha is greater than beta, then what we get is maximum of 1 minus a alpha comma beta. And what we have here is maximum of 1 minus alpha comma 1 is always 1 and alpha is less than or equal to beta. If alpha is greater than beta what we get is maximum of 1 minus alpha comma beta.

Now, note that when alpha is greater than beta, it is max of 1 minus alpha comma beta. So, if it is beta, it is definitely greater than or equal to beta, if it is maximum is 1 minus alpha which is clearly greater than beta, then it is greater than or equal to beta. So, we see that for this pair of implications taken to be as I and F, in the case of BKS in front we do obtain interpolativity and that is because the IFC functional inequality is satisfied.

In the other cases where we have the Kleene-Dienes implication for I and minimum for the F, we see that we saw that it was not interpolative at least for the fuzzy sets A and B that we consider. But that is because it does not satisfy IFC; once again this can be easily shown.

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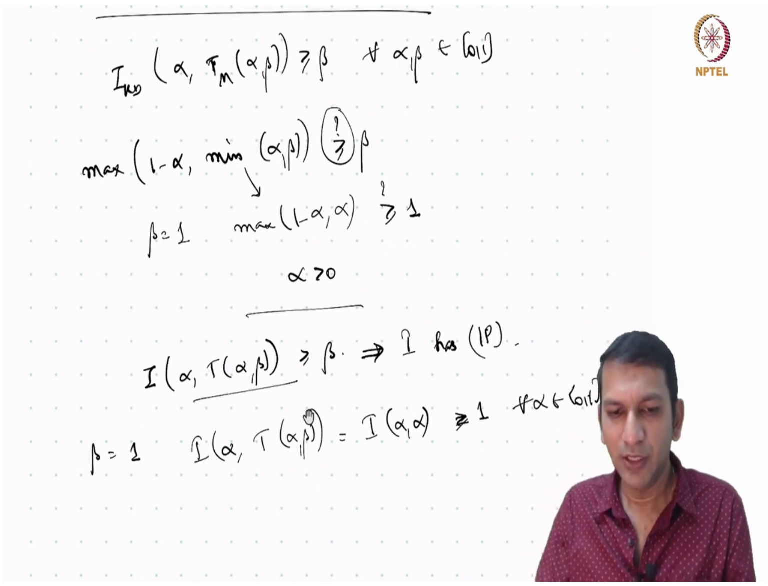

So, we have we need to see whether I KD of alpha and T M of alpha beta is greater than equal to beta, for all alpha beta element of 0, 1. This is nothing but max of 1 minus alpha comma minimum alpha comma beta, we are asking the question is it actually greater than or equal to beta.

Well, clearly if we take beta to be 1 what we get here is max of 1 minus alpha comma alpha. That is what we get here we are asking question is it greater than or equal to 1, definitely not when alpha is greater than 0. In fact, it can be shown that if we have an implication and a T norm, this inequality for it to be satisfied if it is satisfied, this means I in fact, has the identity principle, IP. How do we know this?

Let us assume that this inequality satisfied and let us take beta is equal to 1 and what we have is I of alpha comma T of alpha beta. Now beta is 1, which means that is I of alpha comma alpha it has to be greater than or equal to 1 which means it is 1. So, I of alpha comma alpha is 1, for all alpha element of 0, 1.

So, you see here a Kleene-Dienes implication does not have the identical principle. So, with this implication if we take F to be a T norm, it is not going to satisfy IFC and hence it will also not satisfy interpolativity.

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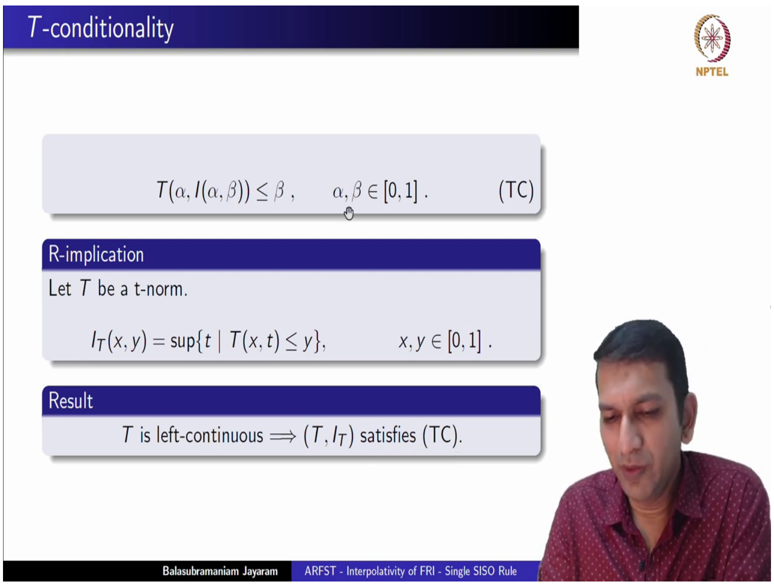

