

Approximate Reasoning using Fuzzy Set Theory
Prof. Balasubramaniam Jayaram
Department of Mathematics
Indian Institute of Technology, Hyderabad

Lecture - 33
Fuzzy Inferencing Schemes - A Visual Illustration

(Refer Slide Time: 00:16)



Hello and welcome to the first of the lectures in this week 7 of the course titled Approximate Reasoning using Fuzzy Set Theory, a course offered over the NPTEL platform. In the last week of lectures, we were introduced to the basic schema of fuzzy inference system and we went on to see one major type of fuzzy inference mechanism or fuzzy inference system which is that of fuzzy relational inference.

We looked at how to perform the inference in the presence of a single rule be it single input single output or multi input single output and also in the case of multiple rules where we brought in 2 different types of inference strategies that of first infer then aggregate and first aggregate then infer. In this week, we will look at the second major type of fuzzy inference systems that of similarity based reasoning.

In the first lecture of this week, we will have a gentle introduction to fuzzy inference systems specifically one type of similarity based reasoning. We will largely go with a Visual Illustration of the entire inference process itself.

(Refer Slide Time: 01:40)



Well, before that if we were to trace back in history and look at who could be considered as the fatherly figures of fuzzy set theory definitely Professor Lotfi Zadeh without any hesitation can be attributed to bringing in fuzzy set and its applications into the stream of engineering.

He was a professor of control theory in UC Berkeley and with the seminal paper in 1965 he showed that fuzzy sets can be brought into use in engineering strips, as a control engineer he brought them into use into control systems. In one of his prefaces to a book he had divided fuzzy logic or what he called fuzzy set theory as fuzzy logic into 2 major types or broad divisions that of fuzzy logic in the broad sense and fuzzy logic in the narrow sense.

Fuzzy logic in the broad sense he said refers to all the engineering applications where fuzzy set theory is being used and according to him fuzzy logic in the narrow sense can be thought of as the logical aspects, the algebraic and analytic aspects of fuzzy set theory itself.

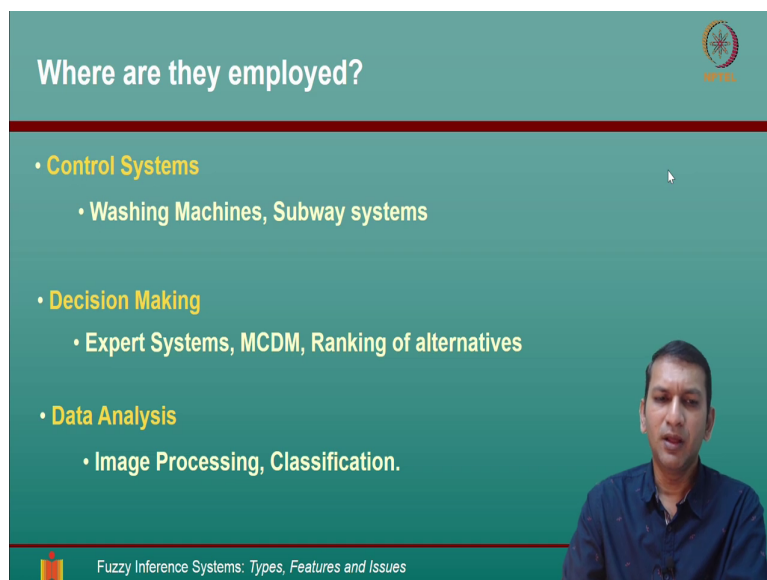
And, if one way to trace back in history as to who could be consider as a fatherly figure for this almost unarguably it has to be Jan Łukasiewicz who during the early 20s proposed multi valued logic and discussed explored them. So, these perhaps are the 2 fatherly figures for fuzzy set theory looked at in the sense of theoretical or explorations or in applicational aspects.

(Refer Slide Time: 03:34)



With a short introduction let us get into fuzzy inference systems. We have seen fuzzy relational inference, but we will look at one particular type of similarity based reasoning visually.

(Refer Slide Time: 03:52)



Before that, we should also understand where fuzzy inference systems themselves are being utilized. We know why the need for fuzzy set theories itself arose, but now we are discussing where are we using fuzzy inference systems. As was mentioned professor Zadeh himself was control systems engineer which meant initially it was applied in control systems.

These days you hear about fuzzy logic washing machines, fuzzy logic camcorders and what have you. But, perhaps one of the earliest systems to adopt using fuzzy set theory was the subway system and some of the cities in Japan especially that of Sendai. Next came decision making quite; obviously, and naturally because we are dealing with reasoning and fuzzy set theory still plays a major role here in the form of being incorporated into expert systems, multi criteria decision making, ranking of alternatives and so on.

And, of course, when you look at it from the point of view of data analysis, we know that fuzzy set theory fuzzy inference systems form part of what they call the soft computing. So, they play a huge role in data analysis specifically in the areas of image processing in terms of image segmentation, the edge detection and also in classification.

(Refer Slide Time: 05:19)




Having said this, we said that as a control engineer professor Zadeh brought fuzzy set theory into control systems and at that time he had explained his motivation for bring in fuzzy set theory.

Of course, a control engineer would be able to better present it and better we will better understand it, but only so that we have a idea of what used to happen with classical rule based systems and how fuzzy set theory helps us in an immediate sense. Let us look at a water down version of a classical rule based system and identify some things which we can immediately iron out by the introduction of fuzzy sets.


(Refer Slide Time: 06:04)

Classical Rule-Based Control

- A Control System controls the output of a system
- It maps “inputs” to “outputs”
- The relation between i/p and o/p is given as:
if - then rules



Fuzzy Inference Systems: Types, Features and Issues





So, in classical rule based control we want to control a system and have a control system there, what does a control system do? It controls the output of a system. So, given an input, it controls the output based on the input. So, in that sense it is a mapping between inputs and outputs. Quite clearly for us now this relation between input and output is given in the form of if – then rules, classical if – then rules.


(Refer Slide Time: 06:40)

Classical Rule-Based Control of an AC

- Goal:
 - Room temperature ~ constant
- Working:
 - Thermostat senses the room temperature
 - Adjusts the rpm of the motor
- Inputs : Varying room temperatures - T (deg)
- Outputs : Speed of the motor - S (rpm)



Fuzzy Inference Systems: Types, Features and Issues

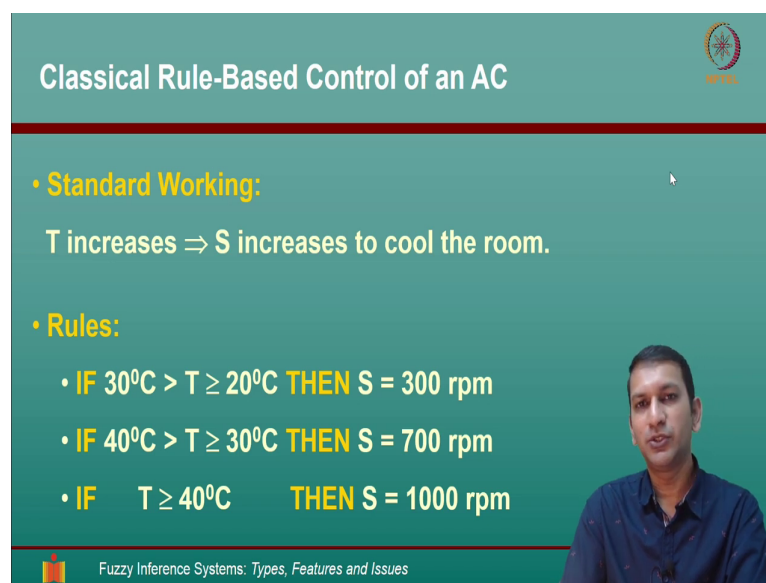


Let us look at an example. So, let us say that we want to implement a control system for an air conditioner. You know what could be the goal of an air conditioner? Ideally it is to keep

the room temperature at a constant. So, now, to help us do this, what are the components present in an air conditioner? The working of it could be captured in just these 2 points.

So, you have a thermostat which senses the room temperature and accordingly, adjust the fan speed essentially the rpm of the motor rotations per unit of the motor. So, the inputs here are the varying room temperatures which the thermostat senses using some sensors and the output is the speed of the motor, the fan speed which it adjusts. So, essentially by measuring the temperature we need to handle or assign an appropriate fan speed for the motor.

