Approximate Reasoning using Fuzzy Set Theory
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Lecture - 32
Fuzzy Relational Inference - Multiple Rules

Hello and welcome to the last of the lectures in this week 6 of the course titled Approximate Reasoning using Fuzzy Set Theory. A course offered over the NPTEL platform.

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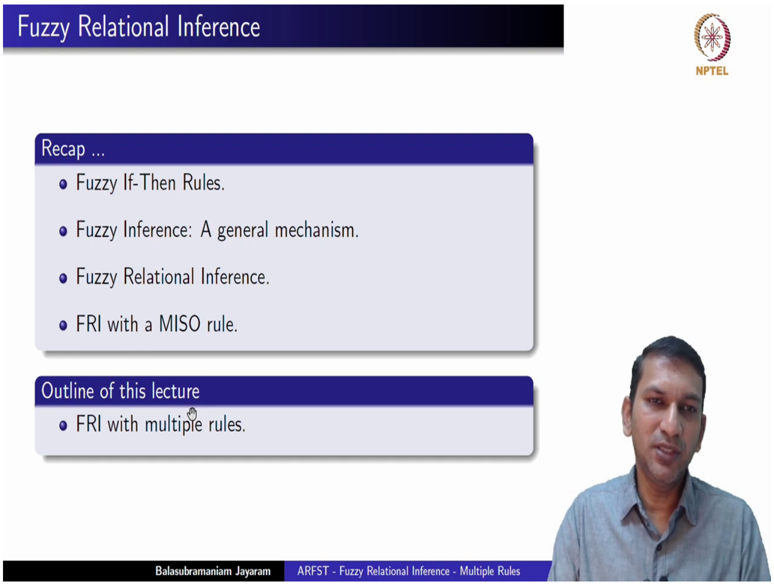

Let us have a quick recap of the topics that we have dealt with in this week. We looked at fuzzy If-Then Rules in depth both from the different perspectives that we can look them at and also in terms of the classification.

We have already seen the impact of 1 particular type of classification whether a fuzzy If-Then Rule is single input single output rule or a multi input single output rule on the fuzzy relation inference scheme itself.

Then, we moved on to looking at fuzzy inference schemes a very general schematic of it and we have been discussing 1 particular type of fuzzy inference which is the fuzzy relational inference. And in the last lecture we have looked even at handling multi input single output rule. In this lecture we will look at fuzzy relation inference when we have multiple rules means a knowledge base which consists of many fuzzy If-Then Rules.

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A quick recap of the mechanism itself.

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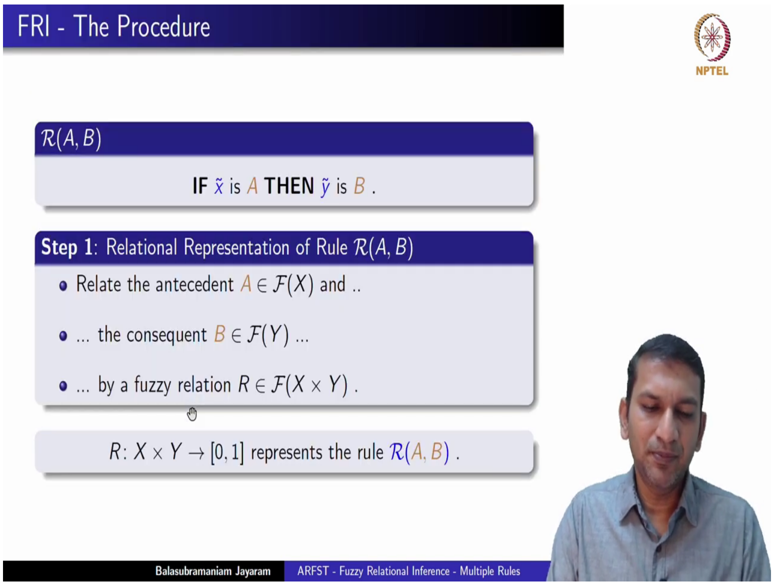

It is a two step procedure we begin by representing the rule by a fuzzy relation which relates the antecedent which is a fuzzy set on X to the consequent which is a fuzzy set on Y and this we capture it as a fuzzy relation on X cross Y.

