Approximate Reasoning using Fuzzy Set Theory
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Lecture - 37
Contrast Enhancement in Images: An FIS Approach

Hello and welcome to the 5th of the lectures in this week 7 of the course titled Approximate Reasoning using Fuzzy Set Theory a course offered over the NPTEL platform. In the previous lecture we had looked at building a Mamdani Fuzzy Inference System using the fuzzy logic toolbox available in MATLAB to approximate a given function. Now, at that time we were wondering is there any need to build a fuzzy inference system to approximate a function that is already known.

Well. Firstly, it is a proof of concept that we can build a Mamdani fuzzy Inference system to approximate a given function of course; it is theoretically known that any given continuous function can be approximated to arbitrary accuracy using a Mamdani fuzzy inference system with appropriate choice of inference operators. Secondly, a system function is not known to us; however, some qualitative properties of the system function may be known to us.

Perhaps we might know that it is an oscillatory function or it might be a monotonic function or we might know about it is behavior in different sub domains or we might know the bandwidth within which the graph of the function might lie. So, using these heuristics we can construct a fuzzy inference system to approximate the system function. In this lecture we will see one practical application among the many in which fuzzy inference systems are employed and found use.

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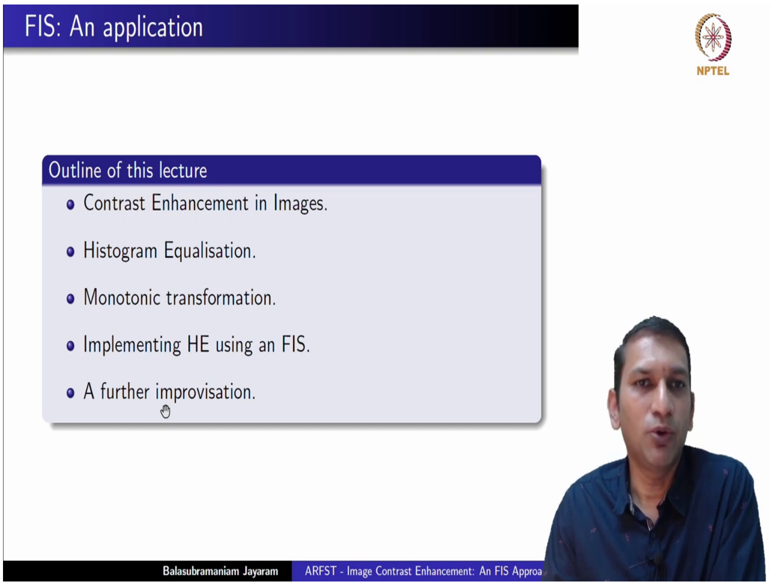

In this lecture we will take a simple application of contrast enhancement in images, a common method that is used especially in the case of grayscale images is that of histogram equalization. We will look at histogram equalization as a kind of a monotonic transformation of the dynamic range of a given picture grayscale image.

Seeing it as a monotonic transformation as a system function which is being monotone we will try to simulate this using a fuzzy inference system. We will compare the results in fact, we will go one step further having seen it as a monotonic transformation we might also be able to come up with further improvisation of original fuzzy inference system and see the results that we obtain.

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Now, let us look at the following images. If you were to look at these 4 images and ask which of them has visual quality appeal, clearly the one at the right bottom trumps everything else. If you wonder why, the one on the left most on top if you see that it is largely dark, the one on the top right is it seems as if it is over exposed and the one on the bottom left is somewhere in the mid-range and clearly the one at the bottom right is quite clear.

It is as if for the other 3 images there is a filter that is kept on top of it perhaps a translucent sheet or a glass. Now the thing is the contrast in these images is actually very less, what do we mean by the contrast? The pixel intensities of these images they occupy a very less range, the dynamic range of the pixel intensities of the 3 pictures which we feel does not have the visual quality appeal is actually very very less.

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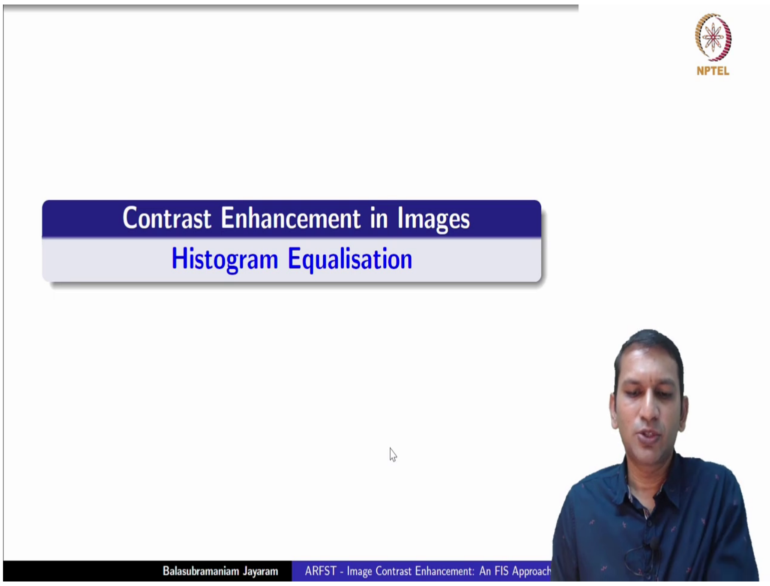

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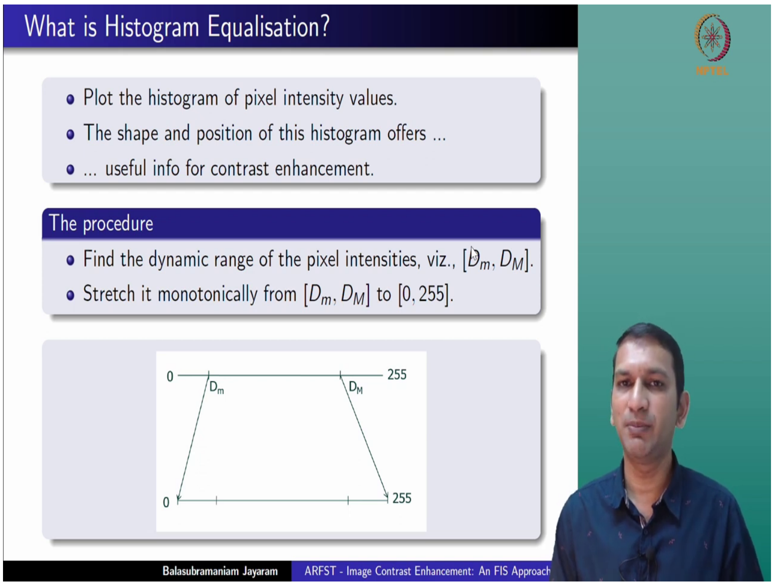

Now, what is histogram equalization? If you plot the histogram of pixel intensity values; the shape and position of this histogram offers useful information about the contrast in the picture and hence this can be exploited used to perform contrast enhancement.

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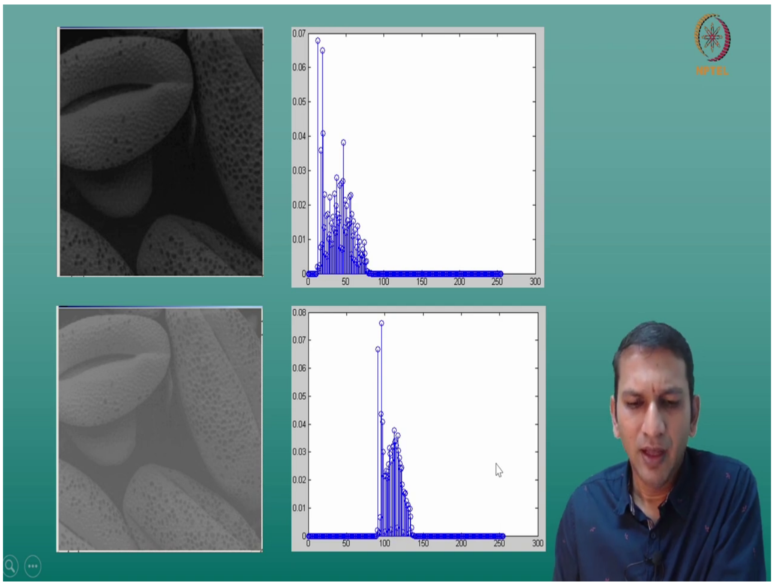

Let us look at the same pictures, if you look at this dark image slightly darker image and look at the histogram of the pixel intensities present in this picture. We see that it is largely towards the darker end of the spectrum so, on the x axis we have the pixel intensities this is a grays scale image. So, we have normalized between 0 to 255 and these are the corresponding pixel intensities once again which are normalized based on the number of points in the image.

So, this bar here represents that the number of pixels with pixel intensity 50 is almost 4 percent of the entire image. Now, if you look at it for this image where we felt largely the pixels are in the somewhere in the middle range neither black nor white, the histogram of pixel intensities also confirms this that they are in around the middle of the range and you see that they are not well spread. So, the dynamic range both in terms of its width the spread and also in terms of the position they are actually placed in the middle.

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Now, let us look at some actual pictures, if you look at this image of a car we see that the spectrum the histogram which could occupy the entire 0 to 250 interval is actually restricted to between 100 to 200. And in this case once again what looks like some kind of a blurry image if you look at the dynamic range of the pixel intensities which is again is restricted it is not the entire interval between 0 and 255.

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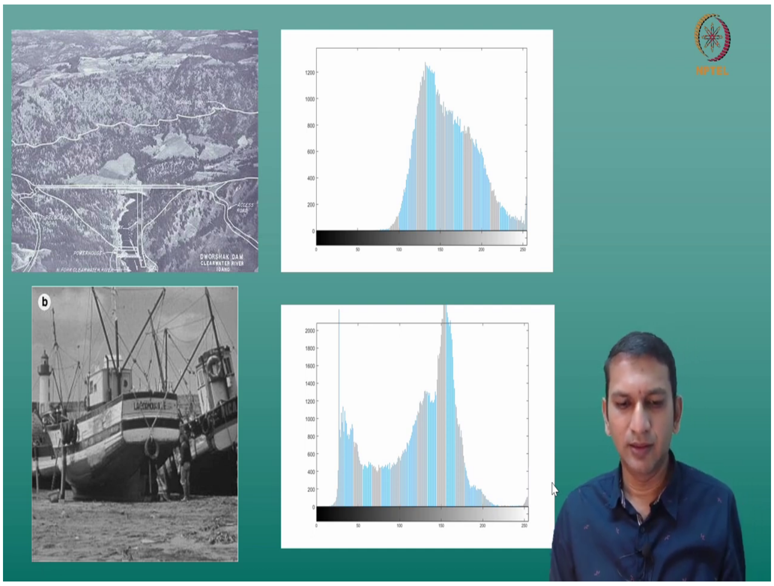

So, is the case, with this image. If you look at this image we believe that the details are quite good in this and if you look at the corresponding spectrum the histogram of the pixel intensities we see that its dynamic range is quite high, it occupies quite an amount of the interval 0 to 255. Now, the idea behind histogram equalization is this.

