Approximate Reasoning using Fuzzy Set Theory
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Lecture - 04
Fuzziness vs Probability

Hello and welcome to the 4th of the lectures in this week under the course titled Approximate Reasoning using Fuzzy Set Theory. A course offered through the NPTEL platform. In this lecture we would look at the essential difference between Fuzziness and Probability.

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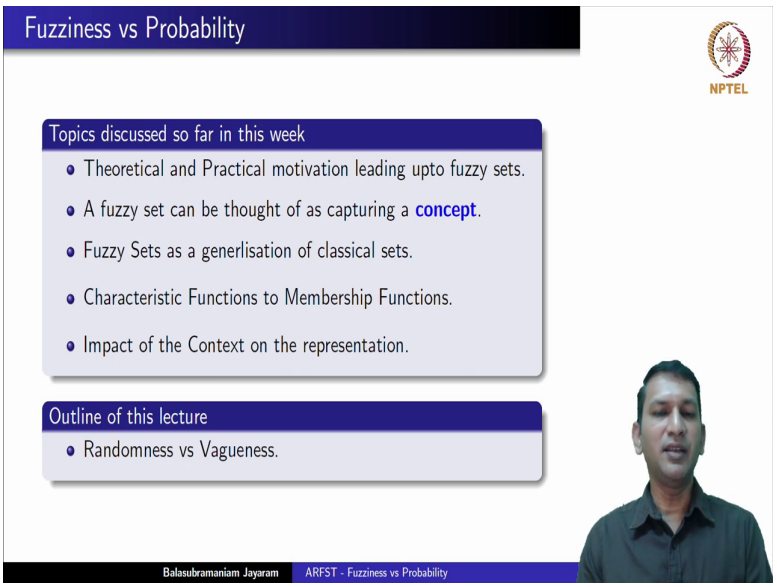

Let us take a quick peek into the concepts that we have covered over the course of this week. We look at the theoretical and practical motivation that led up to introducing fuzzy sets. We now know that a fuzzy set can be thought of as capturing a concept. We have seen fuzzy sets as a generalization of classical sets essentially moving from the characteristic function which was a function from the domain to the set with zero on one to membership functions which is a mapping from the underlying domain to the entire unit interval zero one.

We have specifically seen the impact of the context on the representations that we can obtain.

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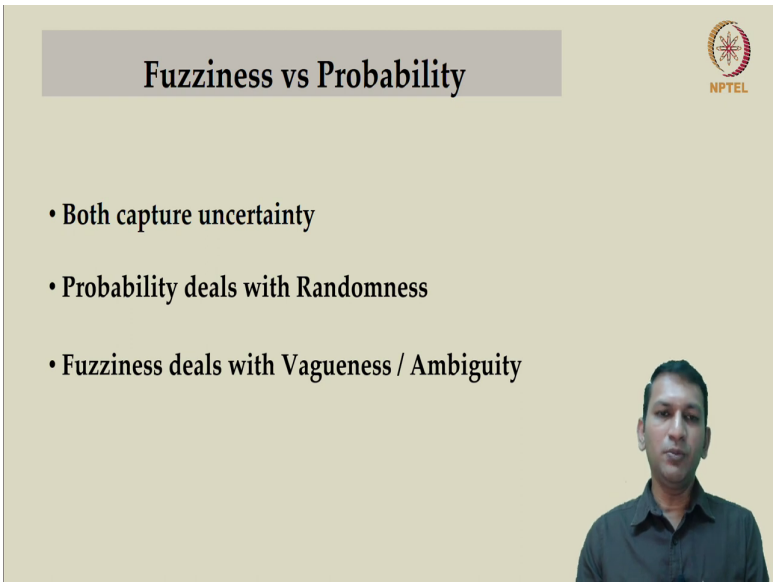

In this lecture, we would deal with two types of uncertainty which are randomness and vagueness. Both fuzziness and probability capture different types of uncertainty, while probability deals with randomness. Fuzziness deals with vagueness or ambiguity. I allow me to explain this in the rest of this lecture.