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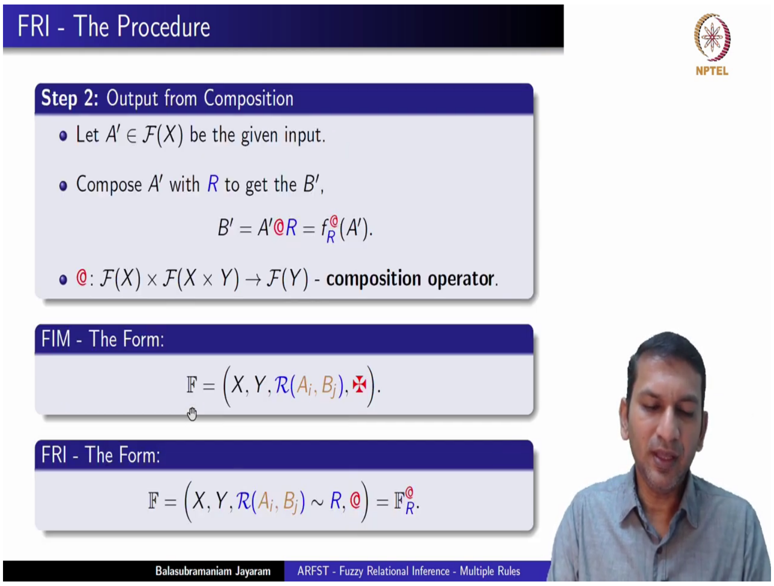

In the next step given an input A dash we compose the input with the relation that represents the rule and we obtain an output.

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Well, we will revisit 1 particular illustrative example that we have seen and then move on from there.

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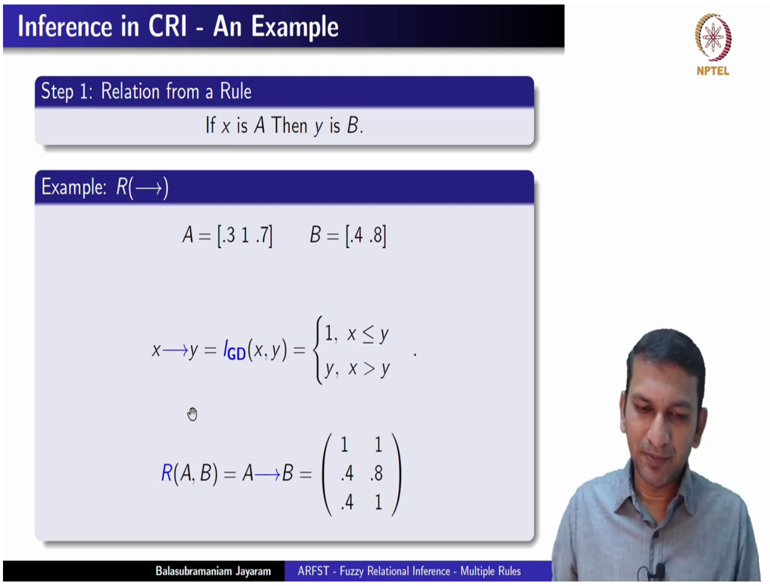

So, we have a rule first step is to represent it as a relation for instance let A and B be given as these vectors and now, we are going to use an implication essentially the Godel implication to relate A and B to obtain the relation. We have seen this before that it will turn out to be this perhaps we will do this once more.