(Refer Slide Time: 07:36)



Classical Rule-Based Control of an AC

- **Standard Working:**
T increases \Rightarrow S increases to cool the room.
- **Rules:**
 - IF $30^{\circ}\text{C} > T \geq 20^{\circ}\text{C}$ THEN S = 300 rpm
 - IF $40^{\circ}\text{C} > T \geq 30^{\circ}\text{C}$ THEN S = 700 rpm
 - IF $T \geq 40^{\circ}\text{C}$ THEN S = 1000 rpm

Fuzzy Inference Systems: Types, Features and Issues

How would we do this? So, if we were to capture it, there is a function behind the working of the system. So, if T denotes temperature and S denotes the fan speed, then there is a function mapping between T and S. So, you could say S is equal to F of T. This function obviously, we do not know or even if we know we need to be able to simulate it, we need to be able to approximate using the control system. But, what we do know qualitatively is they have a monotonic relationship as T increases we want S also to increase.

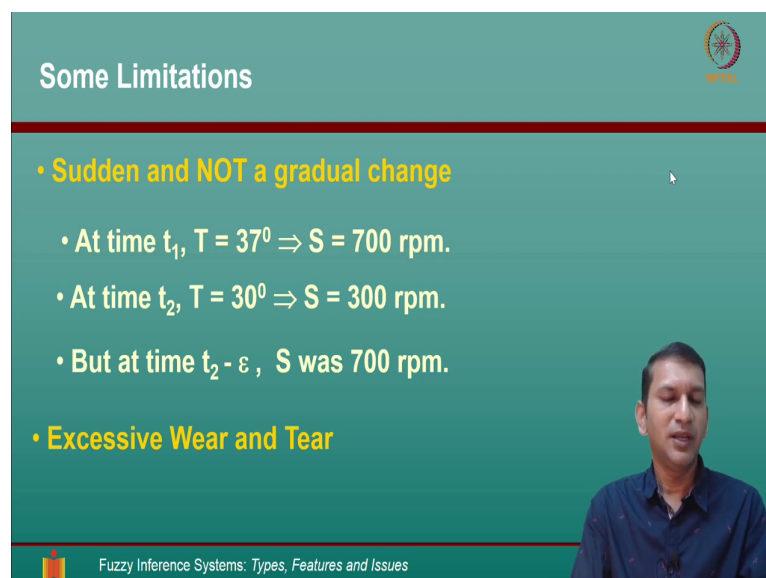
So, perhaps we could come up with rules of this type if T is between 20 and 30 degrees then the fan speed should be 300 rpm. Similarly, if T is between 30 and 40 degrees, then the fan speed should be 700 rpm and if it is more than 40, then the fan speed should be 1000 rpm. So, there are few things that perhaps we should note here.

Firstly, if you look at these rules, these are classical if – then rules because you have if you recall what we have done in one of the earlier lectures in the last week we started with a classical if – then rule, where it is of the form if X belongs to A, then Y belongs to B. You could look at T belonging to 20 to 30 degrees as T belonging to the interval 20 to 30, which is essentially a subset of the domain whatever we are considering as the temperature domain.

And S is equal to 300 rpm. So, you could look at 300 rpm as a single tensor. So, it is more like if T belongs to A 1, then S belongs to B 1. Of course, in this case B 1 is just a single tensor. So, we are looking at really classical if – then rule. Second thing that we should note here is given a T how does it work? All it does is, it checks much like a couple of lines of code that you would probably write in a basic programming language like C. If T belongs T is between these 2 values, then S should be this first thing this value.

And you will see that there is no overlapping here. So, you would just use if-else-if else if so that kind of a structure you have here and that is how the inference has been done. So, the inferencing with classical if then rules is very clear straightforward and there is no parallelism because we do not allow for overlapping sets. So, only one of these rules comes into play for a given temperature.

(Refer Slide Time: 10:10)



Some Limitations

- **Sudden and NOT a gradual change**
 - At time t_1 , $T = 37^\circ \Rightarrow S = 700$ rpm.
 - At time t_2 , $T = 30^\circ \Rightarrow S = 300$ rpm.
 - But at time $t_2 - \epsilon$, S was 700 rpm.
- **Excessive Wear and Tear**

Fuzzy Inference Systems: Types, Features and Issues

Now, some immediate limitations can be pointed out the first of them is this. Look at it, the change in temperature because that is what finally, we are controlling using this control

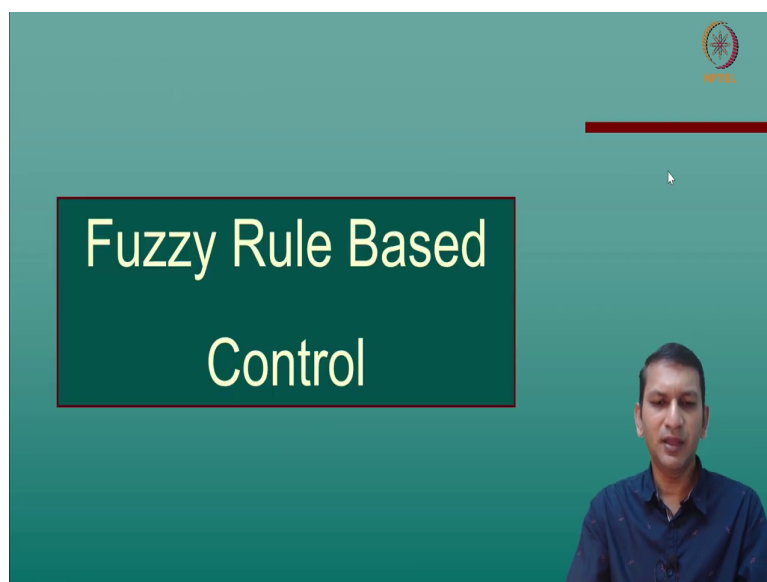
system and air conditioner and that is what we feel also when we keep the air conditioner on the change is sudden and not a gradual one. Why do we say this?

Let us see at time t_1 the temperature is 37 degrees and according to the rule the fan speed should be 700 rpm. So, after a certain time of time period it starts to cool down and the temperature drops to 30 degrees. The moment it drops to 30 degrees at time t_2 , the second rule kicks in and it says that no we should run it at 300 rpm – that is the first rule that what we see what we saw in the earlier slide.

But, what is interesting now is consider the time $t_2 - \epsilon$ for some small epsilon at that point of time the fan speed were was running at 700 and now, suddenly you have shifted from 700 to 300. It is not a slow gradual change and hence accordingly, the temperature variation also will be sudden and not gradual. Not only is the temperature variation not gradual, this also leads to excessive wear and tear of the mechanical parts.

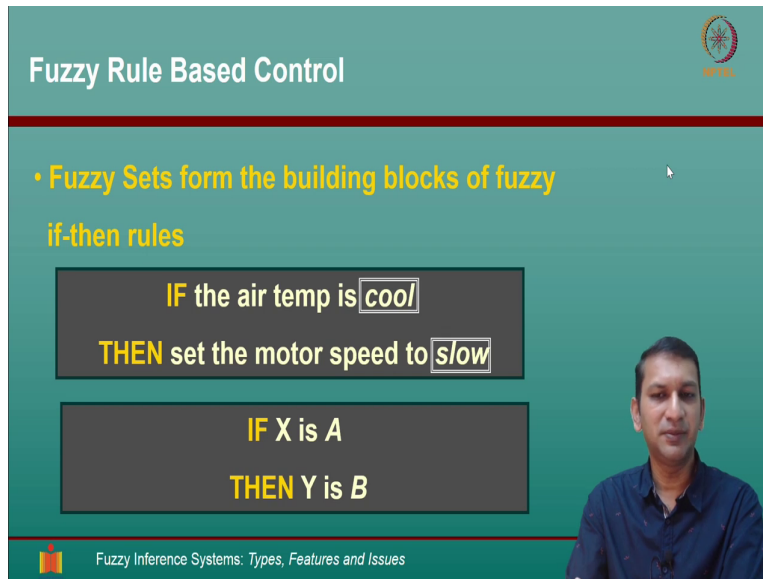
So, not only are you not having very nice comfortable feeling inside the room with the air conditioner on, but in the long run it probably is detrimental also to the functioning parts inside this. So, these are some simple limitation that you could show with classical rule based control, but let us see how we could manage this by introducing fuzzy sets here.

(Refer Slide Time: 11:59)



Let us look at fuzzy rule based control.