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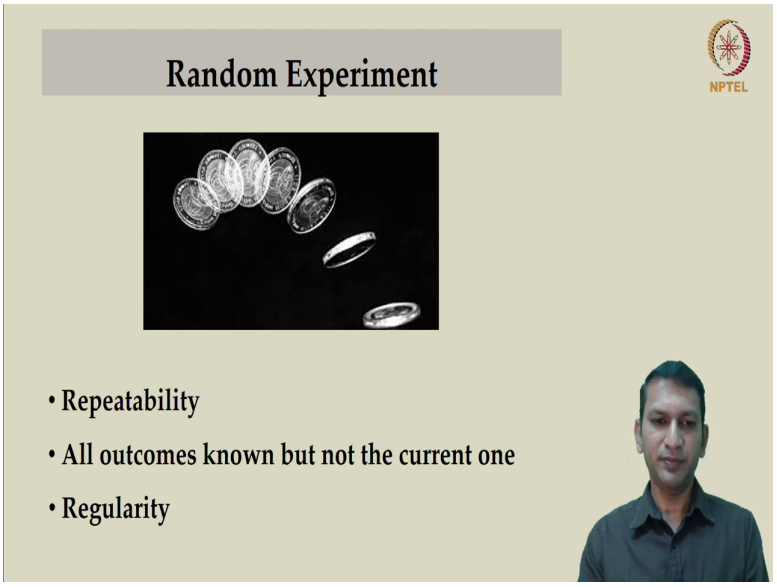

When we talk about probability we assume there is a random experiment that is being conducted. Now, what kind of experiments qualify to be called as random experiments? There are three properties that such an experiment should possess. The first of them is

repeatability, second from the experiment all possible outcomes must be known even though the one that is currently going to come out is not really known.

Finally, we should also ensure the experiment possesses a regularity or statistical regularity; that means, after a few trials large enough trials a pattern should emerge.

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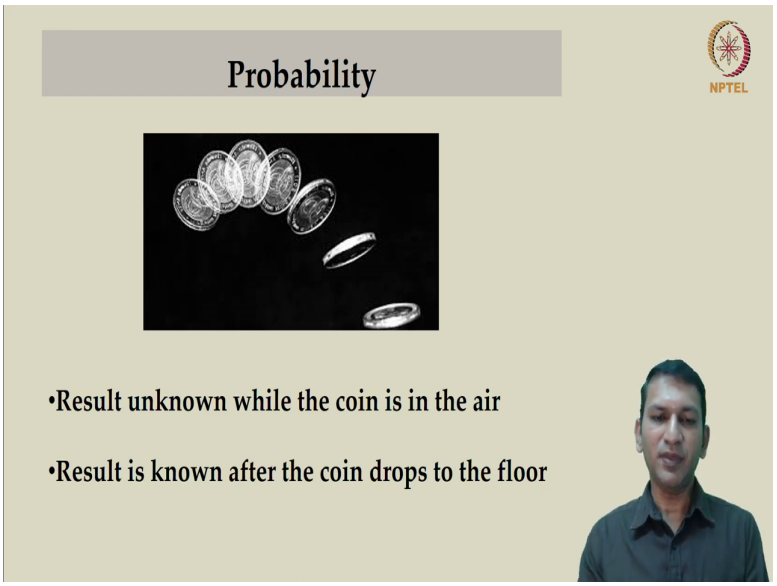

Now, what is probability? Let us consider the basic coin toss experiment which as we know is or can be qualified as a random experiment. Interestingly, when you toss a coin the result is unknown while the coin is in the air. However, once the coin drops to the floor it is easy to verify whether you got a head or a tail. So, probability exists before the experiment or hypothetically even without doing the experiment. Now, how does this compare with fuzziness?