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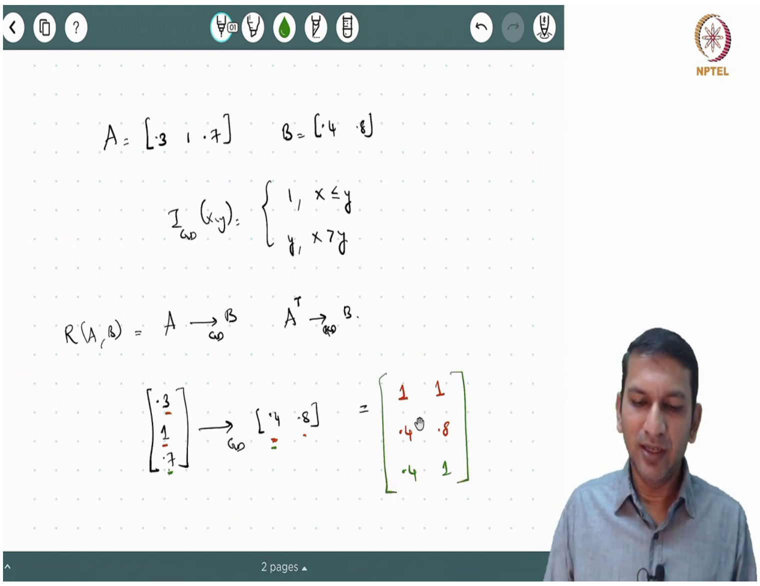

So, what we have is A here which is 0.3 1.7, B is 0.4 0.8 and the Godel implication we know is essentially 1 if x is less than or equal to y and y; that means, x greater than y.

So, what we want to do is, we want R of A, B to be related by the Godel implication; that means, A and B we wanted to be related by Godel implication. So, we have seen what it means is taking the outer product with respect to the Godel implication B. Simply put what we do is we take 0.3, 1 and 0.7 apply the Godel implication 0.4 0.8.

Now, let us fix 3 here we are comparing 3 to 0.4 under the Godel implication we see three is less 0.3 is less than or equal to 0.4. So, this becomes 1 and 0.3 with 0.8 once again it becomes 1 and you will see that is exactly what we have is in the first row.

Now, let us look at 1 and 0.4 we know that 1 is the left neutral element of the Godel implication so; that means, you will get 0.4 and 0.8 and finally, if you look at this 0.7 and look at 0.4. 0.7 is greater than 0.4. So, you would get 0.4 here, but 0.7 is less than 0.8.

So, this will become 1. So, this is essentially the relation that you have got from the rule using the Godel implication.

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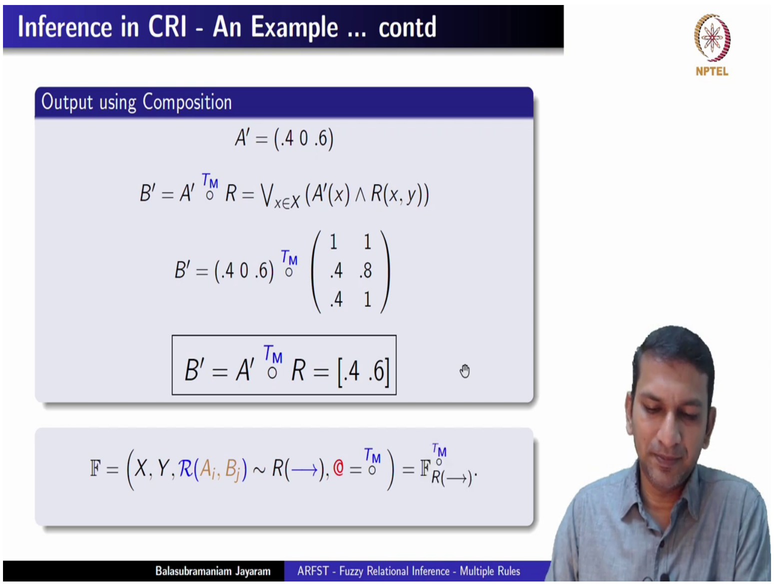

Now next step is given an A dash we need to compose and obtain an output. Since it is CRI we are going to use the sup decomposition in this case we have chosen the minimum T norm which means the formula looks like this and when you compose it with this we would get the output like this.

Well, once again it is very easy to see that in this composition all we are doing is we are taking max among the min. So, if we look at this row into this column, the minimum of this is 0.4 minimum is 0 minimum is 0.4. So, the max of it is 0.4 similarly 0.4 and 1 it is 0.4, 0 and 0.8 it is 0.6 and 1 it is 0.6 max upon is 0.6.