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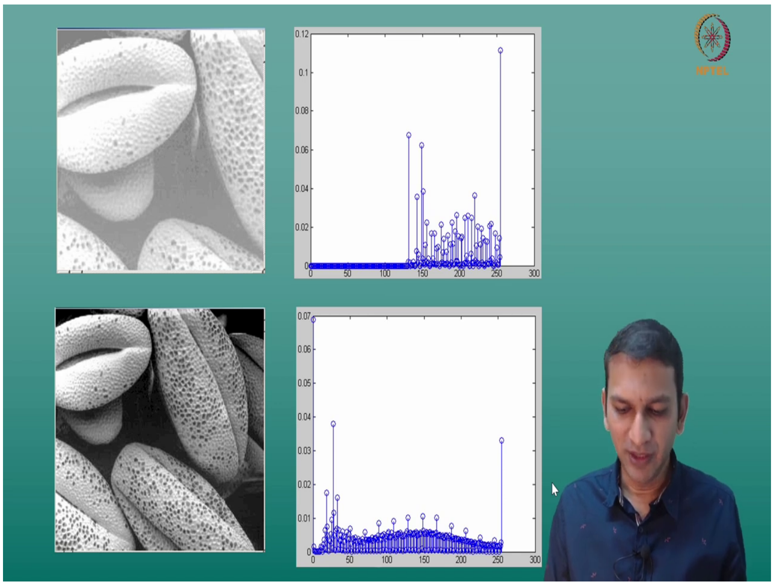

Once again let us look at the images that we began with. So, a darker image its histogram of pixel intensities is closer to the lower end of this boundary 0 to 255 interval, the grayish image somewhere in the middle and the almost over exposed image towards the higher end of the 0 to 255 interval.

However, what you need what we might be interested in is not only the position, but also the spread if you look at these images their positioning wise they are either close to lower end of the interval or the middle or higher end of the interval; however, their spread is quite limited. Now, look at this image which we felt was quite a good quality in terms of visual appeal. If you look at the corresponding histogram of pixel intensities we see that it is quite well spread almost occupying the entire 0 to 255 interval.

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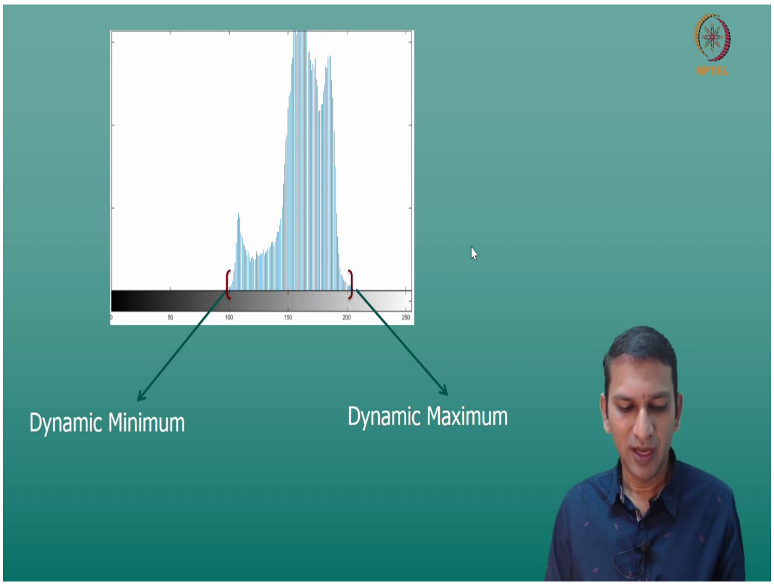

So, taking cue from this given a pixel an image we look at the corresponding histogram of pixel intensities let us assume this is the histogram of pixel intensities for a given image we find the dynamic range of this histogram. By the dynamic range we actually mean the values the pixel intensity values that are actually assumed in the image.

So, this is the dynamic minimum and this is the dynamic maximum and what we do is after finding the dynamic range of this pixel intensities we essentially stretch it monotonically from this dynamic range to the entire 0 to 255 interval. Histogram equalization essentially does this monotonic transformation of this dynamic range of a picture to the entire 0 to 255 interval. So, visually this is exactly what it does. Now, let us look at the results of applying histogram equalization to some of the pictures that we have seen.

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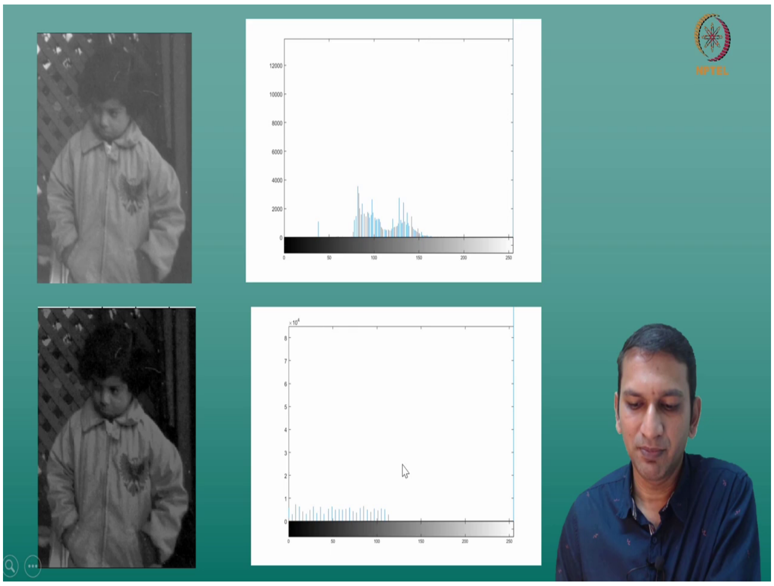

So, for this picture we have seen that it is of low contrast and this is the corresponding histogram of pixel intensities. This is the histogram equalized image already you can see the quality has improved we are able to make out some details in the image and if you look at the corresponding histogram of pixel intensities it is quite well spread.

You might notice that there are lot of pixels almost at the white end of the spectrum and those are again retained. So, the dynamic range of the modified or the histogram equalized image is essentially the entire 0 to 255 interval.

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And for this image having these cars this is the original histogram of pixel intensities of the corresponding original image that we consider and you see that this is the histogram equalized image. Already you can see details in greater depth and if you look at the corresponding histogram of pixel intensities we see that the dynamic range has actually expanded.

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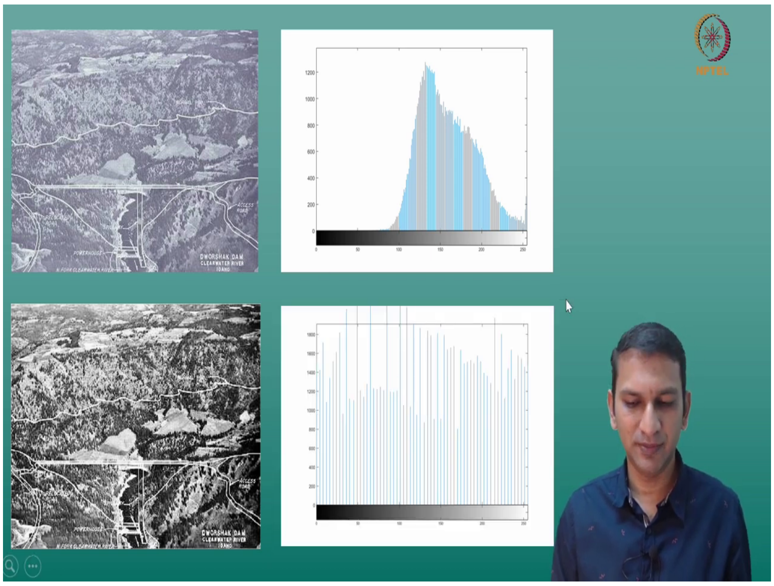

For this picture once again what we have is this is the histogram of pixel intensities, the dynamic range is much better than earlier, but it could be even greater, histogram equalization does that wherein you see greater detail in the images. Now, having seen histogram equalization as a monotonic transformation from the dynamic range to the entire 0 to 255 interval, we understand this is the system function that we want to capture. Now, the question is can we capture this by a Mamdani fuzzy inference system?

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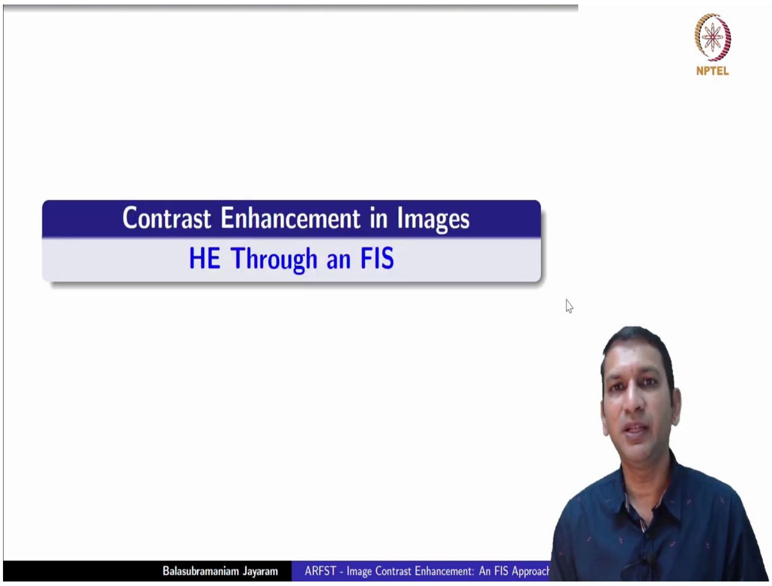

And let us see what it means to build a Mamdani fuzzy inference system to capture this idea. Now, we may not be implementing histogram equalization exactly, but we have taken the idea from here that it is essentially increasing the dynamic range that of pixel intensities contained in the picture can we implement this monotonic transformation using a fuzzy inference system; that is what we will look into presently.

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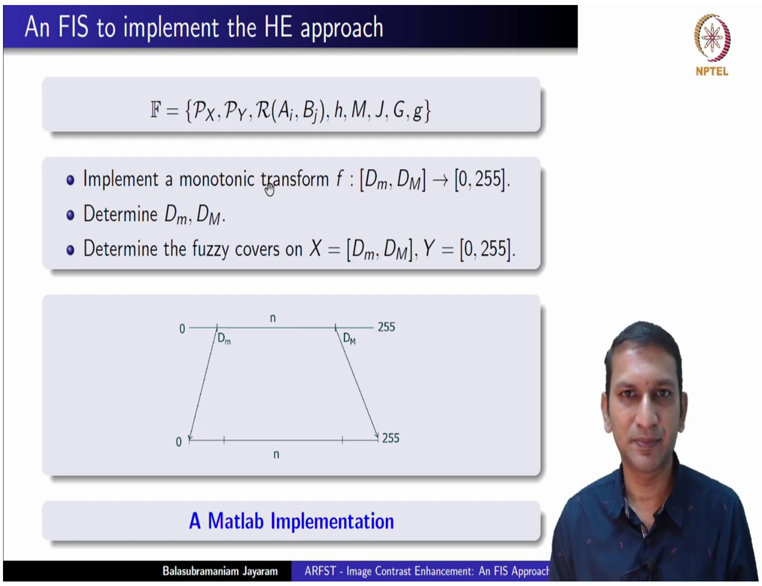

What does it mean to implement such a fuzzy inference system? Well, we know that a Mamdani fuzzy inference system which is a similarity based reasoning system to implement;

that means, specifying all these things. We need to specify the fuzzy covers on the input and output domains, build the rule base where the antecedents and consequence come from the corresponding fuzzy covering on X and Y.

And choose the corresponding operators a fuzzification, a matching function, a modification function, an aggregation function and finally, a defuzzifier. We know that in this case we need to implement a monotonic transform from the dynamic range D m D M to the entire 0 to 250 interval it would also be helpful if it were an on to function. So, that we really increase the dynamic range to the entire interval of 0 to 255.