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If I if we pick up this coin and toss while we might discuss and debate on whether we would get a head or tail at the end of the toss imagine the coin has fallen down and this is the side of the coin that we are seeing. Now, we need to decide whether it is a head or a tail. Now, the experiment is over after the experiment there is no randomness we are able to see the outcome, but now we are not able to decide the outcome, it could have been this coin that you picked up or this coin or this coin.

Now, we are wondering is it a head or a tail. Here in fuzziness it is not lack of information, but it is not a being able to decide even with all the given information that is what is leading to the uncertainty.

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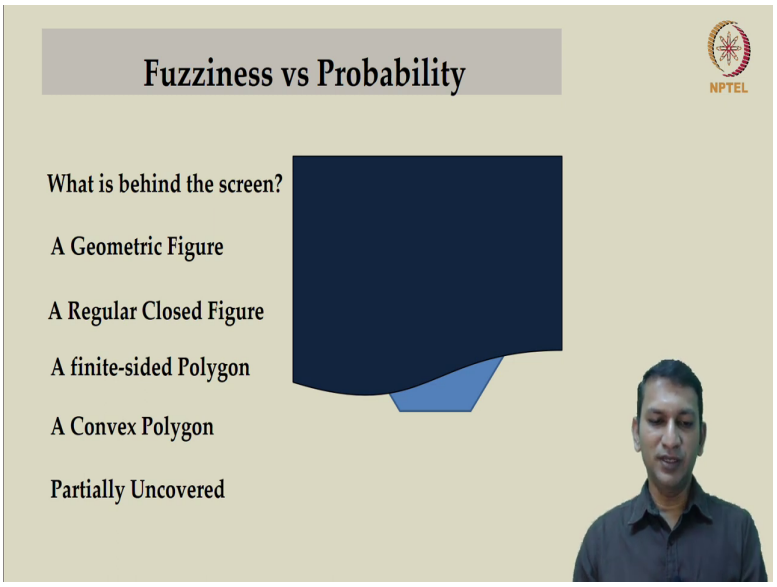

Let us play a game. We have a screen and now a question is asked to us what is behind the screen? Now, it could be any one of the zillion things that we have in the universe, it could be a picture of a child eating ice cream, it could be a picture of a scenic place. It could be a portrait of a great personality or it could be a snapshot from a sports video, it could be any one of a million possibilities.

So, if you are asked to guess what is behind this screen then the entire universe that whatever you could photograph and put behind the screen forms the sample space; however, let us see whether we can get more information. Now, if you are given the information, then what is behind the screen is actually a geometric figure. Now, this reduces the sample space considerably.

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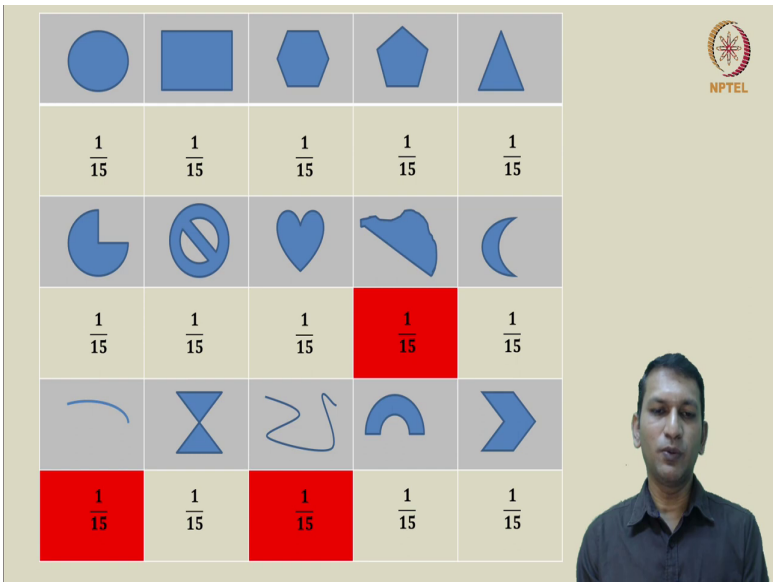

Now, given this information let us pick 15 such geometric figures and consider it in the present scenario. Let us begin with the simplest of the shapes, the circle a rectangle, a hexagon, a pentagon, a triangle. Let us be little more adventurous and pick some more such figures including the heart shape the crescent and some free flowing curves.