So, this is how we obtain the output. Now in the general scheme the form of this particular CRI we could write like this to build the rule base we are using an implication in this case the Godel implication and we are using the sup T composition where T is TM ok.

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Well, we have seen how to do it with MISO Rule also.

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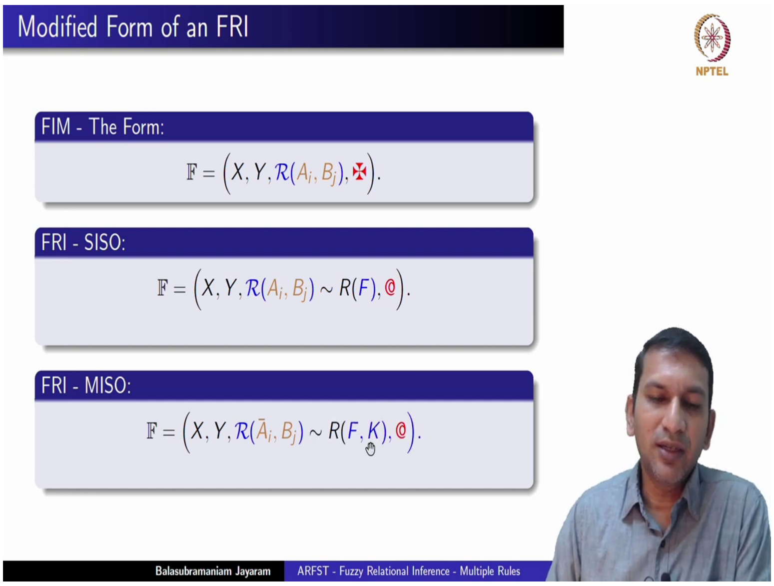

We just look at it in terms of the form this is the general quadruple form that you can look at any general fuzzy inference scheme itself as coming from X and Y the input and output domain and we have a rule base and you have an inference operation.

We saw in the case of FRI with SISO a Single Output Single output single input single output rule all we needed were two things one an operation to capture the relation between the antecedent and the consequent and the composition. In the case of MISO we also wanted another operation to combine the antecedents.

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Well, now let us go to the next step which is how to deal with multiple rules.

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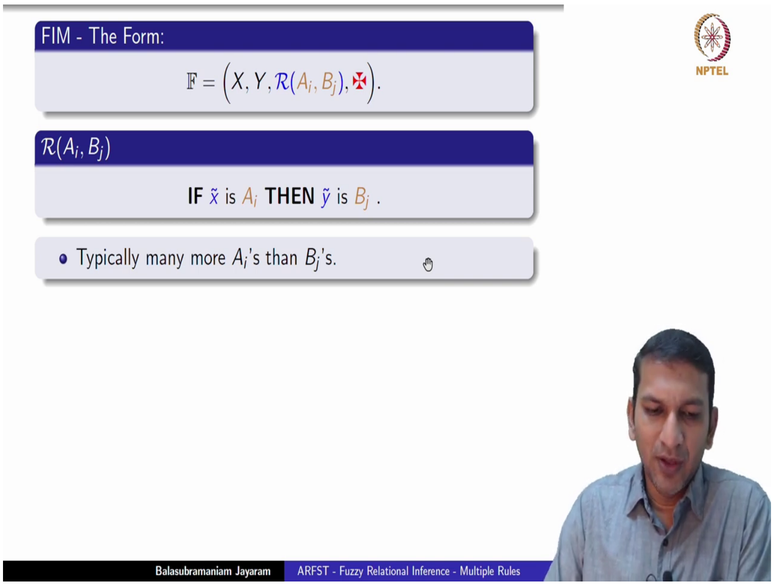

So, now, if you have noticed we have always written the general form for a rule base because that is how we abstracted it from the general schema. So, we are already have multiple rules even though we have discussed only for a single rule case.

Now, we have indicated as A i and B j the indice index sets could be different; however, typically we have many more Ai's than Bj's which was quite common because we want to look at fuzzy inference mechanism as a function mapping F(X) to F(Y); that means, we typically should not be having less number of Ai's and more number of Bj's then they it would not be a mapping as you might you would have to map Ai to many Bj's.