So, it means to determine the dynamic range the endpoints of the dynamic range and find fuzzy covers determine the fuzzy covers on the input domain which for us is the dynamic range the interval D m D M and the output which is essentially the entire interval 0 to 255. Now, how do we do this? Earlier we used histogram equalization to stretch it from the dynamic range to the entire 0 to 255 interval expand the dynamic range.

In our case now, what we will do is take the input dynamic range, this interval and build n fuzzy sets on it. Similarly we will find n fuzzy sets on entire interval of 0 to 255 and relate each of these fuzzy sets on the input domain to the output domain as rules. And use these rules the rule base that is constructed this way in the Mamdani fuzzy inference system to approximate or to come up with this dynamic range expansion. How do we do this? Let us look at a particular MATLAB implementation of this idea.

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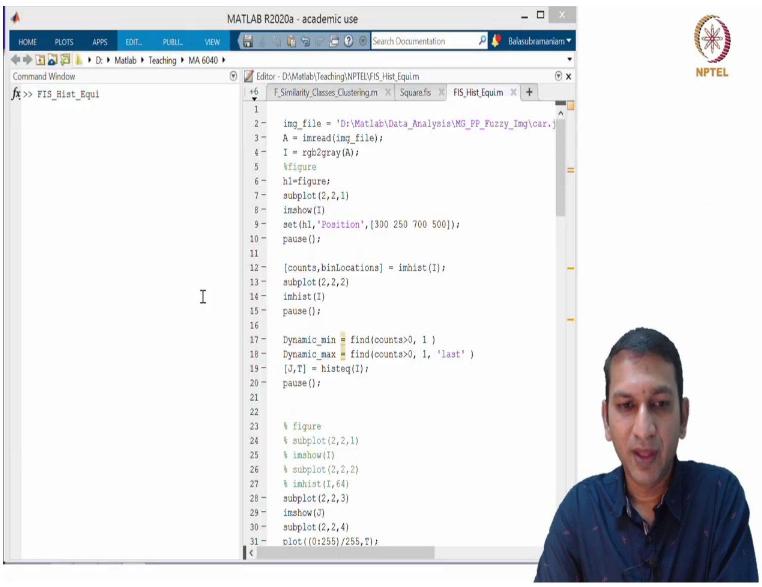

Well, let us look at one particular implementation of the idea that we just discussed.

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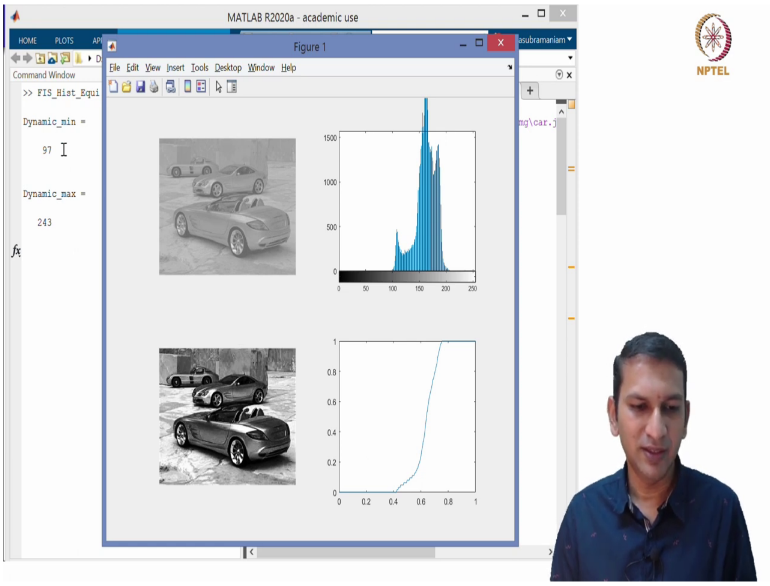

Let us consider the image of the cars that we have seen there is the corresponding histogram of pixel intensities, let us apply histogram equalization on this. Now, before that notice here that the dynamic min and dynamic max of this histogram of pixel intensities is essentially 97 and 243.

Well, looking at the figure we see that essentially it is around 200 or so, but there are also some pixel intensities pixels taking the value 243. So, this is the data that we have got from the histogram of pixel intensities. Now, let us apply the histogram equalization on this image, we have seen this image, which shows much more detail about of what is there in the picture and what we have here is essentially the monotonic transformation that the histogram equalization has performed on this image or on the pixel intensities of this image.

You will see once again that around 40 percent of 0 to 255 of course, these are the x and y axis are normalized. So, 0 to 255 is mapped to 0 to 1 on both x and y axis. So, around 97 would imply around 40 percentage 0.4 on this axis and you see here around 200 is essentially around 80 percent or 0.8 on this axis.

We see that before 0.4 up to 97 it is almost 0 and above 200 or so, it is almost 1; that means, we have extended pulled the dynamic range from 97 or 200 plus to essentially entire 0 to 255 and we see that it is increasing monotonically; that means, any pixel intensity value between 97 and 200 is going to be mapped in an increasing way.

So, you see here that a pixel value of 97 is perhaps mapped only to pixel value of 25, but the effective range has been increased and so, the contrast that is present in the image is actually getting better. Now, so, these are the two values that we want 97 and 243 and using this we will build our fuzzy inference system. So, now, that we have the dynamic range which is the interval 97 to 243 this constitutes the input domain x for us.

Next, the y domain the output range is already fixed it is between 0 to 255 that is how much we would like to expand this x into the question now is how to find the fuzzy covering on this? We will take n equispaced intervals on both the dynamic range x and also on the entire 0 to 255 interval and using these equi spaced points we will build our fuzzy covering.

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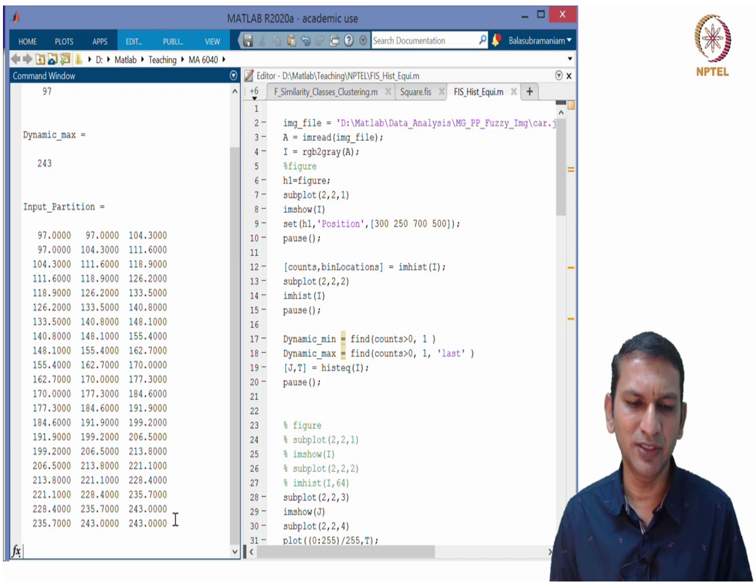

In this particular implementation what we have quoted here is to use triangular fuzzy membership functions both on the input and the output domains. So, you see the input partition consisting of 21 such fuzzy sets we have taken 20 equi spaced points between 97 and 243. So, you have got 21 fuzzy membership function which are triangular membership functions these will form the 21 antecedent on x.

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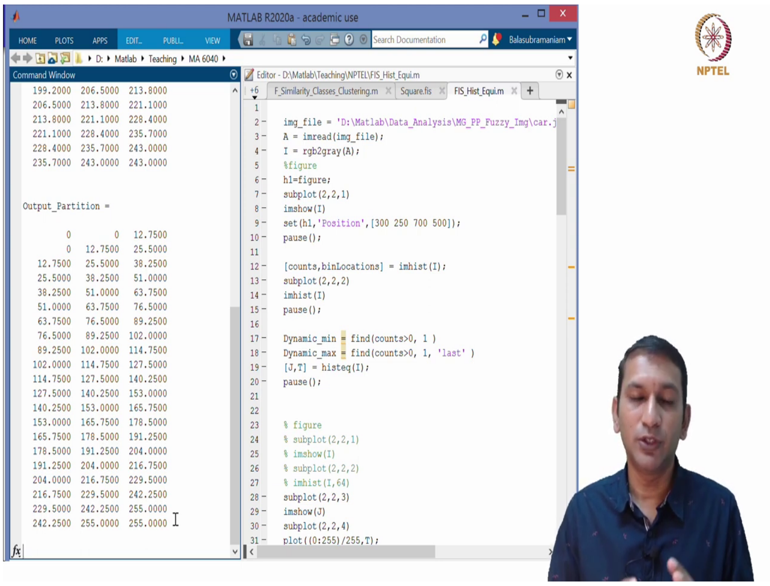

And similarly, the output space has also been partitioned into using 21 triangular membership functions. Now, using these values as the LC and R the left center and the right point for the triangular member function we can build a fuzzy inference system, allow me to show you the actual fuzzy inference system that we have built.

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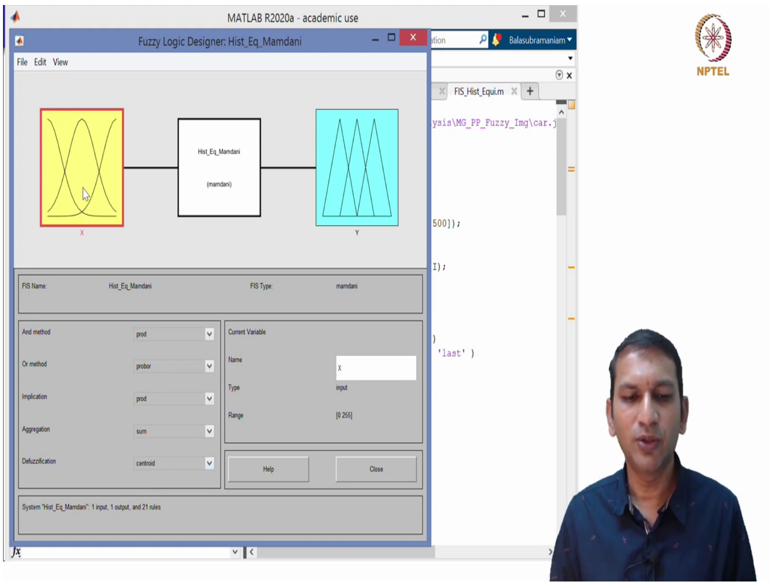

So, we see here that when you are using Mamdani fuzzy inference system implementing it in the fuzzy logic toolbox using MATLAB the H by default is fixed as a singleton fuzzification we have this and or method combined together to give us the matching function, for the modification which is shown here as implication we have taken product, for the aggregation the sound which is essentially the (Refer Time 18:58) and for defuzzification we are using centroid defuzzifier.