A symbol that closely resembles the r class once again a free flowing curve something that resembles a horseshoe magnet and a thickened arrow perhaps. Now, with the given information I am considering only these 15 figures as part of our sample space, we need to make a guess. Now, in the absence of any other information uniform prior is what you assign to each of these which means, the probability with which the figure hidden behind the screen could be a circle is 1 by 15.

So, let us start filling these boxes with the corresponding probabilities. Now, we get a little more information that it is a regular closed figure. Now, this information allows us to revise or reduce our sample space. Now, we are given the information that it is a regular, closed, curve.

Immediately we see that this is not a closed curve, so is the case with this. Now, since we are given the term regular let us understand this in terms of common usage which means you could immediately exclude this figure. So, now, from the 15 we are immediately able to exclude three of them with a given piece of information; however, now out of this 15 we have reduced the sample space to 12 and this calls for re-evaluation of probabilities.

Once again using uniform prior we see that the probabilities for each one of these geometric figures turns out to be 1 by 12. Now, we are given yet another piece of information that it is a finite-sided polygon.

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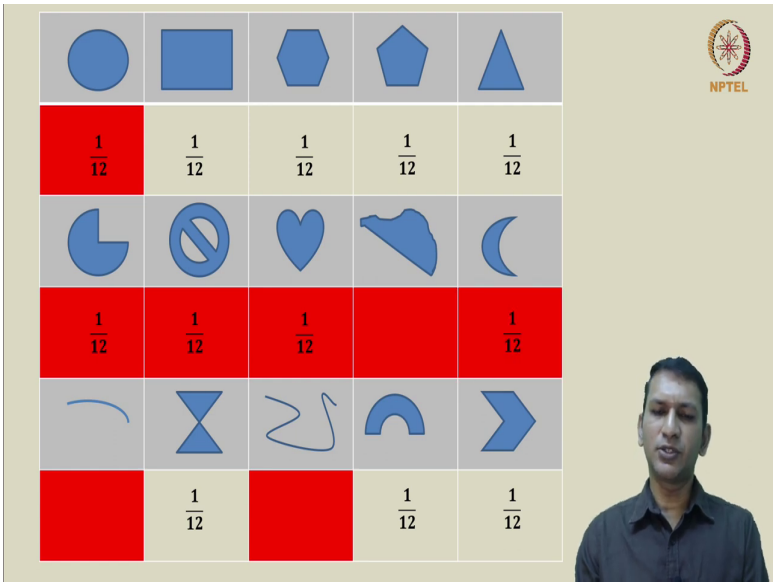

Once again going back to this table we see that this immediately eliminates a circle this figure, this the heart shape the crescent and as also the horseshoe method. So, we reduce the sample space from 15 to 12 now a further 6 of them are excluded. Now, we are left only with 6 out of the original 15 figures that we considered in the sample space.

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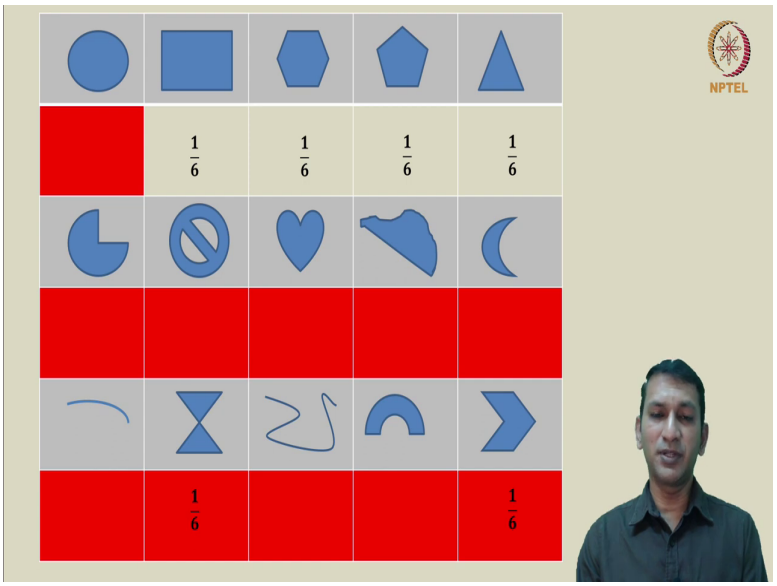