(Refer Slide Time: 12:01)



Fuzzy Rule Based Control

- Fuzzy Sets form the building blocks of fuzzy if-then rules

IF the air temp is **cool**
THEN set the motor speed to **slow**

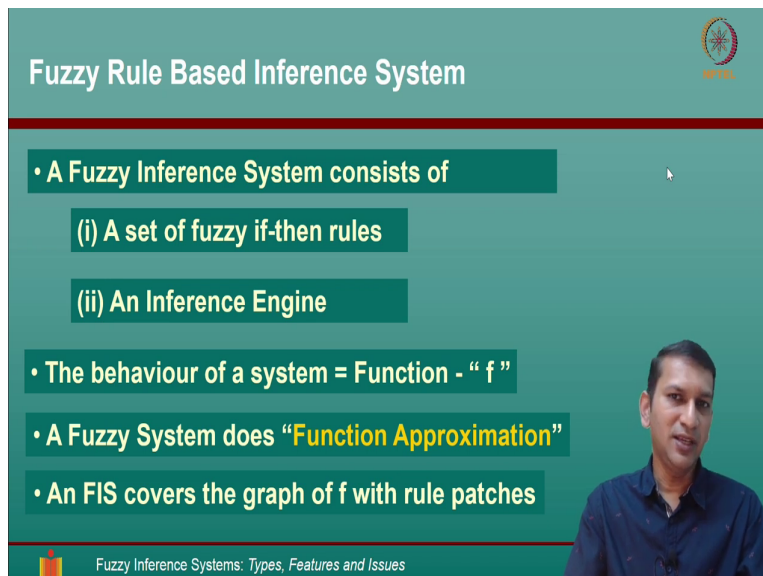
IF X is A
THEN Y is B

Fuzzy Inference Systems: Types, Features and Issues

The slide features a teal background with a red horizontal line. A small circular logo is in the top right corner. A man in a dark blue shirt is visible in the bottom right corner of the slide frame.

Obviously, fuzzy sets form the building blocks of fuzzy if-then rules. We have seen this if air temperature is cool then set the motor speed to slow; this is one type of fuzzy if-then rule. A typical general schema would be if X is A then Y is B; in this case cool is A and slow is B, these are the fuzzy sets.

(Refer Slide Time: 12:27)



Fuzzy Rule Based Inference System

- A Fuzzy Inference System consists of
 - (i) A set of fuzzy if-then rules
 - (ii) An Inference Engine
- The behaviour of a system = Function - "f"
- A Fuzzy System does "Function Approximation"
- An FIS covers the graph of f with rule patches

Fuzzy Inference Systems: Types, Features and Issues

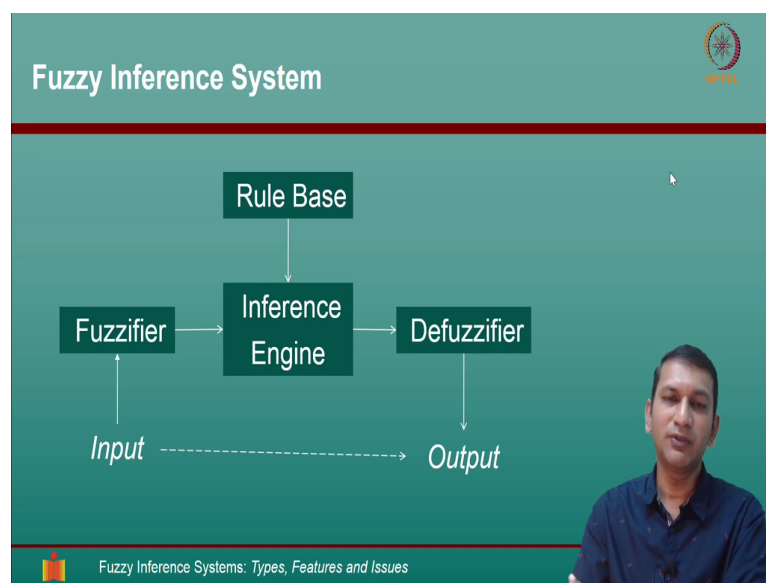
The slide features a teal background with a red horizontal line. A small circular logo is in the top right corner. A man in a dark blue shirt is visible in the bottom right corner of the slide frame.

Well, what is a fuzzy inference system you know this it consists of 2 things: first a set of fuzzy if then rules a set. So, rule based and an inference engine. So, one type of inference engine is what we have seen in the last week of lectures that of fuzzy relational inference.

The behavior of a system can be indicated by a function f because we are giving an input and expecting an output, now we are in the setting of control systems. What does a fuzzy inference system do? It does function approximation. It tries to approximate the behavior of the system simulate the behavior of the system. We have a perfect idea of what that system, how that system should function given this temperature, what should be the fan speed and it is trying to approximate it.

Of course, often we do not know this function f ; we are only trying to approximate this even without knowing it. An FIS the fuzzy inference system covers the graph of f with rule patches the last 2 points are extremely important and we will keep coming back to this throughout this lecture. Firstly a fuzzy inference system does function approximation. How does it do it? It does it by patching up the function itself the unknown function itself in terms of rule patches, as we go along this will become clear to all of us, ok.

(Refer Slide Time: 13:57)



So, now generally what do we have? We are given an input and we want an output. How do we do this? We are given some ground truth because we cannot arbitrarily assign output. So, the ground truth the knowledge about the domain is given in terms of a set of if then rules, the fuzzy if-then rule base.

Now, we want to use this rule base and intelligently reason some output for this given input and that is where inference engine comes into picture the knowledge and the intelligence. So, this input is given to the inference engine and we are expecting an output. Remember, what

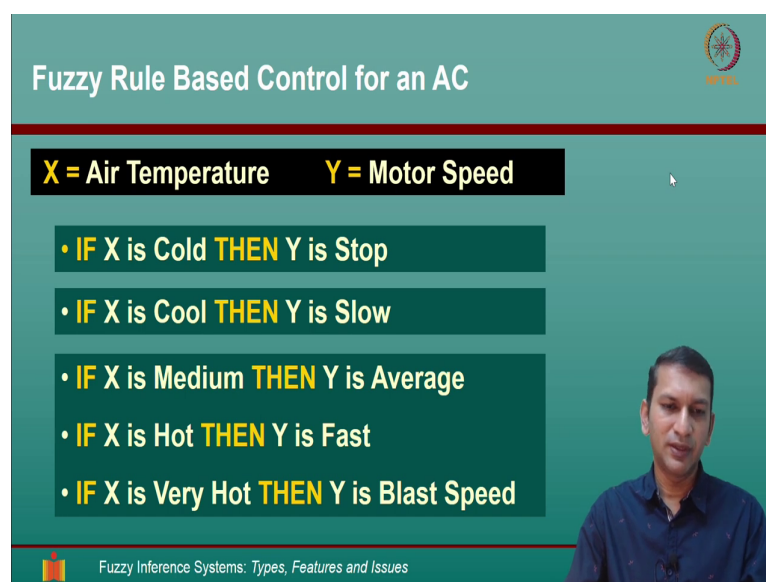
we have seen earlier was we were actually giving a fuzzy set as an input to the inference system.

Whereas, now we are talking about control system and we are actually talking about temperature and we are when we are sensing the temperature that the sensors will not be a fuzzy set. It will might probably say 32.3 degrees or 37 degrees. So, often inputs are either real numbers or real vectors and there is a need to fuzzify them. We will see these in detail, but in this general schema let us also have this component.

So, typically if the input is not a fuzzy set it is fuzzifier and given to a inference engine. What does the inference engine do? It talks to the rule base, takes the help of the rule base and gives us an output; either an output in the form of a real value or real vector or perhaps just a fuzzy set like in case of fuzzy relational inference we have seen we get B dash which is finally, defuzzifier into obtaining an output.

We will see all of these components in play presently. So, this is the overall scheme of any fuzzy inference system. These are the component that you would have ok.