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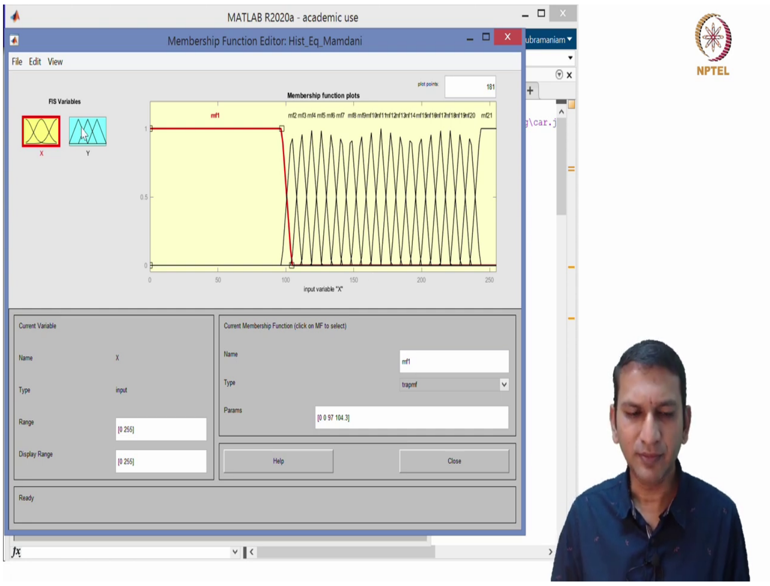

Let us look at how the input fuzzy sets are. So, now, this is the fuzzy covering that we have on the x domain you will see here we have mapped it from 0 to 255; however, effectively it is between 97 and 243 and these are the 21 membership functions which form the fuzzy covering on the interval 0 to 255.

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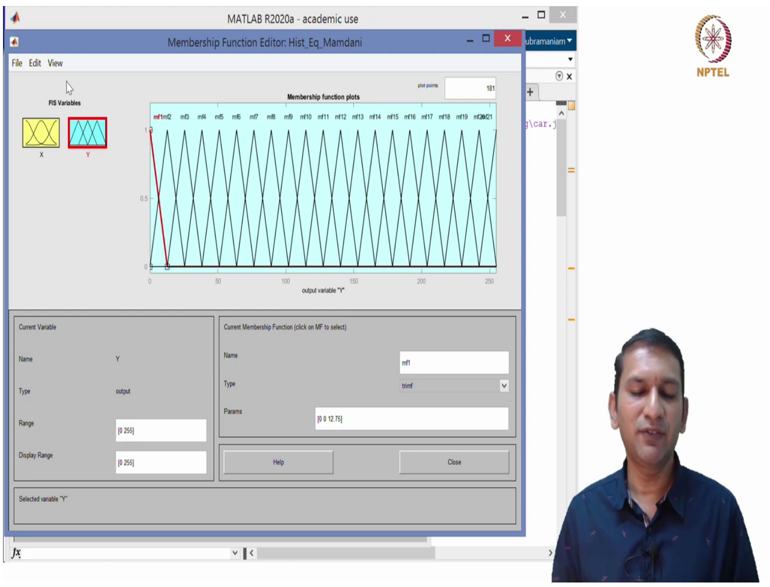

Once we have fixed up the number of output membership functions then on 0 to 255 using triangular fuzzy sets, it is straightforward these are the membership functions that we are going to have and all we now need to do is fix up the rules.

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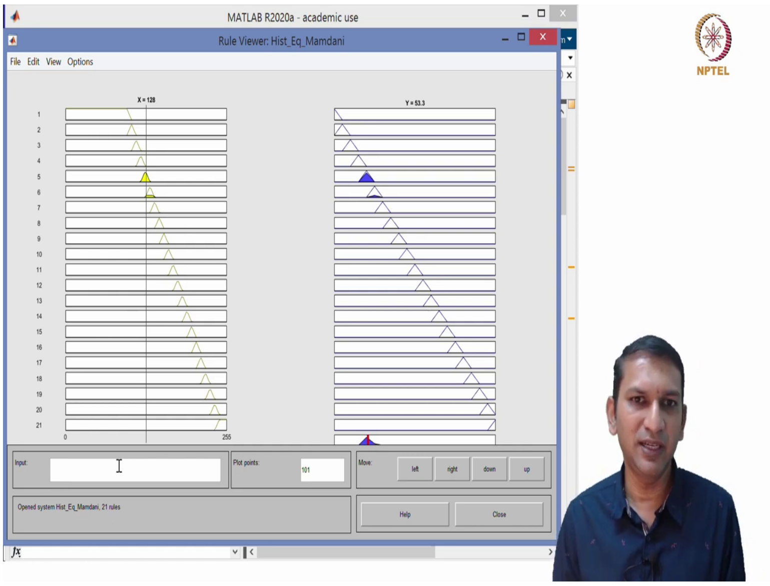

The way we have created the fuzzy coverings it was clear that we are going to map a one to b one and so on and that is what we see here. So, you see here there are these 21 rules 21 fuzzy sets on the input domain and 21 fuzzy sets on the output domain which are related.

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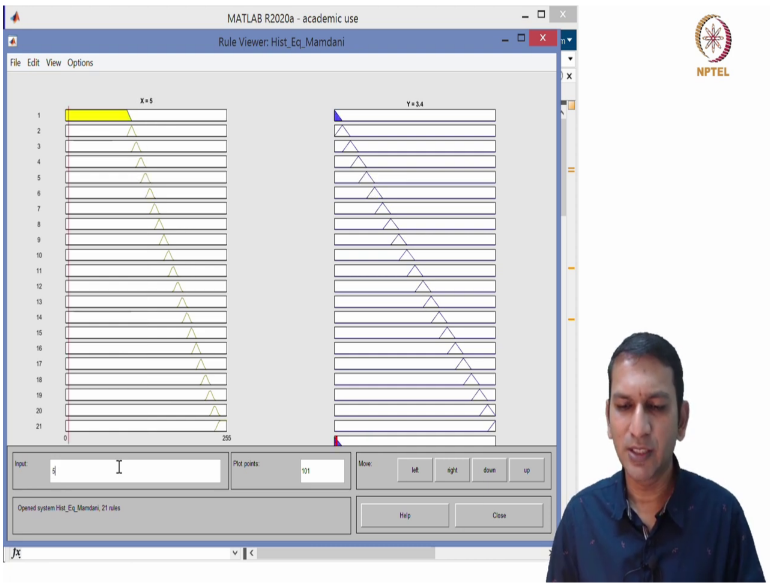

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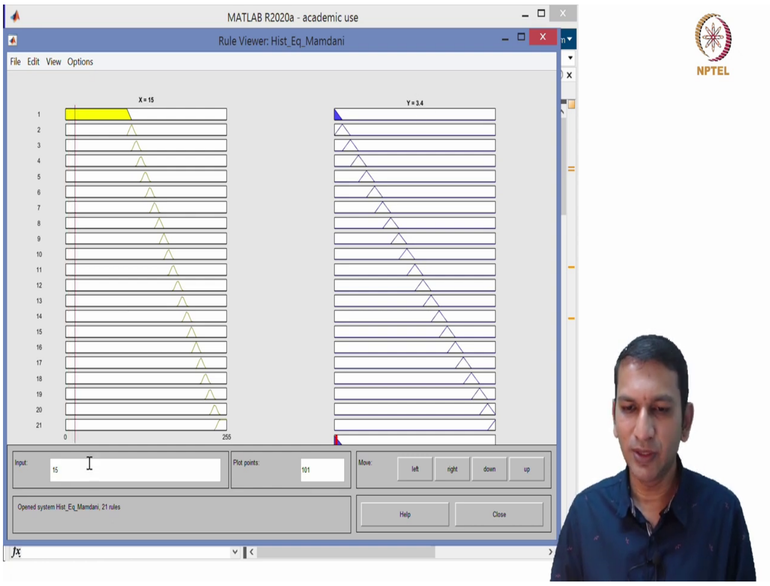

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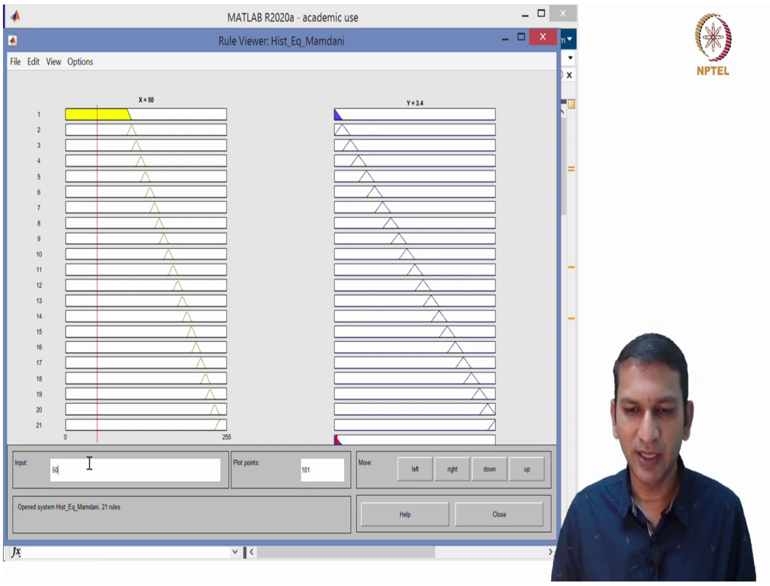

Now, let us give some values here and see. So, if you for this fuzzy system if you give a pixel intensity value of say 5 what we expected it should be 0, but it is somewhere very close to 0 3.5 and if you give say 15 once again it is 3.5, if you give 50 once again it is 3.4 slightly more up perhaps. In fact, it can be checked up to 96 it remains at 3.4 which is very close to 0, now from 97 onwards it starts to move up.

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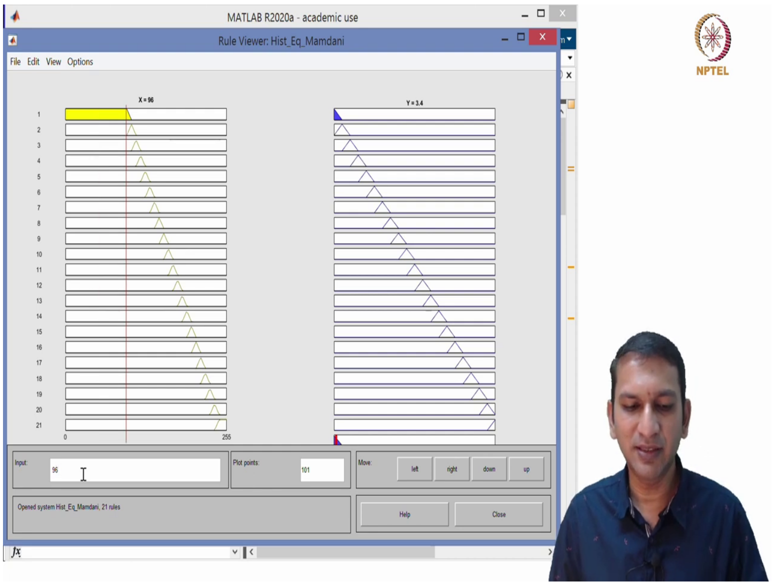

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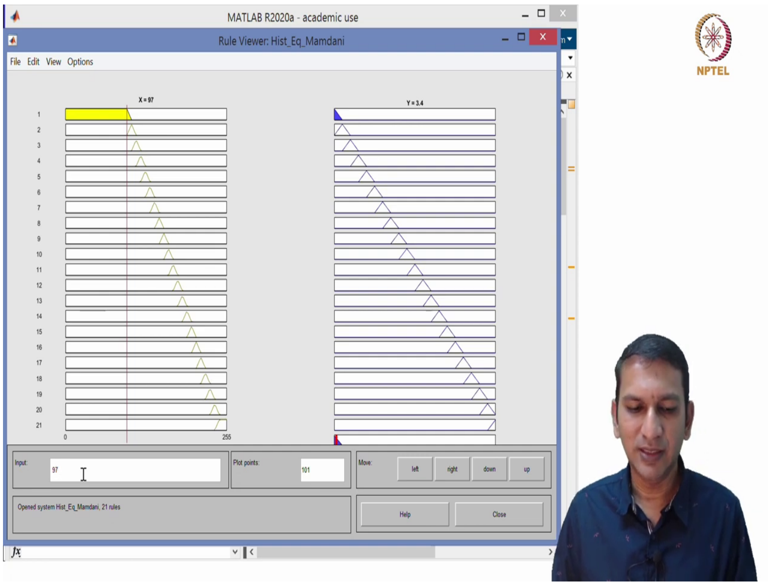