Once again this calls for re-evaluation of probabilities again using the uniform prior it is one sixth for each of these figures. Notice that, with more and more information coming on way the probabilities that we have assigned on each of these objects is getting re-evaluated and reassigned.

All of them to begin with they had 1 by 15 as their value some have gone down to 0 and got eliminated some came up to one twelve again got eliminated and some currently are remaining at 1 by 6. Now, let us see whether we can get some more information. Now, we are given this piece of information that it is actually a convex polygon. Now with this information we are able to immediately exclude these two options. So, further reduction by two from a sample space of current sample space of 6.

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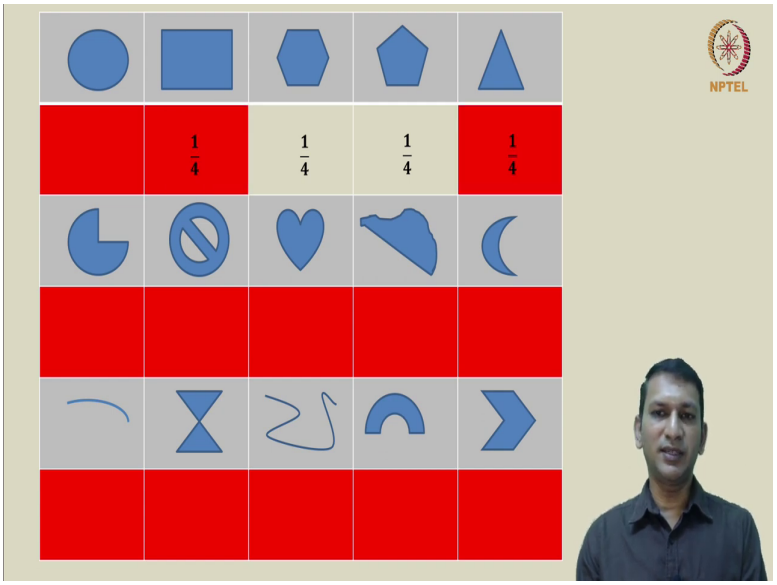

Once again re-evaluating the probabilities we see that its one fourth on each of the 4 figures that remain do we have any more information coming our way. Well, we now are actually given going to be given a sneak peek into what is hidden behind the screen. Now, the moment we see this figure we know that we can go back here and exclude 2 among these 4 the rectangle and the triangle.

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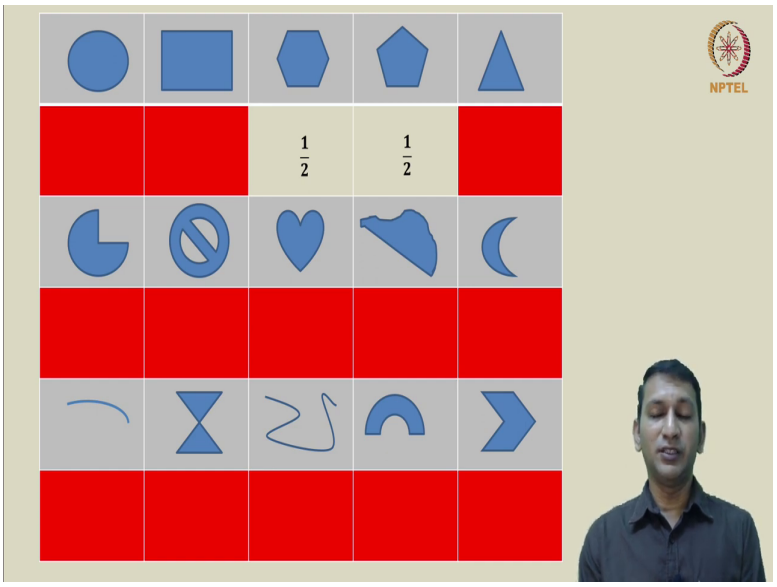