We will come to talking about rule bases complete rule bases, past rule bases consistent rule bases little later, but for now it is typical that we have more Ai's than Bj's.

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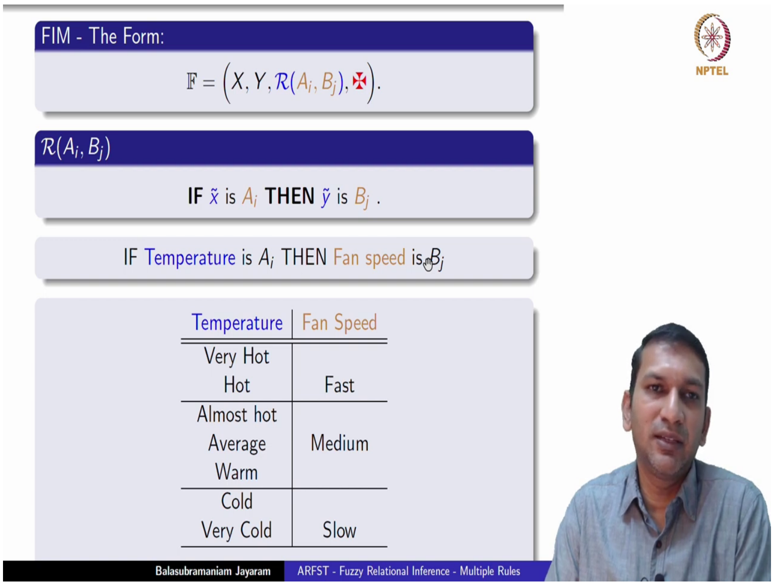

For instance, consider this group if temperature is Ai then fan speed is Bj. Now temperature is the linguistic variable fan speed is the linguistic variable these two take values over the fuzzy sets on x and y.

So, now if you look at what are the linguistic values these two linguistic variables can take. Perhaps it might look like this the temperature can take linguistic value values like very hot almost hot average warm cold and very cold well fan speed can assume the linguistic values fast medium and slow. This is what you have extracted given the domain knowledge and you will clearly see there are more linguistic values the temperature can assume than fan speed.

And perhaps it might even assume I said we will look at rules the each rule actually capturing some part of the domain locally this kind of an interpretation we will see soon enough perhaps in the next week of lectures when we are discussing. Similarity based reasoning for the moment it suffices to know that the index such i and j may not be same, but typically i tends to be the cardinality of i tends to be greater than equal greater than the cardinality of j.

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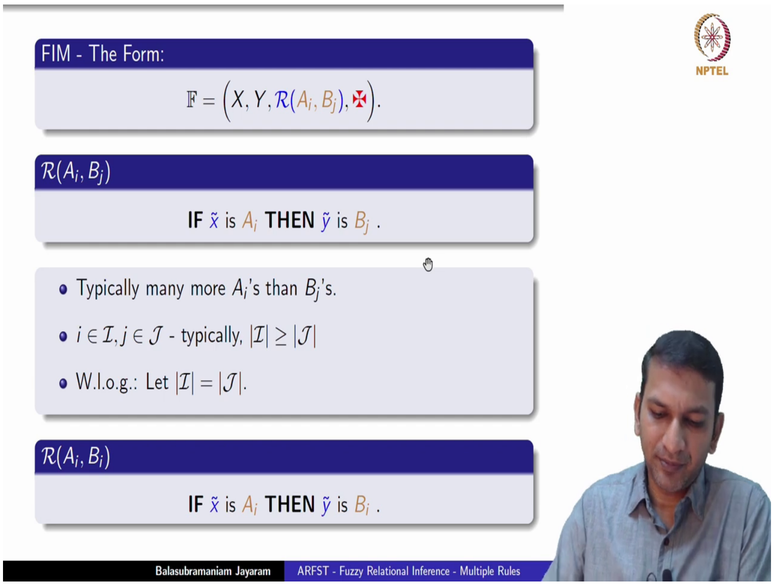

So, this is what happens, but without loss of generality for this lecture we will assume that the cardinalities are same it is only to help us with a notation. So, thus we will write the rules as if x tilde is Ai then y tilde is Bi.