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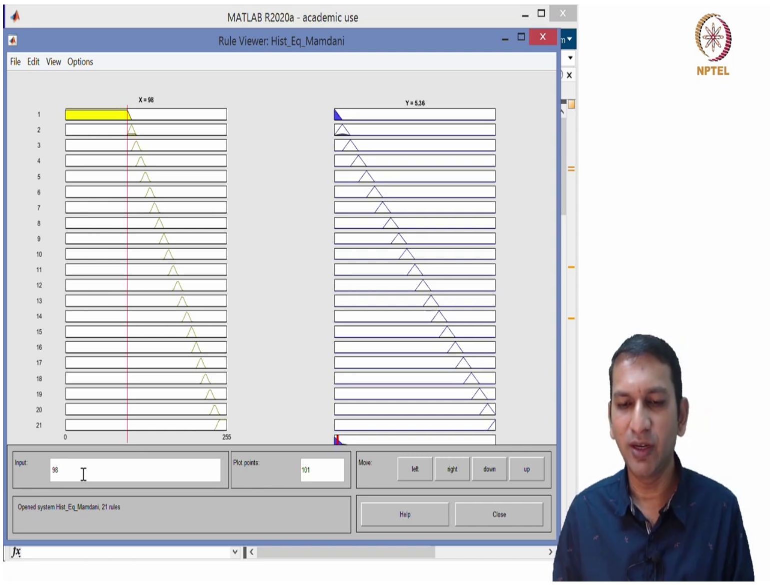

So, till 97 it is 3.4 if you put 98 you see already it is moving to 5.36.

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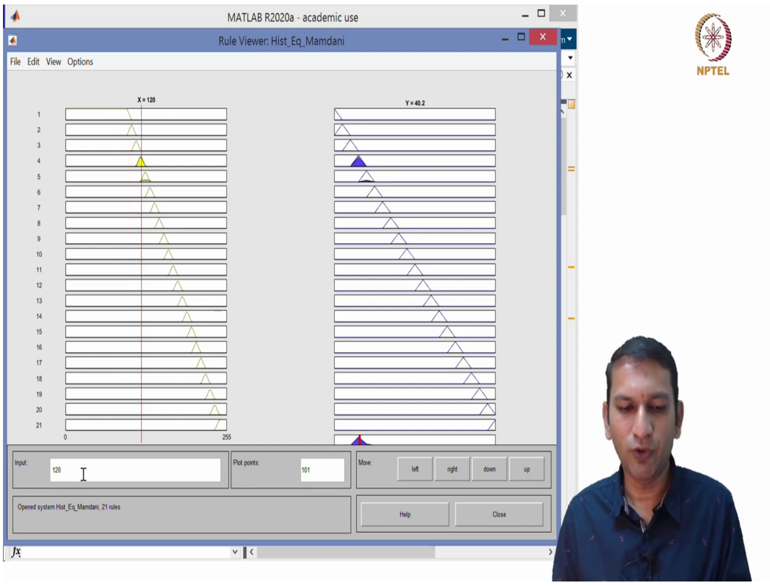

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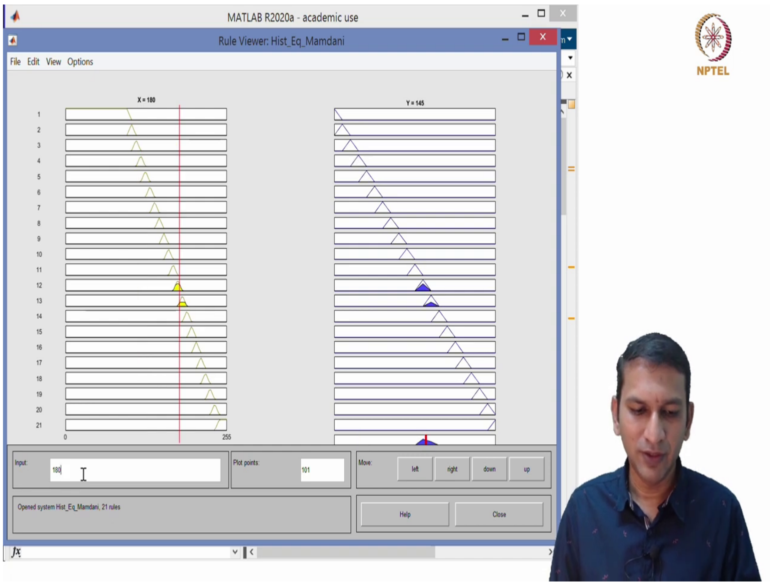

So, if you give 120 it has gone up to 40, if you give 180 it has gone up to 145. So, essentially we are stretching it between 97 and 243 we are going to stretch it.

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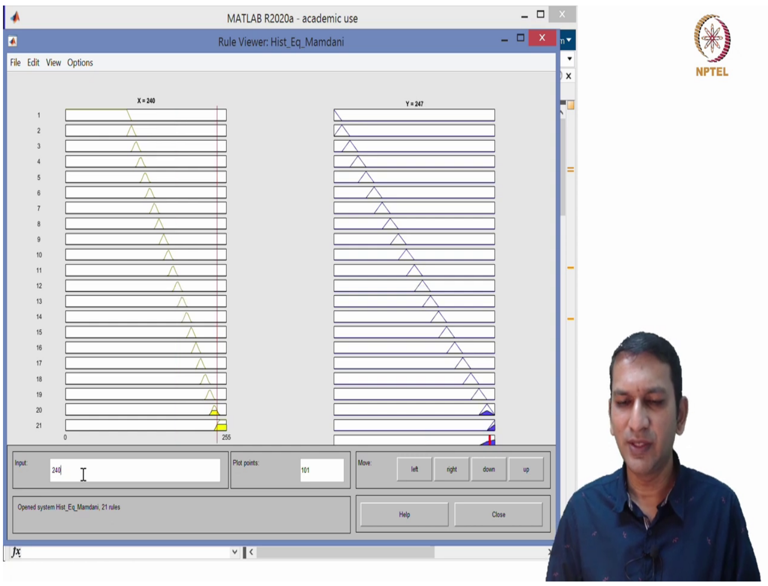

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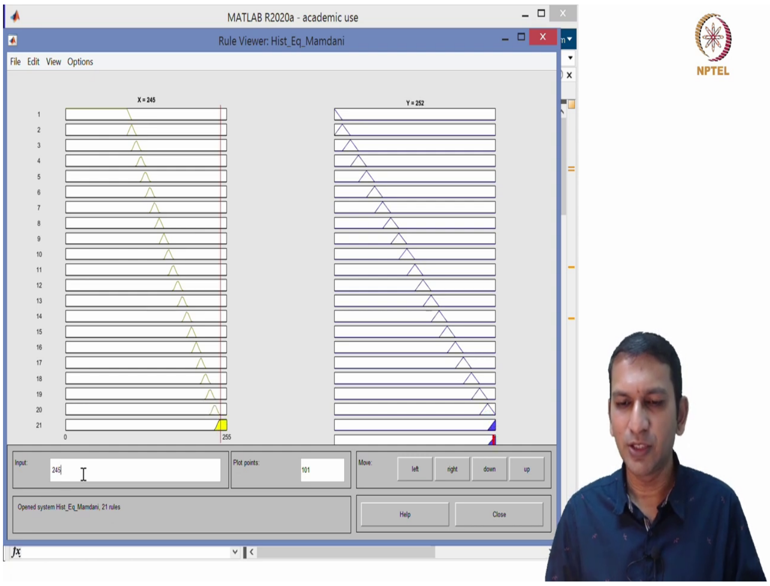

Let us look at what value 240 takes we see it has gone up to 247, if you go to say 245 it has gone to 252. So, it is very close to 255 which is the higher end of the spectrum between over the interval 0 to 255 after that almost it remains a constant whether you take 250 or not this is how it is.

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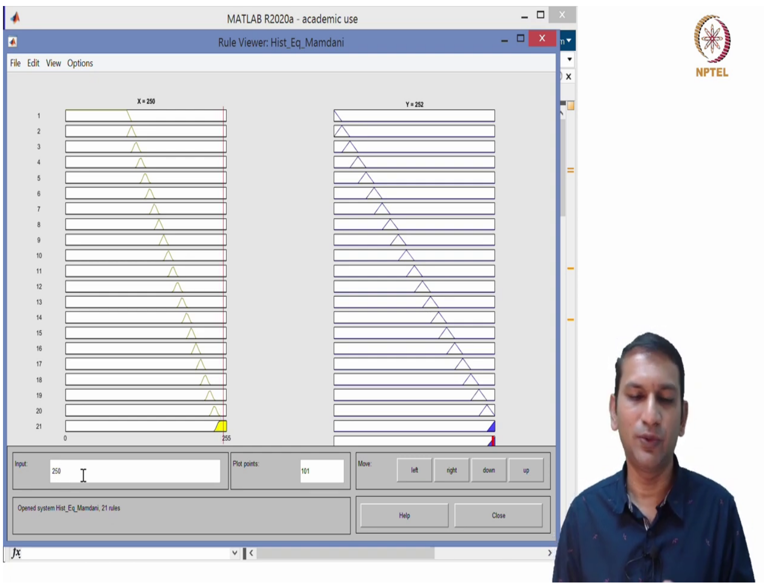

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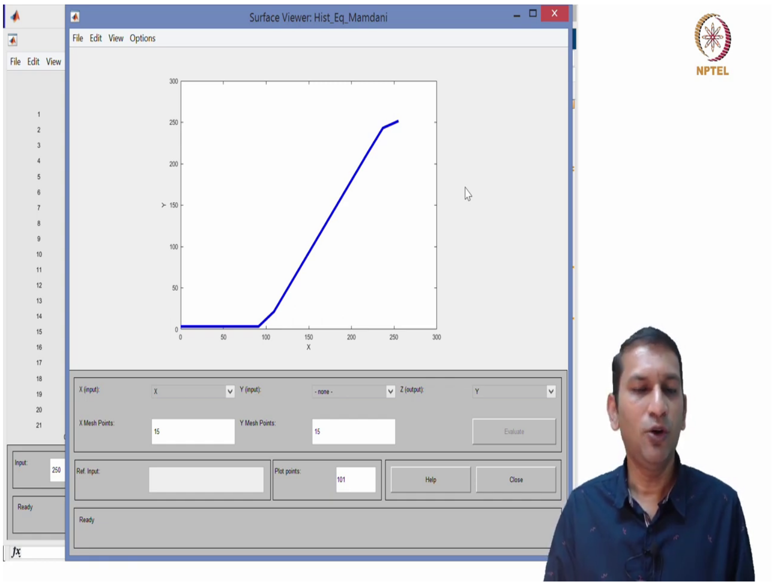

Now, let us look at the surface the function that it is trying to map, you see here up to 97 it is almost 0 and up to 243 or so, it is coming very close to 250 and it almost remains the same or slightly monotonic there. So, this is the system function that we have captured using our fuzzy inference system.

So, once again we had the idea that to be able to stretch the dynamic range we need a monotonic function whose domain is the actual dynamic range of the given input image, which needs to be stretched to the entire 0 to 255 interval which becomes our output domain y and how we did it was finding the dynamic range partitioning it finding a fuzzy covering on both x and y and relating them as antecedent and consequence in rules and using the Mamdani fuzzy inference system.