(Refer Slide Time: 15:48)

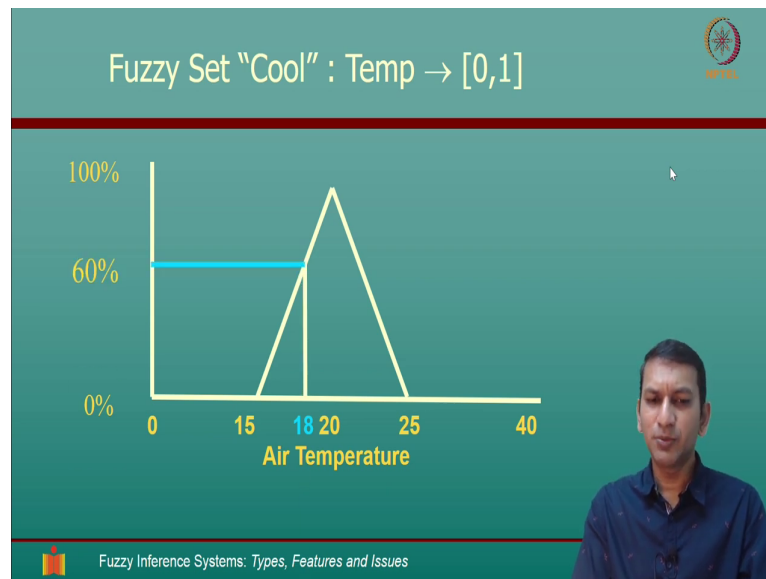


The slide is titled "Fuzzy Rule Based Control for an AC". It features a teal background with a dark red horizontal line. At the top right is a small circular logo. Below the title, a black box contains the text "X = Air Temperature" and "Y = Motor Speed". Below this, five rules are listed in a dark green box with yellow text: "• IF X is Cold THEN Y is Stop", "• IF X is Cool THEN Y is Slow", "• IF X is Medium THEN Y is Average", "• IF X is Hot THEN Y is Fast", and "• IF X is Very Hot THEN Y is Blast Speed". In the bottom right corner, there is a small video inset of a man speaking. At the bottom left, there is a small orange icon and the text "Fuzzy Inference Systems: Types, Features and Issues".

So, let us stay with this example of building a fuzzy controller for an air conditioner. So, we have identified that input is air temperature output is the motor speed and motor speed is what we want to control. By controlling the motor speed, we want to control the air temperature in the room.

So, we would come up with some rules, let us say that these are the rules that you are coming we have come up with if X is cold, then Y is stop. So, fan speed it fan need not rotate; If X is cool maybe it has to operate slowly just to circulate air and the rest of the 3 rules that we have here are quite common sensing. If it is medium – average speed, hot – fast speed, very hot – blast speed. So, these are the 5 rules that we have in this system for the moment.

(Refer Slide Time: 16:36)

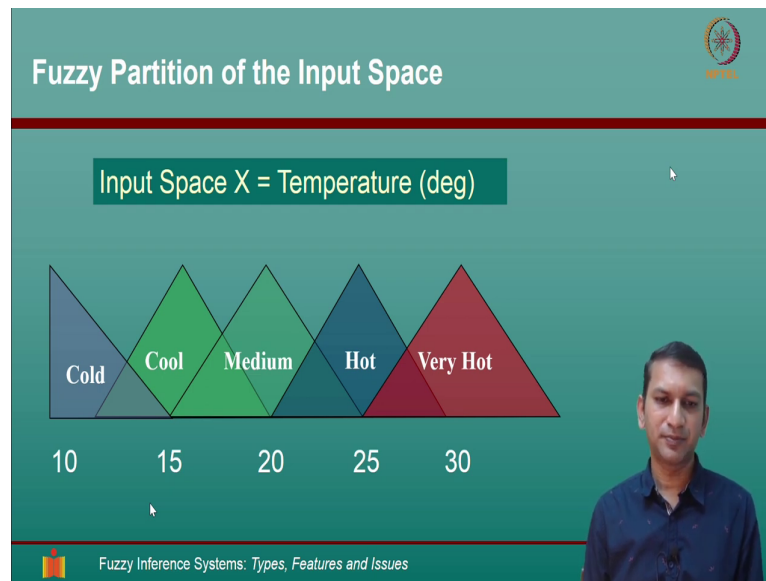


Now, we saw that fuzzy sets from the building blocks. So, once again if you are talking about these fuzzy sets cool, fast, average, medium, then we need a domain on which it could be defined. So, we need an appropriate domain and we need an appropriate way of defining it also. So, if instead of fuzzy set if we are gone for classical sets, we would have some interval like this.

So, T falling within that interval is what the classical if then rule would capture, but in the case of now we are discussing fuzzy if then rules. So, then we are using fuzzy set and we know that given any value which falls in the support of this fuzzy set we can find it is membership value. In this case it is 0.6, ok. So, what is the first step? So, we want to control this, we have come up with the rules.

These rules we know capture the working of the system we still do not know exactly what is that function f which maps X to Y . But, we more or less have an idea of how the system should function and that is what we have actually expressed in the form of linguistic rules.

(Refer Slide Time: 17:45)



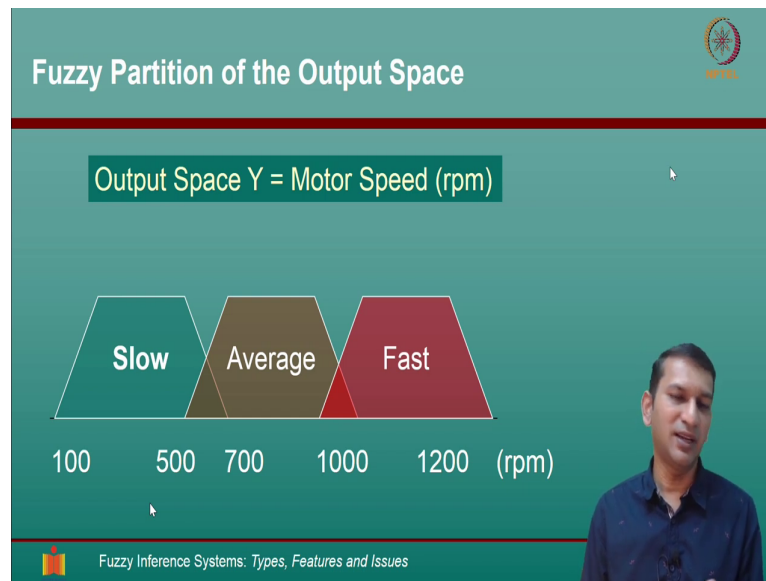
First step is to partition the input space come up with a fuzzy partition of the input space. What do you mean by that we understand fuzzy partition more general sense of fuzzy cover is a collection of fuzzy sets whose support will cover the entire domain that is all that is expected of a fuzzy curve.

So, now here the input space is the temperature domain. Let us for the moment assume that it varies from 10 to 30 volt many more. Now, on this we need to define how cold looks like, how cool looks like, how medium looks like and so on and so forth. So, perhaps let us assume that for us presently cold is this kind of a triangular fuzzy set; that means, at 10 it is definitely cold, but at 13 it is not really that good maybe cold only to the extent of 0.5.

The next concept is that of cool what you see here cool perhaps 12 is not cool, but from 12 to 20 everybody is cool, but to a certain degree and immediately you see that we are actually overlapping cold and cool; that means, the point here maybe it is 13.5 it is both cold and cool. This could not have happened in the classical if then rules because we actually have non overlapping sets that is how we do the partition there using those classical sets.

We will look at a set of rules their completeness parse rule spaces in the perhaps the next lecture. So, the for the moment we are trying to capture these concepts cold cool hot very hot in terms of fuzzy sets on the chosen domain of X which is the temperature domain. So, let us assume that medium looks like this hot like this and very hot like this. So, these are the 5 fuzzy sets that we have defined on the input domain X .

(Refer Slide Time: 19:36)



Next, the logical step is to partition the output space. The output space is Y which consist of motor speed. Let us assume that these are the motor speeds that we have in terms of rpm and for the moment let us have only 3 output fuzzy sets – fast, it should of course, looks look like this somewhere on the right extreme; slow which is looks left extreme and average.

So, let us assume that these are the only 3 fuzzy sets we are going to have on the output domain. Recall in the last lecture of previous week when we were talking about inferencing using FRI fuzzy relation inference in the presence of a multiple rules a rule case. We had said that often it is the case that the number of fuzzy sets linguistic values on X the input domain typically are far more than the number of fuzzy sets or linguistic values that you can have on you would go on to have on the output space Y and this is one such clear example where we have this. 5 input fuzzy sets, but only 3 output fuzzy sets or 5 antecedents and 3 consequence, ok.


(Refer Slide Time: 20:49)

Fuzzy Rule Based Control for an AC

X = Air Temperature Y = Motor Speed

- IF X is Cold THEN Y is Stop
- IF X is Cool THEN Y is Slow
- IF X is Medium THEN Y is Average
- IF X is Hot THEN Y is Fast
- IF X is Very Hot THEN Y is Blast Speed

Fuzzy Inference Systems: Types, Features and Issues

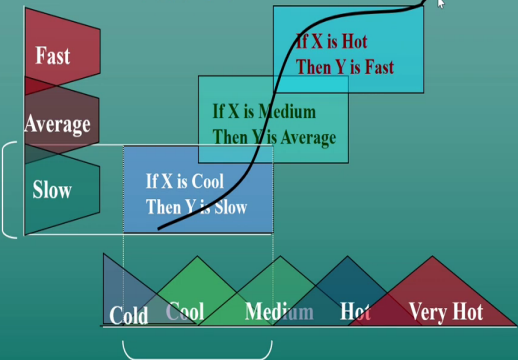


So, now let us look at these rules. So, now, we understand everything here – what is cold, what is stop, what is cool, what is slow so on and so forth.