Now, why did we tag this inequality as IFC and the previous one as TFC? Because, there is a functional inequality involving the T-norm T a T-norm T and then implication line as follows. You will see that if you put F here, this is essentially TFC. This inequality has been studied in the literature under the name of T conditionality. That is why we came up with this tag in the sense abbreviating T F conditionality and I F conditionality.

Now, it has been quite well studied for which pairs of T and I, T norms and implication I this inequality is valid. One family of implications for which this is valid is essentially the R implication. Recall that if we are given a T norm, the corresponding R implication is given as follows. IT of x, y is supremum over T such that T of x T is less than or equal to y.

We have seen its origins, how it comes from writing A complement union B, in an alternative equivalent form in fuzzy in set theoretic terms. Now, the result easily result can be proven which says that, if this T-norm is originally left continuous then the pair T comma I T, the corresponding residual this pair indeed satisfies this T conditionality. But how do we prove this? The proof is not very difficult.

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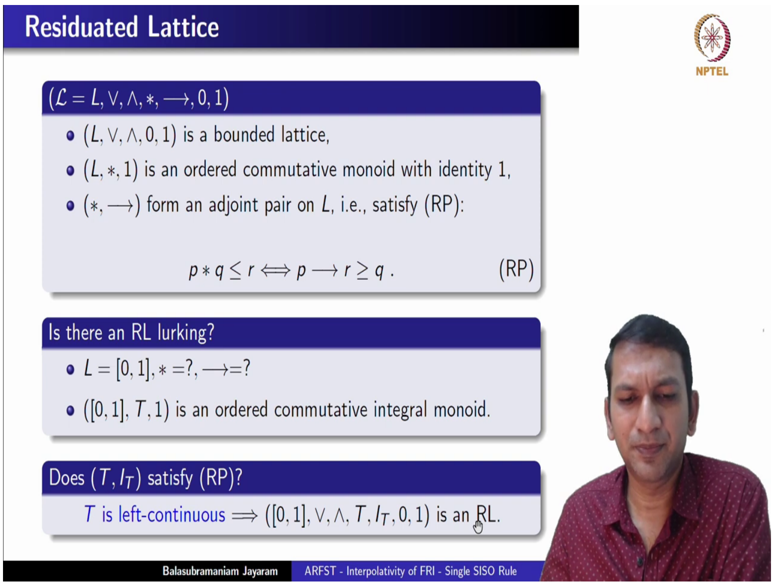

Please recall, we introduced the algebraic structure called the residuated lattice. What is a residuated lattice? It is this following structure, it has a set L, non-empty set L with these two lattice operations join and meet. And two further binary operations star and this arrow along with these two constant 0 and 1, which are elements of l such that L join meet 0, 1 is a bounded lattice.

So, one is the upper bound and 0 is the lower bound and these are the join and meet of the lattice. L star 1 is an ordered commutative monoid with identity 1. So that means, this operation star is compatible with the lattice order imposed on L, it is commutative, it is a monoid; that means, associative and neutral element exist. In fact, the neutral element is 1 and we know that the neutral element 1 is also the largest element with respect to the order given by the lattice.

So, it is an ordered commutative monoid with identity 1 and importantly these two operation star and arrow they form an adjoint pair on l. What does it mean? That means, they satisfy this particular equivalence condition. Whenever p star q is less than r, then p arrow r is greater than or equal to q.

Now, we have seen that if we take L to be 0, 1 and star to be a T-norm and implication to be the corresponding R implication. We can obtain a residuated lattice. It was clear to us earlier too that if we consider the T-norm T 1 0, 1. Then it in fact, this structure is an ordered commutative integral monoid. It is only a question of obtaining a corresponding arrow operation which will satisfy this adjointness property.

And we have seen that if T is left continuous this structure in fact, where this arrow is given by the residual of the corresponding T-norm is in fact a residual lattice.