Well, now let us look at what exactly we get if we apply this fuzzy inference system on the original image. So, remember, the original image can be looked at as a matrix of pixel intensity values. So, we take the image and give each of these pixel intensity values at each of these positions and find out what is the output pixel intensity value given by a fuzzy inference system and map that as an image, map an image with pixel intensity values corresponding to the output of the fuzzy inference system.

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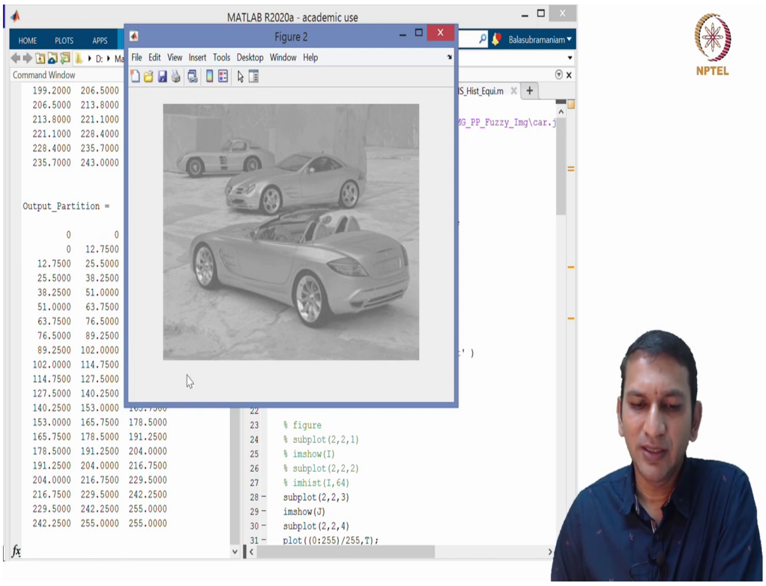

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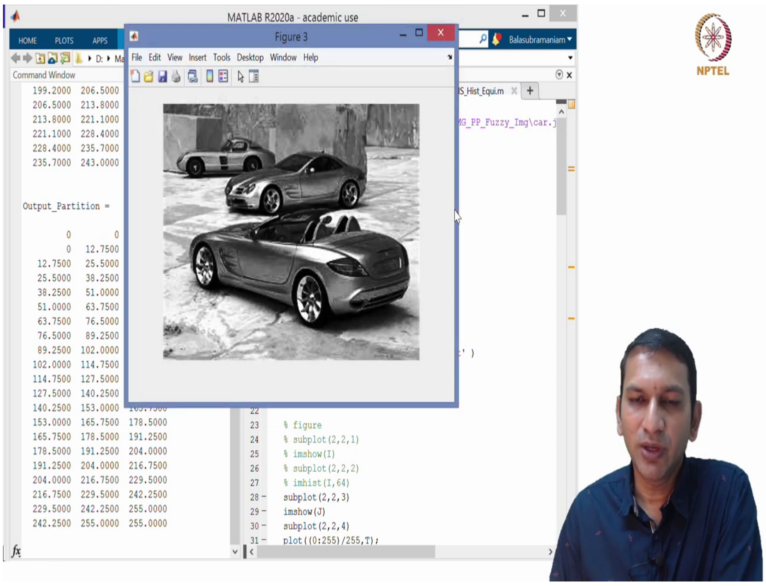

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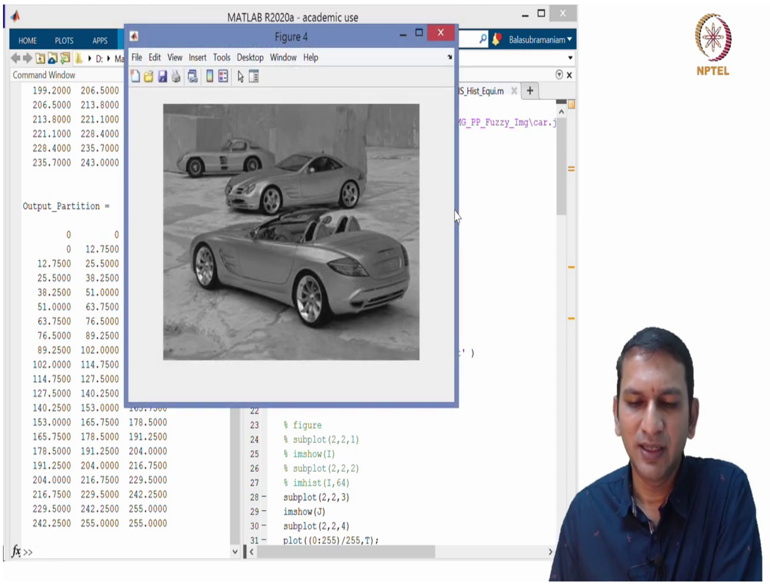

If we do that this is what we get, this is the original image this is the histogram equalized image and what we see here is actually the image that we would get if you imply if you employ the Mamdani fuzzy inference system that you have just built. Let us look at each of these figures in comparison also with along with their histograms of pixel intensities.

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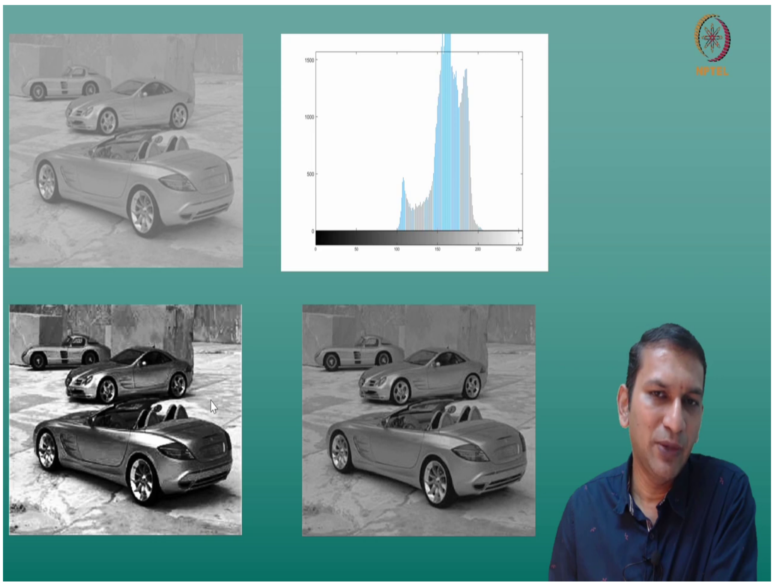

Well, this is the image that we began with and we know this is the corresponding histogram of pixel intensity. The histogram equalized image is given in the bottom left and this is the image we obtain when we apply the Mamdani fuzzy inference system that we have built wherein we have only taken the qualitative nature of the transformation that we want that is essentially monotonic and we wanted to expand the dynamic range of the image.

When we see here comparatively speaking we would see that this image seems little less exposed over exposed than even the histogram equalized image, it is interesting to look at the corresponding histograms.

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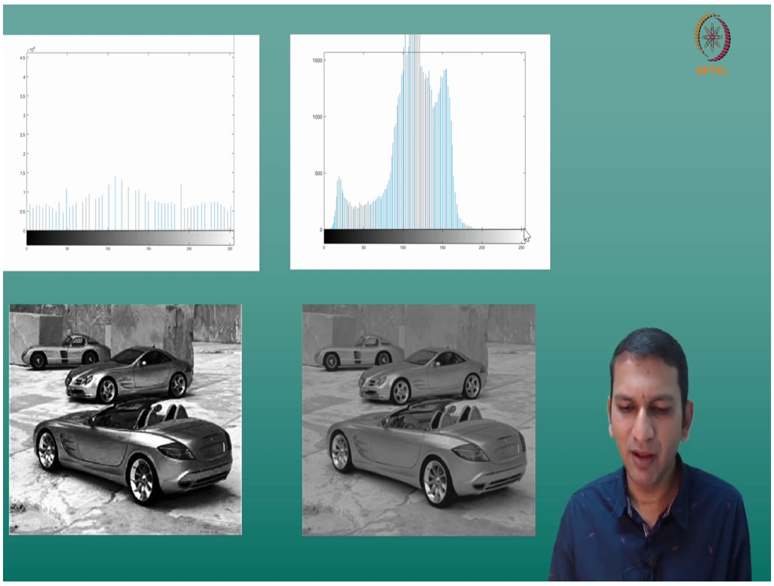

So, for the histogram equalized image what we have obtained is the histogram of pixels intensities is essentially covering the entire 0 to 255 interval. However, if you look at the histogram of pixel intensities for the image that we obtained of out of our fuzzy inference system by applying our fuzzy inference system this is what you see.

Essentially, it is stretching the original histogram of pixel intensities that is the histogram of pixel intensities of the original image even though it does not actually entire I mean extend to the entire interval 0 to 255.

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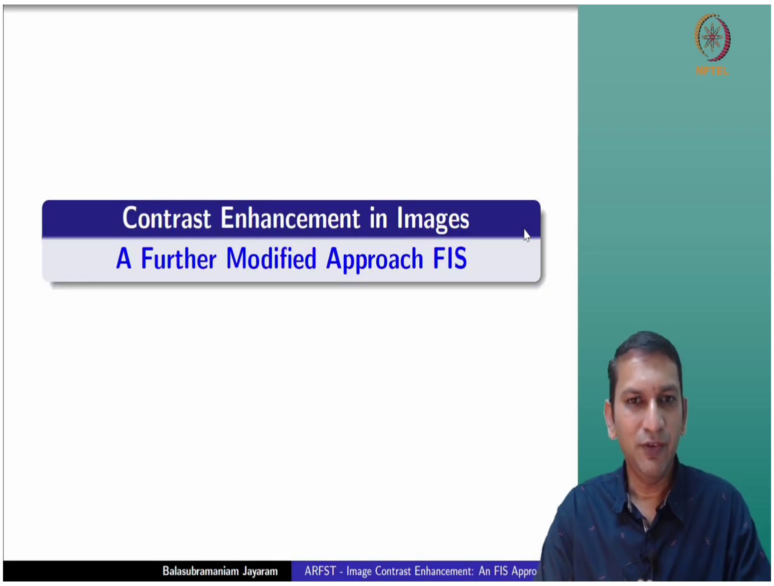

Now, this raises an interesting question, if you are only capturing the monotonic nature of the system function why only this kind of a monotonic function? If you look at it what is it that we have now obtained.

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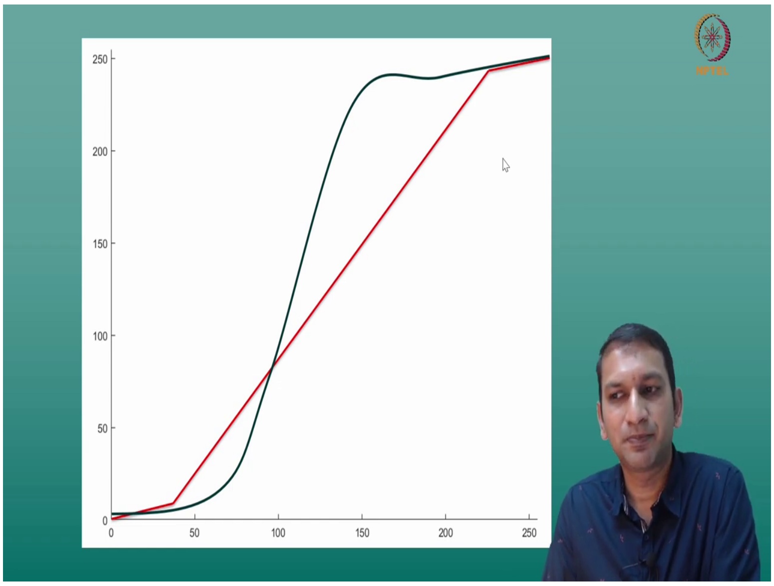

Essentially, this is the given any image assume this is your dynamic min and this is your dynamic max. So, essentially this is the range that we are trying to stretch, it could be stretched in a linear fashion or it could be stretched in a non-linear fashion where the rate of increase can be different at different parts of the domain different parts of the pixel intensity domain, but the question is how do we find in which part we need to increase the rate of increase?