(Refer Slide Time: 21:00)

FIS – Approximating a function

• An FIS covers the graph of f with rule patches
 $f : X \rightarrow Y$




Fast
Average
Slow

If X is Hot Then Y is Fast
If X is Medium Then Y is Average
If X is Cool Then Y is Slow

Cold Cool Medium Hot Very Hot

Fuzzy Inference Systems: Types, Features and Issues



Now, the inferencing itself remember what we said was a fuzzy inference system approximates a function. What is that function? It captures the working of the system. So, there is some particular way in which an air conditioning control system should work, that is what we feel. If this is the temperature this should be the speed at which it should cool if this is the temperature.

So, we have this mapping in mind. We have it only in mind. We do not know exactly if we were if it were known then we could easily code it. So, we want to capture that unknown function more or less using a fuzzy inference system. So, we have the system does operate according to a function f which perhaps we do not know which is a function from X to Y and let us say it looks like this.

Look at this, we are not taking a linear function just only increasing it is non-linear, but a kind of a nice smooth kind of a step functions. So, at every point it increases smoothly and stays for longer and then slowly starts to increase this is the kind of function perhaps we feel is ideally suited to control an air conditioner. Now, how are we going to manage an FIS approximate this function. So, on the X axis you see the temperature domain and the Y -axis you see the fan speed domain.

So, now what have we done we actually have a fuzzy partition on X and fuzzy partition on Y . Now, how do the rules look like? The first rule that we have is the second rule if X is cool then Y is slow. Now, look at the set cool. Who are the people who belong to the set cool? So, essentially we are looking at the support of this fuzzy set cool which is the support it is between these 2 values as outside of it is 0.

So; that means, only those X 's when you are actually measuring the input X a only if it falls within this interval is it going to be considered as cool and only then this rule needs to come into picture because we are in the diff you know region. Now, if what about slow who are all the values who would get some non-zero membership value in the fuzzy set slow.

So, you will see that these are the Y values coming from the domain of Y , domain of fan speed which will actually belong to slow. The support of slow is this. Now, if you look at this entire region how can we interpret this? So, this region consist of all those ordered pairs X comma Y or U comma V such that U belongs to cool to non-zero extent and V belongs to slow to a non-zero extent, that is one way to see it as you know positive example.

Another ways if X is cool then Y is slow. So, our insisting if an X falls here excites cool that mean belongs to cool, then you are insisting that the Y somehow should belong to slow that is what you are insisting ok as a restriction as a constraint. Now, this is what a rule does. So, we said an FIS covers the function with rule patches. So, you see this rule it is actually patching up part of the domain. Let us continue further.

Look at this rule; if X is medium then Y is average. What part of the domain would it cover? Clearly this where on the X-axis you have taken the support of medium on the Y-axis you have taken the average support of average and essentially doing a Cartesian product. So, then every rule you actually take this depending on the supports.

So, now, you see here what we have is this we said an FIS covers the graph of f with rule patches and how are we covering? We are essentially taking the Cartesian product of the corresponding supports and you see that these patches are in fact, capturing the function, the function continues to remain within these patches.

Clearly, if you have more fuzzy sets on both the domain and the range co-domain then, these patches will start to shrink which means you are going to have smaller and smaller patches approximate this approximating these function which is preferable perhaps will be clear in a few slides that we are going to see now. So, the key take away from this graph is an FIS covers the graph of f with rule patches that should be (Refer Time: 25:36) ok.

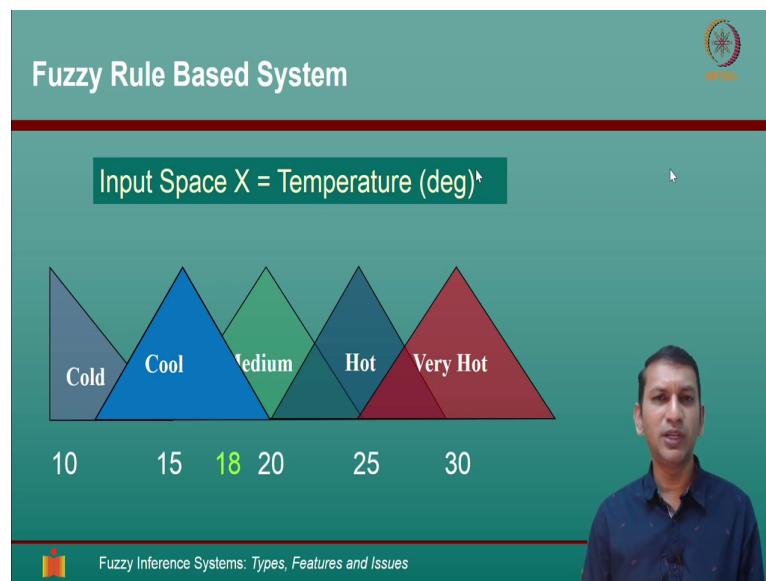
(Refer Slide Time: 25:38)



Now, let us look at how the inference itself is done. Inference is done in a parallel manner. Now, if you think back at fuzzy relational inference, there again it was parallel in the sense that we were aggregating and doing it. So, in the case of (Refer Time: 25:53) clearly it was parallel because A dash was given to each one of those rules, they were excited and they were giving us B 1 dash, B 2 dash so, until B n dash and finally, we aggregated them to get a simple B dash.

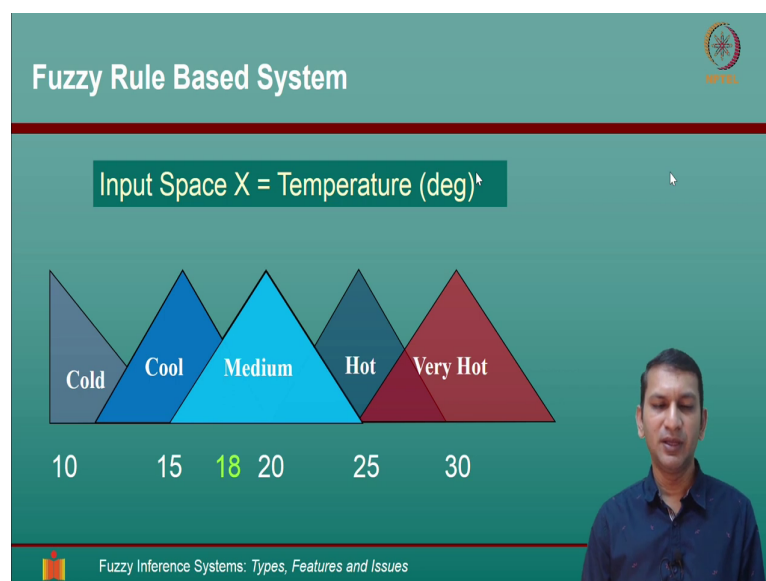
So, here again the inference is parallel of course, because it is a fuzzy inference system.

(Refer Slide Time: 26:08)



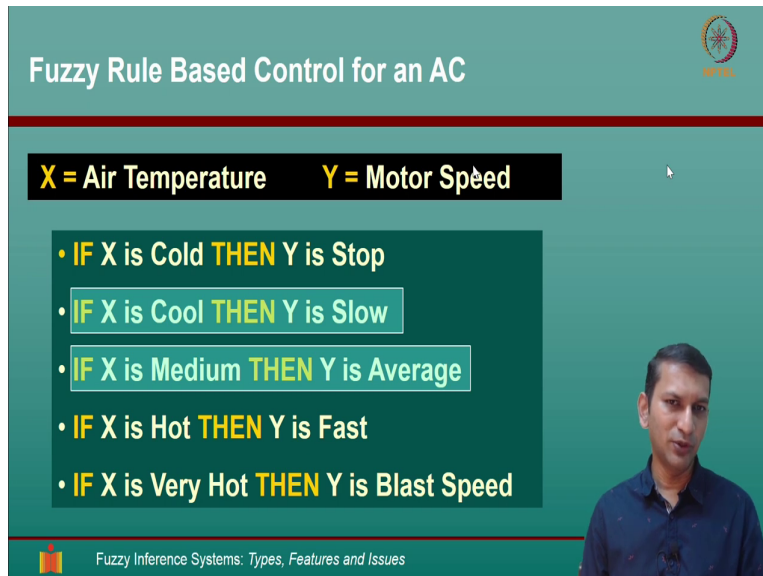
What we have? We have this partition on input domain. Let us assume 18 is the value that is given to us as input; that means, the sensor and the air conditioner sense it is 18 degree centigrade. Now, question is what should be the fan speed that is what we want to do. How do we do this?