Once again re-evaluating the probabilities, now among the choices that we made to be put in the sample space now there are only two such possibilities that of a hexagon or a pentagon and with no further information forthcoming with uniform prior assigned to them it could be either hexagon or a pentagon.

The point that we are trying to drive home here is that with more and more information seeping in we are able to revise our probabilities, but now probabilities are actually assigned to a random experiment. Now, the question is how was this object picked from and hidden behind the screen? Now, this was done in a random manner. So, among the different possibilities randomly one such object was chosen geometric figure was chosen and hidden behind the screen.

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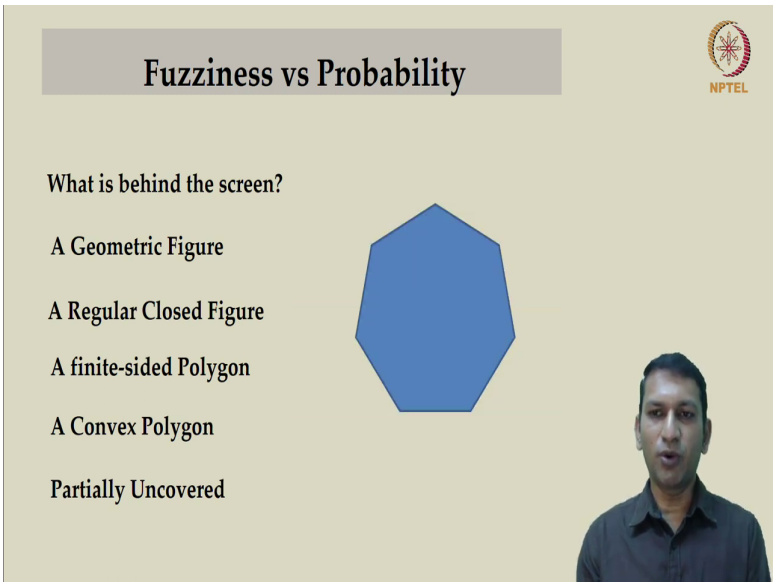

Now, let us remove the screen to see what we have behind the screen. Now, there is neither a hexagon nor a pentagon, but this does not discredit the procedure that we have followed in a sense probability deals with randomness and it is a lack of information that leads us to make play this guessing game.

And with more and more information coming our way we are actually revising the probabilities. Note once we are shown the object we are not guessing anymore we know clearly it is a finite sided polygon with 7 sides.

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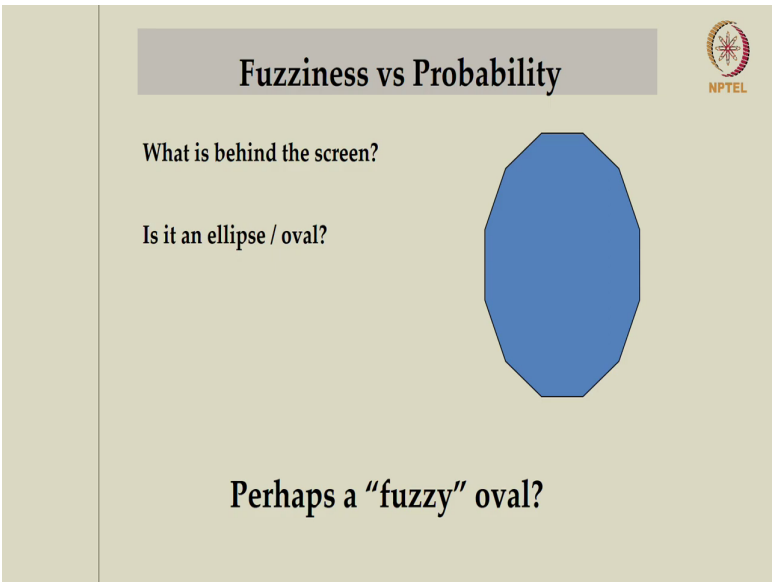