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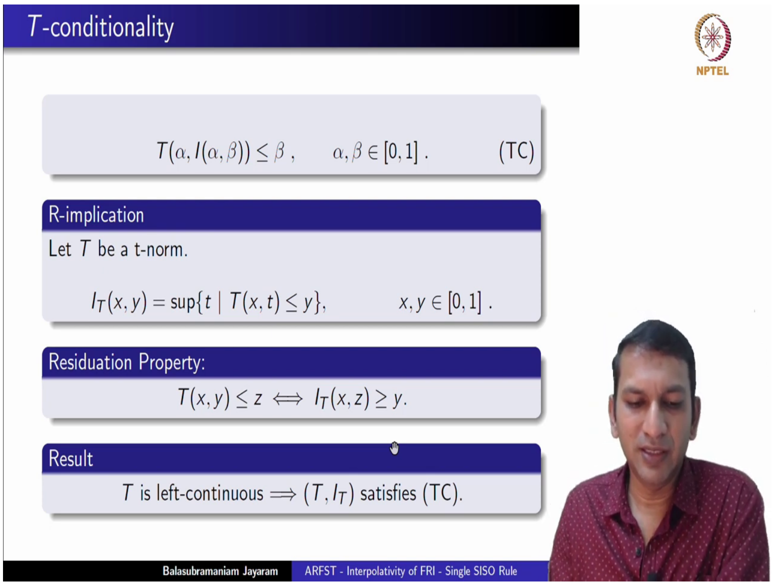

So, now we are interested in this particular functional inequality, which is the T conditionality and we are looking at the implication to be an R implication. We know that this property the residuation property is satisfied by the pair T and I T. It is in fact, clearly and easily from this residuated property, we obtain this result that whenever T is left continuous T of I T satisfies TC.

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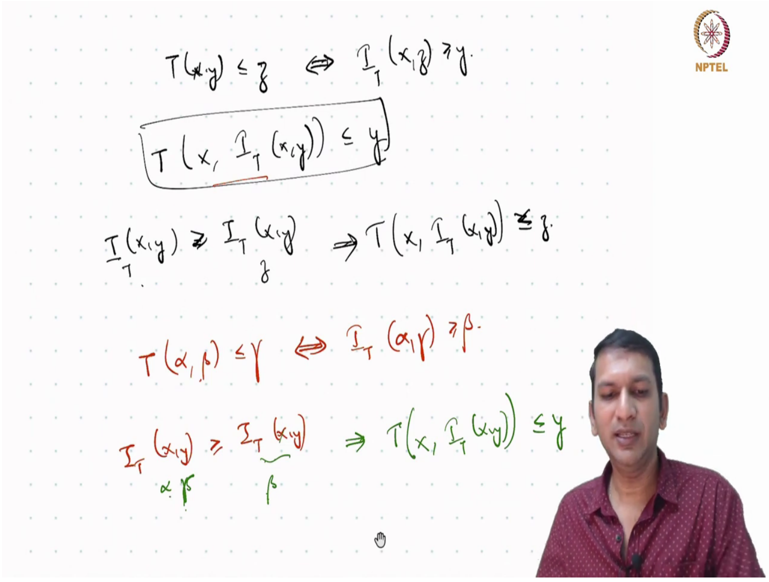

Now, the proof is immediate, perhaps is good to see that, note that we are looking at T and I T we know that the residuation property is satisfied; that means, if T of x, y is less than or equal to z, if and only if I T of x z is greater than or equal to y. And what we need to prove is the T conditionality; that means, T of x comma T of I T of x comma y should be less than or equal to y. So, this is the property that we want to prove.

So, let us assume that for some x, y we know that situated like this, I T of x, y is in fact, equal to I T of x, y this is clear. We can also rewrite this as greater than or equal to. So, now, look at this as x, y and this as the z, which immediately implies T of x comma I T of x, y as less than or equal to z ok. Perhaps one easier way to see this, it is like this if we could write in terms of alpha beta.

So, T of alpha beta is less than or equal to gamma if and only if I T of alpha comma gamma is greater than equal to beta and what we need to prove is this quantity. So, now, let us take I T of x, y we know this is in trivially greater than or equal to I T of x, y. Now, we assume h to be alpha y to be beta and I T of x, y to be y to be gamma and I T of x, y to be beta and this implies T of alpha x comma beta which is I T of x, y is less than or equal to gamma, which is y a necessity conditionality that we want.