Well, what we have done so far is found we have found out the dynamic range of the given picture and we have expanded it. But even when you look at the histogram of pixel intensities we see here while this is the entire dynamic range it appears that the all the pixel intensities are in fact, concentrated in a sub range or sub interval of this dynamic range.

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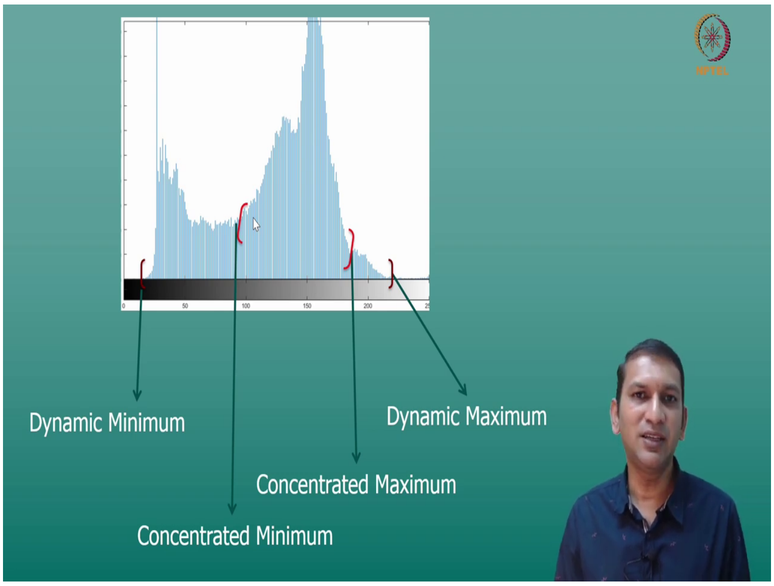

This becomes even more clear in this picture while the dynamic range seems to be between say 20 and 220, essentially this seems to be the that the we could call as the concentrated dynamic range is effectively in a much smaller interval. So, this is your dynamic minimum, this is your dynamic maximum. However, if you see here most of the pixel intensity values large number of pixels most of the pixels their intensities are within this small sub segment of the dynamic range.

Let us call it the concentrated minimum and concentrated maximum, why not stretch this part aggressively? So, while we stretch the dynamic range to 0 to 255 we do not stretch it uniformly the rate of increase of this monotonic function is not uniform, but it is faster over this concentrated dynamic range.

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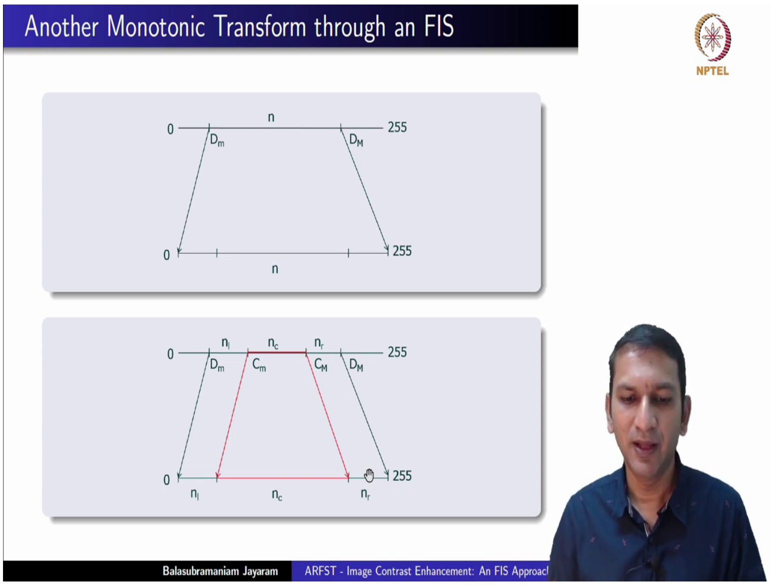

So, essentially the idea is not to just only look at the dynamic range this is what we did earlier we took n fuzzy sets on the dynamic range, n fuzzy sets on the 0 to 255 interval and mapped these as antecedent and consequence coming from this corresponding fuzzy covers, instead why not look at it from this point of view?

Take the dynamic range, take the concentrated range on each of these pieces between D m C m C M C capital M and C capital M D capital M, why not define n l n c and n r fuzzy sets? Similarly, on the mapped domains on 0 to 255 this mapping can be done in different ways perhaps here it is the identity mapping. So, essentially D m is mapped to 0 and C m is mapped to 0 plus C m minus D m; similarly on this side and this range effectively where most of the pixels are concentrated that we actually increase or stretch even further.

If we use this idea essentially the same kind of procedure that we followed to build the previous fuzzy inference system will be followed; that means, we are going to divide this into n l equi spaced intervals have perhaps triangular fuzzy sets on all of these sorry you will have n l plus 1 such fuzzy sets here and n c plus 1 here n r plus 1 here. So, totally you will have n l plus n c plus n r plus 3 fuzzy sets.

Similarly, on this side so, you will relate them as antecedents and consequence and you will have those many fuzzy rules, load it into the Mamdani fuzzy inference system choose the operations carefully and what we will have is perhaps a monotonic function coming out of it, if we apply this how would those figures look like, let us look at it.

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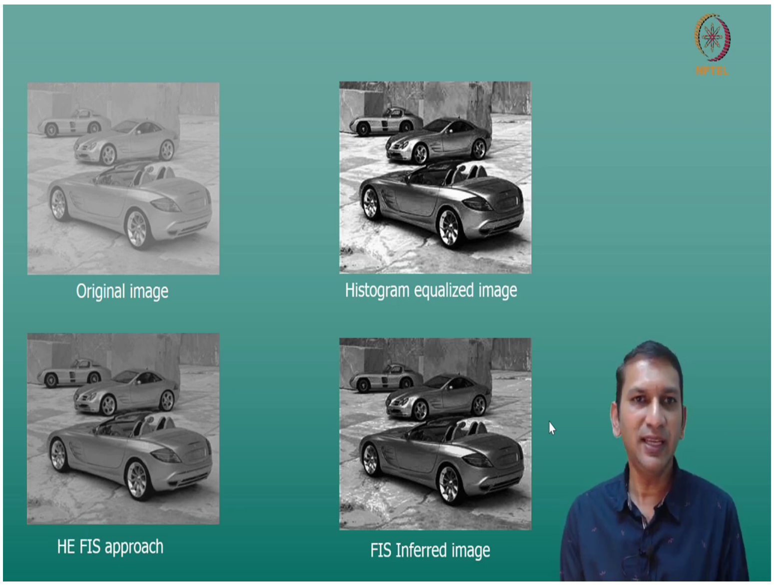

So, this is the idea that we have, let us look at what happens to the image with cars, this is the original image, this is the histogram equalized image, this was the image that we obtained when we tried to take the histogram equalization kind of an approach where we only stretched the dynamic range of the original image.

Now, when we modified it; that means, we also concentrated, on the concentrated dynamic range and stretched it monotonically at a faster rate this is the image that we get. Clearly using the qualitative nature of the system function this could be called as a first cut idea and we have refined this and you can see that among the 4 images that we have got in terms of visual quality perhaps this ranks on top.

Now, all of this we have done pretty heuristically; however, we are taking information from the given image itself. So, in that sense it is dynamic let us apply this idea to also other images.

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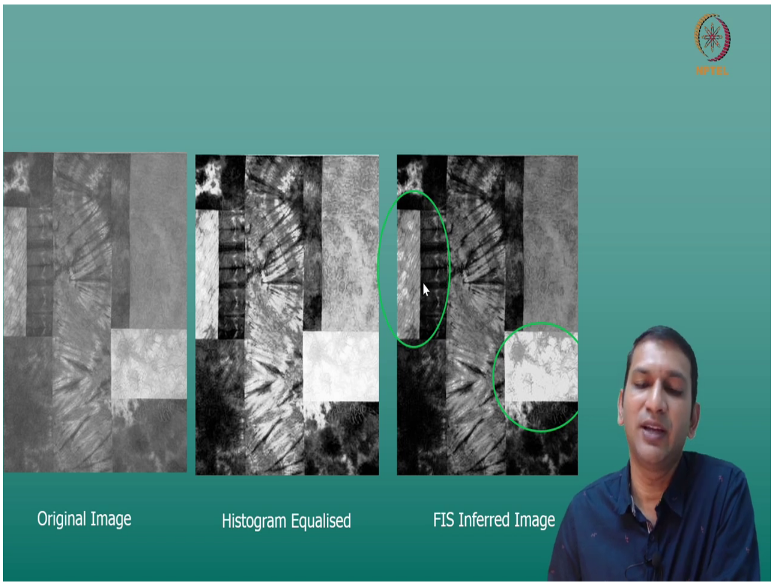

Let us look at this image this is the corresponding histogram equalized image, now this is the image that we have obtained by applying the Mamdani fuzzy inference system as we have just discussed; that means, we not only have the dynamic range, but also making use of the concentrated dynamic range of the pixel intensities that are present in this image.

Clearly you will see that the third image is far better than the first 2 images. In fact, if you look at these two areas the grains here are perhaps standing out at a much greater finer detail than in the other 2 images.

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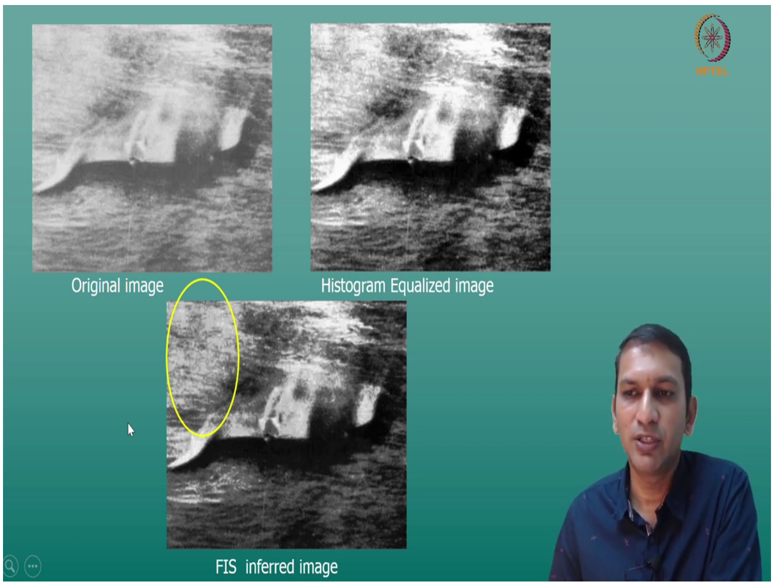

So, if you look at this particular image the histogram equalized image is this and what we obtain out of our fuzzy inference system is this, clearly you can see in some parts of the image it is the details are much better the contrast in enhanced is much better than the original of the histogram equalized image.