(Refer Slide Time: 26:32)



First, we check to what extent 18 belongs to different fuzzy sets it is clear it belongs only to the fuzzy sets cool and medium.

(Refer Slide Time: 26:33)



The slide is titled "Fuzzy Rule Based Control for an AC" and features a small logo in the top right corner. Below the title, it defines the variables: **X = Air Temperature** and **Y = Motor Speed**. A list of five fuzzy rules is presented, each with a condition on X and a result on Y. The rules are: "IF X is Cold THEN Y is Stop", "IF X is Cool THEN Y is Slow", "IF X is Medium THEN Y is Average", "IF X is Hot THEN Y is Fast", and "IF X is Very Hot THEN Y is Blast Speed". The slide also includes a small inset image of a man in the bottom right corner and a footer that reads "Fuzzy Inference Systems: Types, Features and Issues".

Fuzzy Rule Based Control for an AC

X = Air Temperature Y = Motor Speed

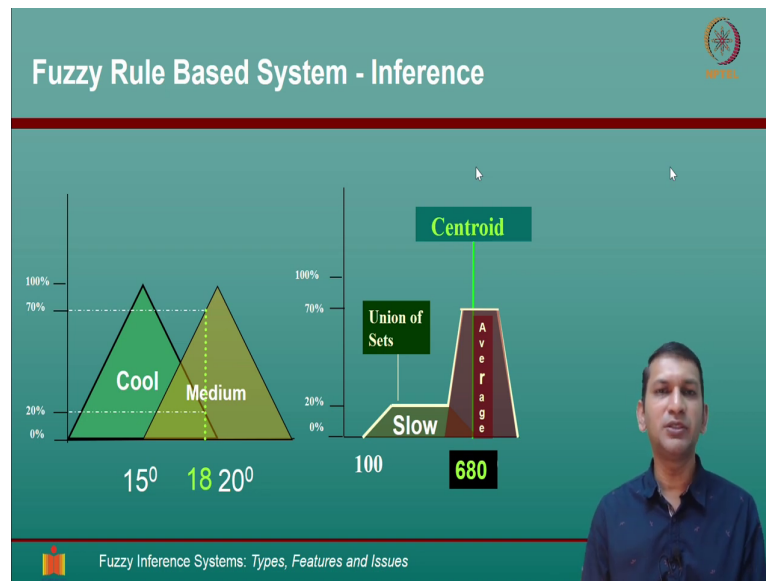
- IF X is Cold THEN Y is Stop
- IF X is Cool THEN Y is Slow
- IF X is Medium THEN Y is Average
- IF X is Hot THEN Y is Fast
- IF X is Very Hot THEN Y is Blast Speed

Fuzzy Inference Systems: Types, Features and Issues

And, to what extent we will find out presently. So, among these 5 rules since the given input is 18 and it belongs only to cool and medium, these are the only 2 rules that come into play of course, we know that there are we have lot of knowledge, but what is useful or important, appropriate, relevant for the problem at hand that is what we are going to pick up.

So, given that the input is 18 degrees so, these are the 2 rules that coming into play because 18 belongs to only both cool and medium.

(Refer Slide Time: 27:08)



Now, the inference itself so, it is sufficient to consider these 2 fuzzy sets cool and medium on the X domain and slow and average on the Y domain. Now, we have given 18 degrees; now, let us look at to what extent 18 belongs to both of these fuzzy sets to medium it belongs to degree 0.2 and to cool it belongs to degree 0.2 and to medium it belongs to degree 0.7.

Now, what do we do? We take this remember cool is related to slow and medium to average we take this value the extent to which 18 degrees is captured by the concept cool, we take that value essentially we will see in the next lecture that it is called the similarity value. It tells you to what extent 18 belongs to cool we are taking this value and we are modifying the corresponding consequent which is the slow.

How are we modifying? Perhaps we will just cut it off, we threshold it. If we are looking at mean kind of an operation we will just threshold it. So, you see here using the extent to which a given input belongs to the antecedent of rule to that extent we are modifying the consequent and in this case when we talk about modifying I am just we are just only thresholding it.

So, similarly the consequent for medium is average and 18 belongs to degree 0.7 in medium. So, we will take this value of 0.7 and accordingly, we will threshold the average fuzzy set. So, now given an input it has exacted 2 rules and you have 2 fuzzy sets as output, but what we need finally, is one value which can be told to the motor and say ok run at the speed that is what we want to do. How do we do this?

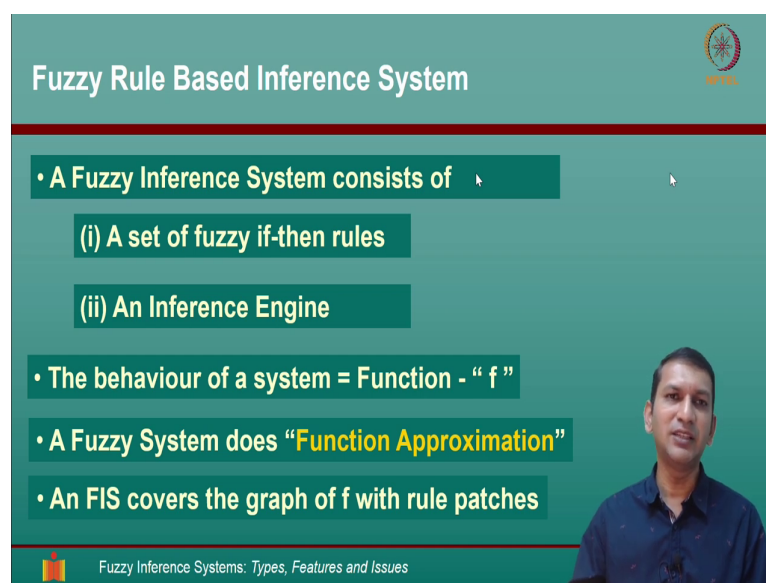
First we take the union of sets. Remember union means we are aggregating this. So, you might think that it is already trying to look like B 1 dash and B 2 dash and we are aggregating it. If you think like that, we are not very far away from the truth; we will see this in a few lectures from now. So, this is the union of sets out the local outputs are modified slow on average and now we have aggregate into a global of fuzzy set.

From this we need a value, how do we do this? This is where the defuzzifier comes into picture. Now, this is a fuzzy set we want to go from a fuzzy set to a value typically within the support of this fuzzy set. So, we apply a defuzzification or defuzzifier we say. Let us assume that we are applying the centroid defuzzifier here and what do we get? The centroid of this entire fuzzy set is here perhaps and it says it is 680. So, that means, for 18 degree input, we are actually getting 680.

Now, you see that it is neither 700 nor 300. So, now, as temperature goes below, it is probably going to go towards 300 more and as temperature rises above it will not be cool or medium it will go to the furthers fuzzy sets further in the extreme, right. So, maybe it is hot or very hot and accordingly instead of slower average it will be fast which means again the motor speed is going to move to the right of your screen and, which means it is increasing the motor speed.

So, you see that we have nicely captured this monotonicity relationship using these partitions.

(Refer Slide Time: 30:36)



Fuzzy Rule Based Inference System

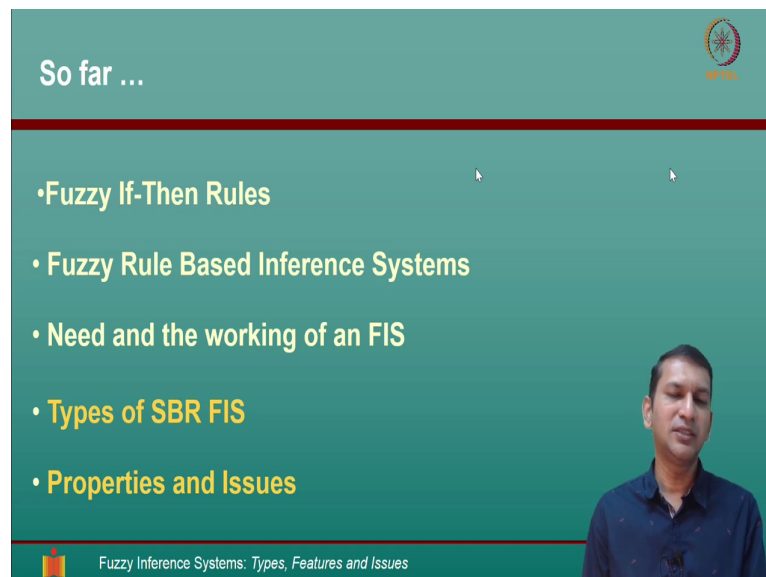
- A Fuzzy Inference System consists of
 - (i) A set of fuzzy if-then rules
 - (ii) An Inference Engine
- The behaviour of a system = Function - “ f ”
- A Fuzzy System does “**Function Approximation**”
- An FIS covers the graph of f with rule patches

Fuzzy Inference Systems: Types, Features and Issues

Once again, quick recap: what is the fuzzy inference system? It consists of 2 things the knowledge base in the form of fuzzy if then rules and intelligence in the form of an inference engine. We are trying to capture the behaviour of a system which we assume is a function f mapping X to Y .