So, clearly for this family of implications, R implications obtained from left continuous T-norm we see that the corresponding T and the residual do satisfy T conditionality. Well, so if you use a left continuous T-norm for in the composition of CRI and use the residual implication for F that is to obtain the relation from the rule A implies B, then we see that CRI is in fact, intrapolative. When the antecedent is normal, because F already has the neutrality property and T conditionality will be satisfied by the pair T comma I T.

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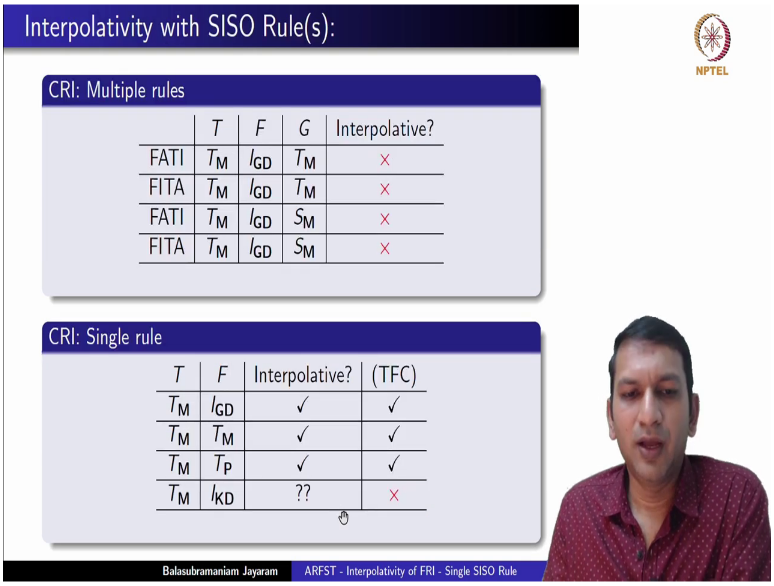

Well look at this we have discussed only the case of single SISO rule. We need to discuss the case of multiple rules, but a quick recap of what we have seen in the previous lecture. You see here when we consider the CRI for a single SISO rule this pair minimum and Godel implication they satisfy TFC and hence they give rise to interpolativity.

However, in the case of multiple rules when you consider the same pair for T and F and even in the case of different aggregation operators G and different inference strategies FITA or FATI, they are not actually interpolative.

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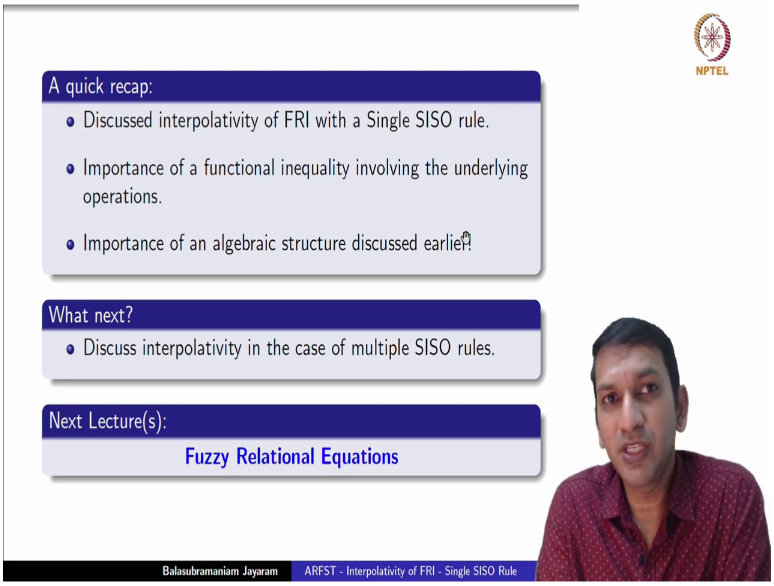

Which means we need to discuss little deeper in the case of multiple SISO rules. In this lecture we have looked at interpolativity of FRI, with a single SISO rule. What is very interesting is the important role played by functional inequality involving the underlying operations. And also happy one of the algebraic structures that we have discussed earlier that of residuated lattices has surfaced here.

But this allows us to only discuss interpolativity in the case of single SISO rule. What we want to discuss is the interpolativity in the case of multiple SISO rules and this is what we will take up in the next lecture, which will lead us to discussing fuzzy relational equations in depth. Glad that you could join us today for this lecture and hope to see you soon in the next lecture.

Thank you again.