I now going to play a different game with you. Now, once again we have a screen and there is something hidden behind the screen. Now, we are asked the same question what is behind the screen? We are not going to be given information in bits and pieces. In fact, not even partial uncovering of the screen we actually would remove the screen, we are actually being shown the object. Now, the question is it an ellipse or oval? Clearly, you would say that it is neither an ellipse nor an oval, but you would perhaps say it is more or less an ellipse or perhaps a fuzzy ellipse or a fuzzy oval.

Now, in probability due to lack of information we are making a guessing game as to what could be hidden behind the screen, what is that object and we are without the full information we are trying to decide whether that object belongs to the class of heptagons or pentagons or one such regular geometric figure. Whereas in fuzziness on the contrary we have all the information about the element about the object.

But now we are wondering not whether it belongs to the set of ellipses, but to what extent it could belong to that set. Much like what we have seen in one of the previous lectures about the ball with different shades of blue and we are left we were left wondering whether it belongs to the set of blue box.

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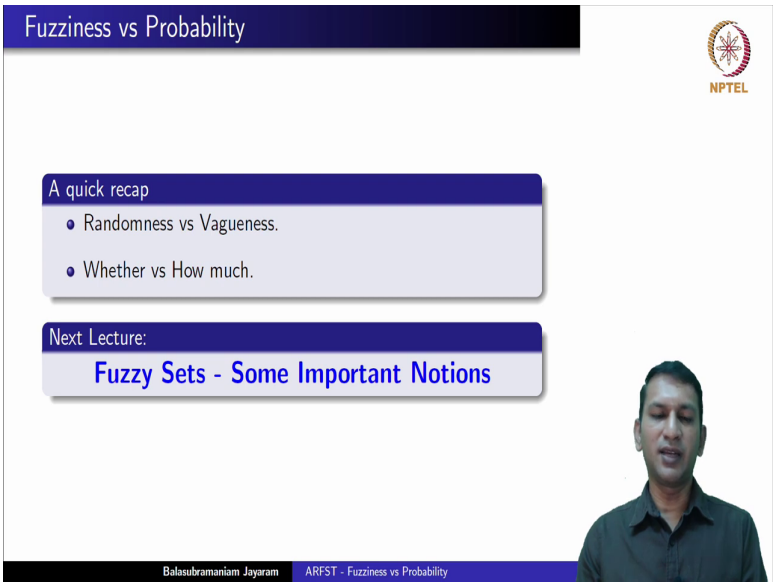

To recap both fuzziness and probability deal with two different types of uncertainties. While fuzziness deals with vagueness that comes about because of lack of clear boundaries between sets probability deals with randomness. So, it is a question of whether it belongs in the case

of probability without all the information to how much it belongs in the case of fuzziness having given all the information.

With this we come to the end of this lecture. In the next lecture we will look into some important notions related to fuzzy sets. We will see a few types of classifications of fuzzy sets some components of fuzzy sets and as we know fuzzy sets being a generalization of classical sets we there is a need to also import or export the different concepts properties of classical sets to the setting of fuzzy sets for instance the case of subset or that of cardinality.

We will see some of these notions only, but only those that are relevant for the rest of the course.