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Well, when you have multiple rules and when you want to apply a fuzzy relation inference there are two inference strategies. What are they?

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The first one of them is called first aggregate then infer it is typically abbreviated and called as FATI strategy what do we have?

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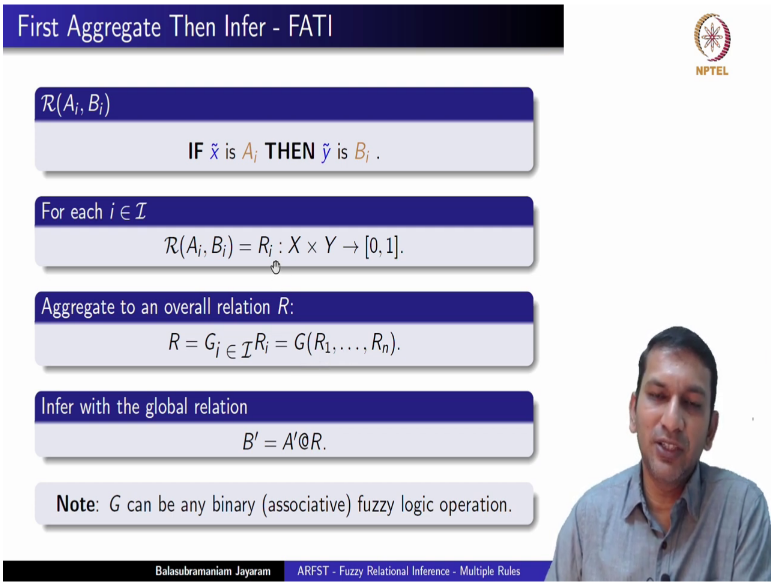

We have a set of if then rules first what we do is for each of them we obtain a relation R i just like if you had a single rule you obtain relation similarly for each of those rules we obtain a relation R i R 1 R 2 so, on till R n then we aggregate all of these relations into a single relation using an operation G.

So, you have n relations representing these n rules we aggregate all of them into a single overall relation R then we infer with this global relation. Now given an A dash we would just compose it with the global relation R to get the B dash. So, first we aggregate all the relations then we infer so, that is where it gets its nomenclature from its first aggregate then infer.

Note that this G operation can be any binary associative fuzzy logic operation because what are we doing? We are actually come aggregating Ri's which are fuzzy relations which are essentially fuzzy sets on x cross y.

So, this is the general procedure the inference strategy of FATI first aggregate all the rules and then infer with a given input.

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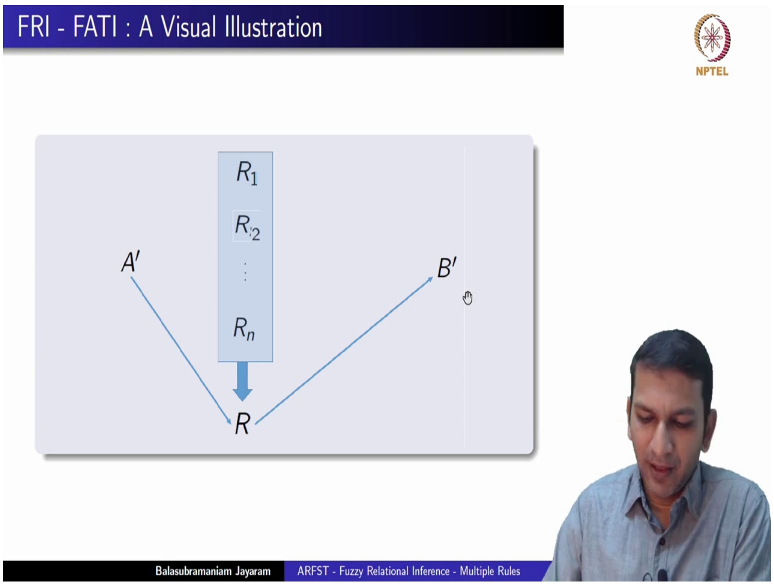

Which only if we look at it we have A dash we have these n relation and what we are interested is in obtaining B dash. Remember these Ri's are capturing each of these rules and each rule has some local knowledge about the domain that is under consideration.