A fuzzy system does function approximation. It just only tries to approximate this system behavior. Most importantly, it does this by covering up the function. We do not know the real function, but we are covering it up with patches wherever we think the function rule fall using the corresponding rules.

(Refer Slide Time: 31:09)



The slide has a teal background. At the top left, it says 'So far ...'. At the top right is a small circular logo. In the center, there is a bulleted list of topics. In the bottom right corner, there is a small video inset showing a man in a dark blue shirt. At the bottom left, there is a small icon of a book and the text 'Fuzzy Inference Systems: Types, Features and Issues'.

- Fuzzy If-Then Rules
- Fuzzy Rule Based Inference Systems
- Need and the working of an FIS
- Types of SBR FIS
- Properties and Issues

Fuzzy Inference Systems: Types, Features and Issues

So far, we have seen how fuzzy if-then rules are being used. Fuzzy rule based inference system we have seen this in the last week of lectures too, but in this lecture we have seen the need and working of an FIS it is very different from fuzzy relational inference that you would have noticed already. We are going towards similarity based reasoning. We are related to it.

Already we have used some terms which we are familiar with aggregation, modification and so on. What is left? Let us quickly have a look at the different types of fuzzy inference systems which are based on similarity based reasoning; some properties and issues – some desirable properties and some issues that we would face.

(Refer Slide Time: 31:59)



What are the types of SBR-FIS?

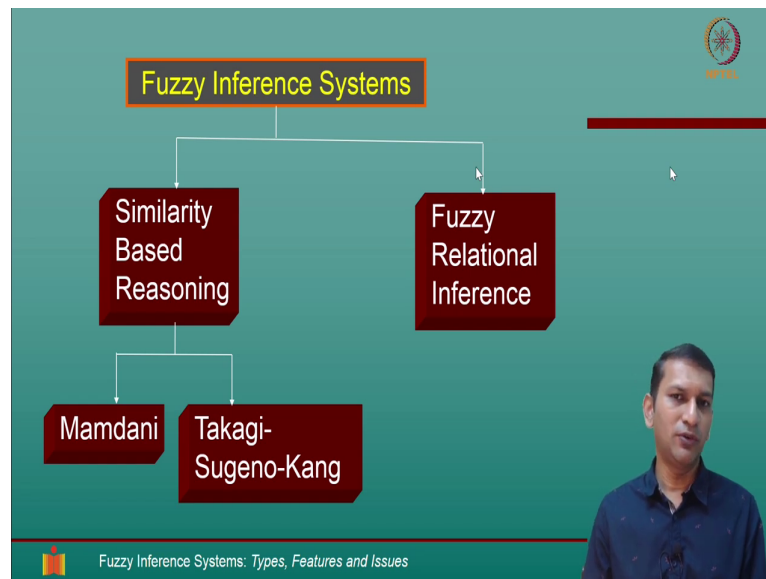
(Refer Slide Time: 32:01)



To understand that we have to look at 2 practitioners who have come up with these models the first of them is Abraham Mamdani during this mid 70s, he had come up with a few papers dealing how you could do similarity based reasoning, now what we understand the similarity based reasoning. He had proposed his form of fuzzy inference which is now call the Mamdani inference system or Mamdani form of fuzzy inference system.

A little later Professor Michio Sugeno he proposed an alternate way of doing similarity based reasoning call the Sugeno Takagi-Sugeno fuzzy system it is also called the Takagi-Sugeno-Kang fuzzy system or abbreviated as TSK fuzzy system. These are the 2 pioneers who proposed this major and to this take a very useful models of similarity based reasoning fuzzy inference systems.

(Refer Slide Time: 33:07)



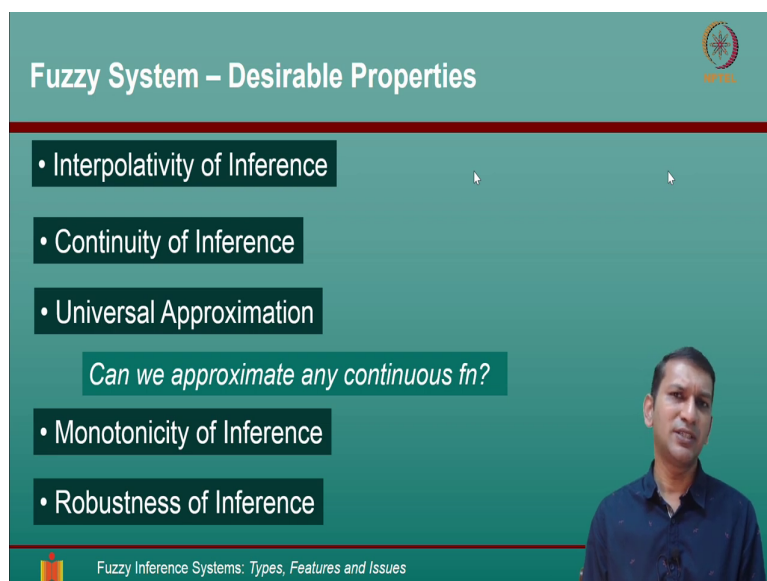
What are they? So, if you look at fuzzy inference system that is also repeated many times there are 2 major types. First is the similarity based reasoning, other is the fuzzy relational inference, which we have seen in the last week of lectures and similarity based reasoning there are 2 important reasoning schemes fuzzy inference systems that are proposed by Mamdani and the one proposed by Takagi-Sugeno-Kang. These are 2 things that we will see going forward in the next week of lectures.

(Refer Slide Time: 33:35)



What are the desirable properties?

(Refer Slide Time: 33:42)



Just like in the case of fuzzy implication we said that these are the desirable properties IP identity principle, ordering property exchange principle. In the case of fuzzy inference systems what do we expect? Well, we expect that it interpolates the inference is interpolative. What do we mean by interpolativity? We will define this soon enough.

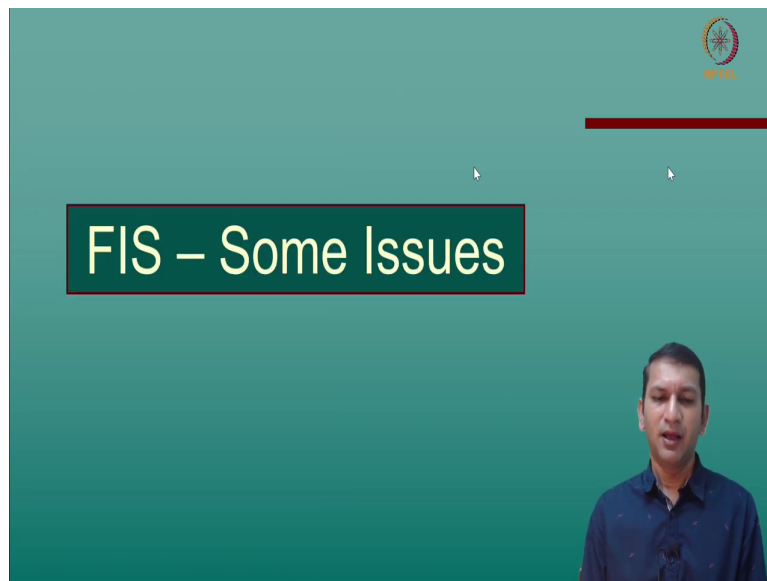
Essentially you are looking at how is the rule base constructed based used sometimes also from data that are coming out of the system. So, now, the way you have constructed the rule base and use the inference system you are asking the question if I give the input from the data I already have input output patch from the data which I have measured if I give this input measurement will it get the corresponding output measurement. So, that is essentially the is the interpolativity.

Continuity of inference: so, what I get the input output mapping is that function finally, continuous; continuous in the sense that we understand over the clear or in the appropriate domain. Universal approximation: so, we said that fuzzy inference systems to function approximation. So, the question now is can we approximate any continuous function? We assumed in this case of an air conditioner a smooth nice looking monotonic curve, but what if it were oscillating hotly or wildly, will we be able to approximate – that is a question that we need to answer.