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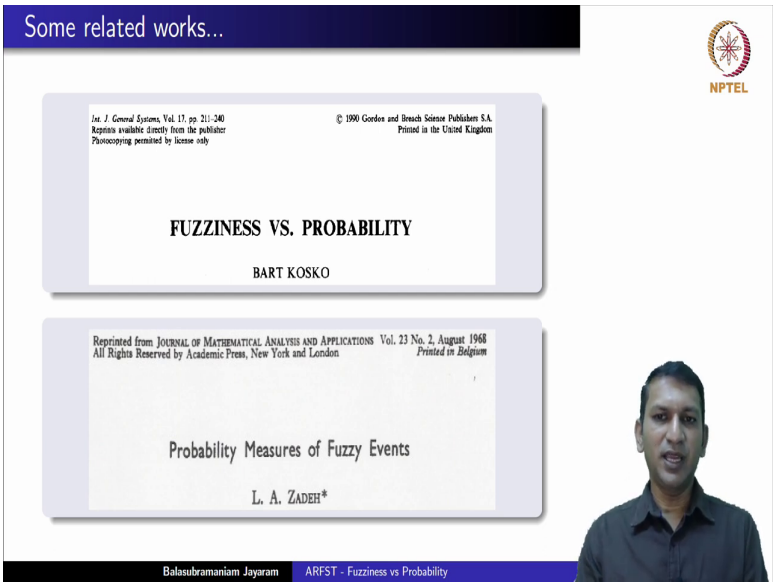

Once again allow me to refer you to some research level articles most of what I have covered in this lecture has been based on this excellent paper by Bart Kasko which is also titled Fuzziness versus Probability. Professor Zadeh himself even as early as 1968 has discussed the relationship between probability and fuzziness.

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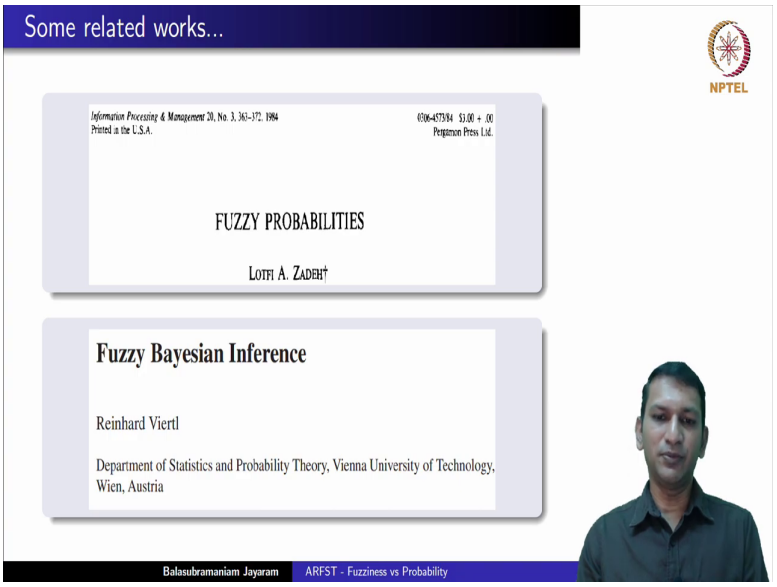

Not only in this paper even a little later almost after 20 years we had another paper under the title Fuzzy Probabilities. In these two papers and other related works he argues that often the information about an event itself is imprecise for instance he gives an example saying that among the 20 tosses of a coin several more were heads than tails and he insists that these can be very nicely captured using fuzziness.

So, he looks at a happy marriage between fuzziness and probability. If you think that these works have been done only 20, 30 years back no these are still relevant and there is a lot of work activity research activity that is being done relating these two areas; one such case end point is that of a recent work done by Professor Viertl on Fuzzy Bayesian Inference.

And of course, if you were to look into other resources on the internet I am sure you will find many many more relating these two very interesting and useful concepts when it comes to analyzing data and making sense of them either in terms of inference or reasoning or decision making. With this we will stop here, thanks for your patient listening and hope to see you soon in the next lecture.

Thank you.