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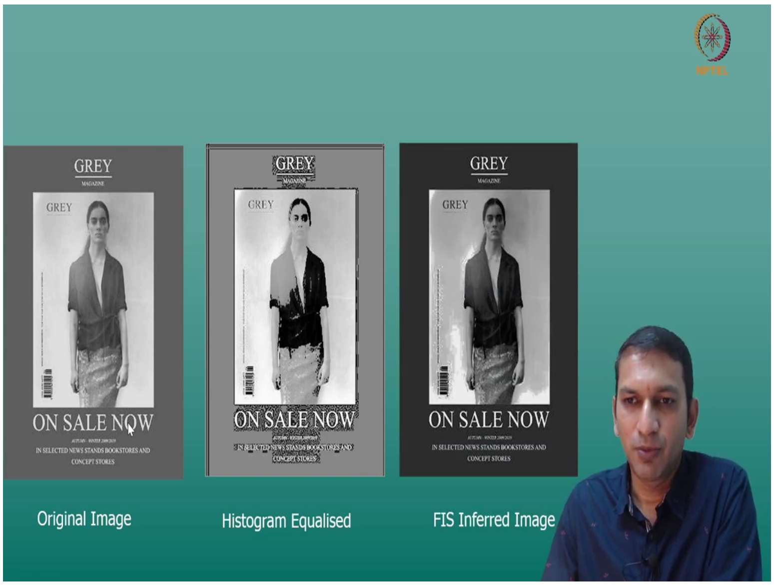

And in this case it is immediately clear visually that the image processed by a fuzzy inference system is much better than the original of the histogram equalized images.

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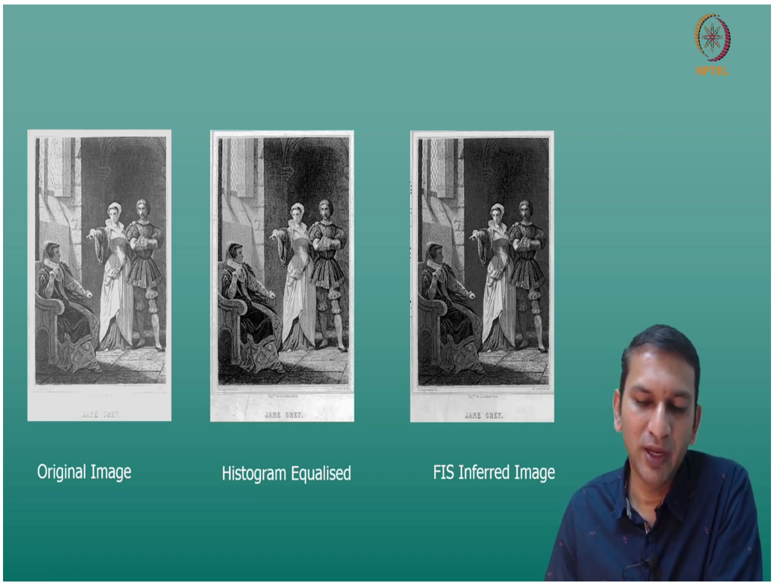

Once again even in this case you might notice here the depth that we have behind the characters is much better from the image obtained by applying the fuzzy inference system.

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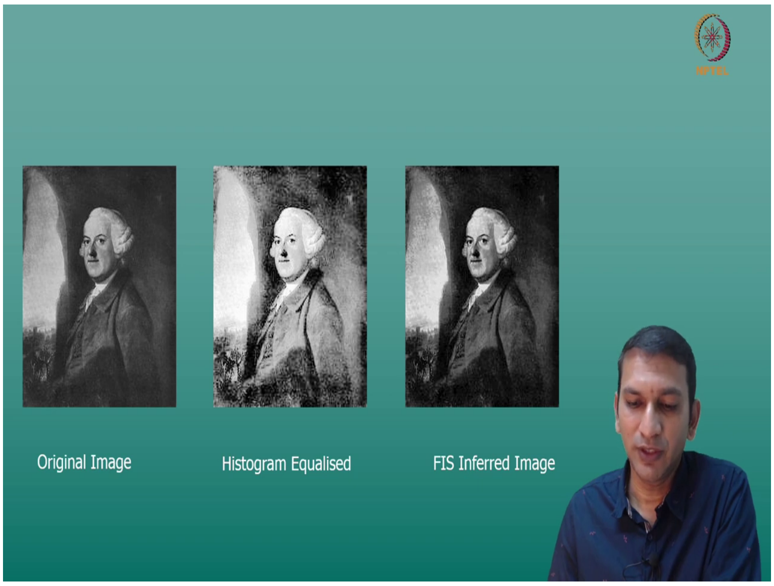

Once again it is immediately palpable that the third image is far superior than the first 2 images perhaps.

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However, we should not leave with the feeling that this fuzzy inference system that we have constructed is better always of course; this is only a first cut and refinement of the first cut.

We will see here that in some cases the histogram equalized image does better than the fuzzy inference system itself.

The fuzzy inference system that we have built of course, as fuzzy set it was from first principles a basic heuristic based approach to building the fuzzy inference system, but if you would like to take many more parameters or more factors into consideration you could build a far better fuzzy inference system, which would do a much better enhancement of contrast given a picture.

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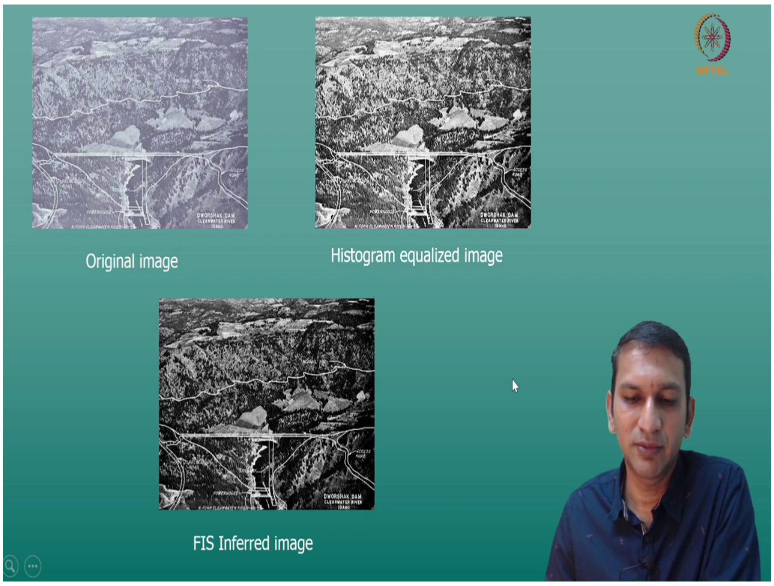

So, this was one of the images that we have seen earlier this is the histogram equalized image and this is what we would get from our fuzzy inference system. Clearly, the details here are much better quality than the previous 2 images of the earlier 2 images that are there on the top.

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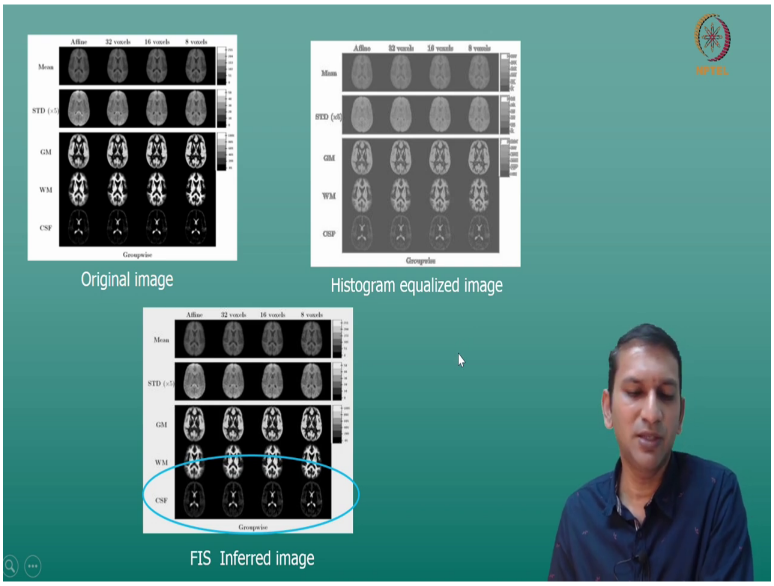

Finally, if you look at this medical scan image we see that what we obtained from our fuzzy inference system processed by a fuzzy inference system seems much better than the original of the histogram equalized image. Especially in the bottom row the details are much clear than clearer than either of the original or the histogram equalized images.

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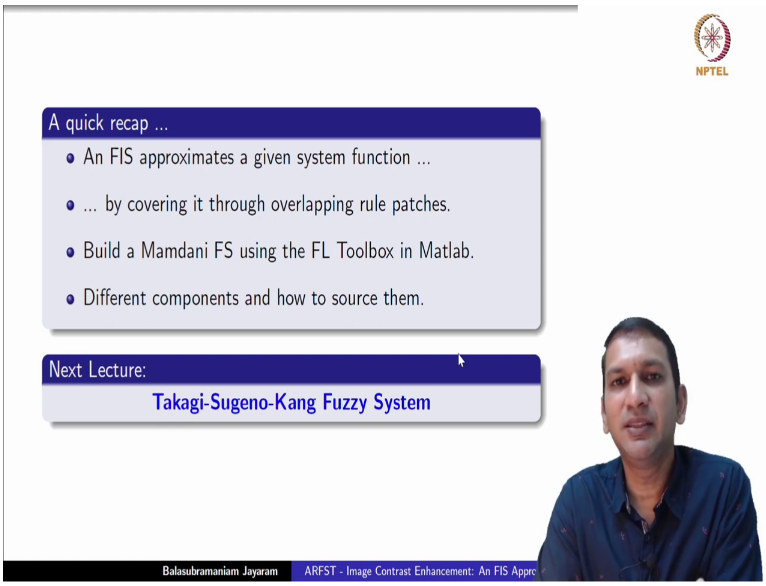

A quick recap of what we have seen in this lecture, we know that a fuzzy inference system approximates a given system function, by covering it with overlapping rule patches, using I that idea in the back of our mind we built a Mamdani fuzzy system, using the fuzzy logic toolbox in MATLAB and we have shown how such a fuzzy system inference system could be built for contrast enhancement in for in a given grayscale image.

Along the way we have seen a practical way a procedural way of how to pick the most important components to build the fuzzy inference system essentially identifying the input and output domains and building the fuzzy covers on these two domains and relating them as antecedents and consequence to make the rule base and finally, choosing appropriate operators for the inference mechanism itself. This is what we have seen in the last two lectures, in this lecture specifically for a particular practical application that we have taken.

This application was chosen only so that it is easier to explain and easier to code and also to demonstrate, but there are many many more such applications where fuzzy inference systems perform very decent and often a good job basically being able to construct a fuzzy inference system from heuristics or the qualitative nature of the system function when other information about the system function may not be available or exact information about the system function or even data from the system function may not be available to approximately.

In the last lecture of this week we will look at the Takagi Sugeno Kang Fuzzy Systems, you might recall there are two major types of similarity based reasoning inference schemes; one is the Mamdani fuzzy inference system, which we have discussed at length both the theoretical aspects and also the implementing and practical application aspects of Mamdani fuzzy inference system. The second major SBR inference scheme is the Takagi Sugeno Kang fuzzy system and this is what we will take up in the next lecture. Glad you could join us for this lecture hope to see you soon in the next lecture.

Thank you all.