Monotonicity of inference – if there is an ordering between in inputs with the same ordering we maintain on the outputs also. For example, for 18 degrees I get 680 as the rpm, when I go to 20 it should actually go higher than rpm because higher than 680 because it has got hotter.

Suddenly, if it falls down, if it says no run it at 650, then there is something wrong with it. Now, finally, robustness of inference – it is slightly different from continuing. We will explain this in the context when we take this up in the coming weeks. So, these are some of the desirable properties of a fuzzy inference system.

(Refer Slide Time: 35:44)



(Refer Slide Time: 35:46)



And, what are some issues well one of them we have seen dealing with the MISO and fuzzy relational inference the computational complexity both storage and the calculation or computing complexity. And, in the same in the scenario of control systems stability is also extremely important. So, instead of starting from one point if you start from another point would it is the system still be stable that is what is the question here. We will not go into this because we are not going to talk about control systems so much, but we will definitely deal with computational complexity.

And, also this combinatorial rule explosion; that means, you have unwieldy number of rules. So, one is computational complexity and this leads to both computational complexity and also storage complexity. So, first and last ones we definitely will look into in during these lectures.

(Refer Slide Time: 36:40)

Fuzzy Inference System – A quick recap...

- Particular type of Similarity Based Reasoning
- Each if-then rule captures knowledge ...
... of some part of the domain
- An FIS does “Function Approximation”
- An FIS covers the function through rule patches

Fuzzy Inference Systems: Types, Features and Issues

Well, a quick recap of what we have discussed in this lecture so far. We looked at a particular type of similarity based reasoning, only visually we will look into the operations in the next lecture. We know that each of the if-then rules captures knowledge of some part of the domain we have seen that.

We know that an FIS does function approximation and it does that by covering the function through rule patches. We will keep referring to this and this idea will also allow us to present many perspectives on the inference system itself and especially about the fuzzy if-then rules.

(Refer Slide Time: 37:22)



A look ahead ...

- Similarity Based Reasoning – the Operations!
- If-then rules as fuzzy points
- FIS as an interpolation of fuzzy points
- Mamdani vs TSK fuzzy systems
- Matlab simulations.

Fuzzy Inference Systems: Types, Features and Issues

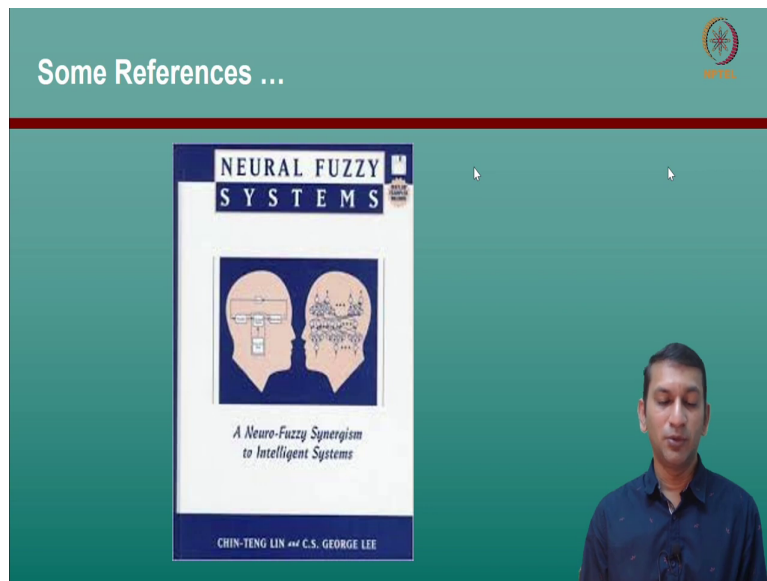
The slide features a teal background with a dark red horizontal line. A list of five topics is presented in white text within dark green rectangular boxes. In the bottom right corner, there is a small video inset of a man in a dark blue shirt speaking. The bottom of the slide has a dark teal footer with a small orange icon and the text 'Fuzzy Inference Systems: Types, Features and Issues'.

Well, what is coming up? We will discuss the operations involved in similarity based reasoning give it a general and formal framework just as the just as how we did for fuzzy relational inference. Interestingly, you look at if then rules as what are called fuzzy points pairs of fuzzy points and once you see it like this you can look at FIS fuzzy inference system itself as an interpolation of these fuzzy points and of course, we look at Mamdani and TSK fuzzy systems in little depth.

Finally, we will like to simulate these things on MATLAB. So, what we mean by simulation? So, we know that fuzzy inference system approximates the function. So, we assume that we take a function assume that it represents the working of particular system and we see how to build a fuzzy inference system which approximates that which essentially means finding a proper corresponding.

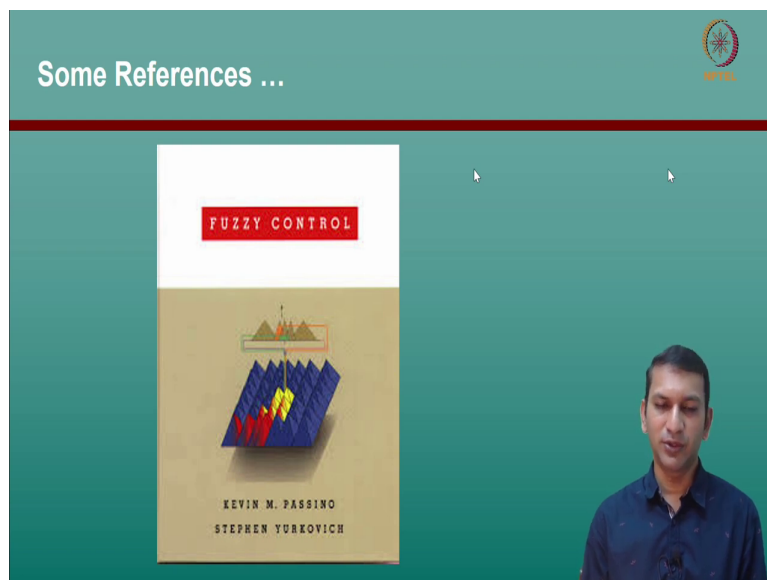
Once the function is given the domain and range are known, then it is about partitioning the domain and relating them nicely relating this fuzzy sets nicely so that you could form rules which make sense in the context and then choosing the operations the operators and performing the inference.

(Refer Slide Time: 38:36)



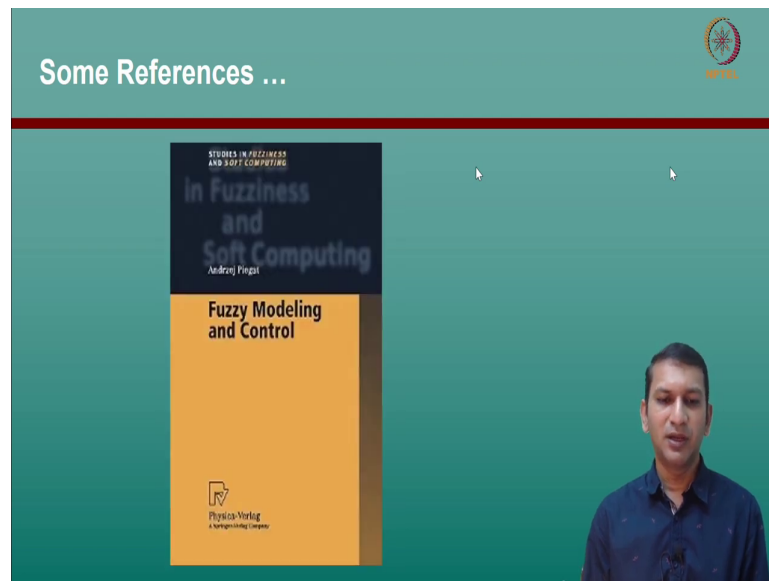
Some references, more than calling them references these are my favorite resources especially in this topic called Fuzzy Inference System, the book by C T Lin and George Lee.

(Refer Slide Time: 38:48)



Also the book by Passino and Yurkovich – these are very very good references. They explain things in much simpler way, a good textbook material and they have lot of topics covered too.

(Refer Slide Time: 39:03)



And, also the book of Andrzej Piegat. As I said, in the next lecture we will look at putting similarity based reasoning in a formal framework what are the operations involved and also a little bit about types of fuzzy rule basis sparse complete and so on. Glad you could join us for this lecture and hope to see you soon in the next lecture.

Thank you again.