What we first do is combine all of them into a single R and then use A dash to composite with R to obtain the B dash. So, this is essentially how we do the inferencing.

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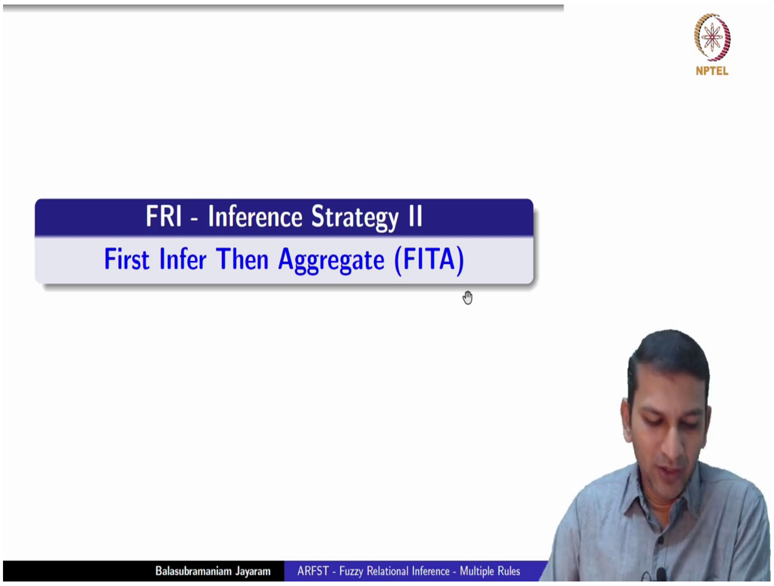

Now there is also an alternate strategy which says first infer then aggregate now what is this strategy?

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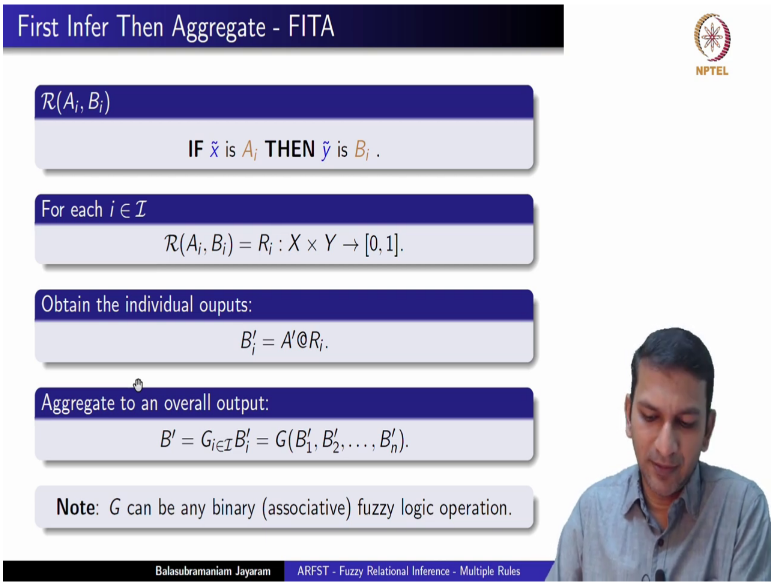

Once again we are given multiple rules for each one of them we obtain the relation that represents the rule.

Now, instead of aggregating all the rules first what we will do is, we will obtain the individual outputs; that means, given A dash we assume there is each one of those rules is separate the relations are there we are actually composing A dash with each one of these Ri's and obtaining the corresponding Bi dash it is not the B dash it is Bi dash. So, locally we are inferring then we aggregate this to an overall output once again using an operation which we have denoted it as G.

So, this what it does is it aggregates all this B a dashes from B 1 dash B 2 dash until B n dash. Note that these Bi dashes they are all fuzzy sets on y.

So, in that sense once again we can use any binary associative fuzzy logic operation.

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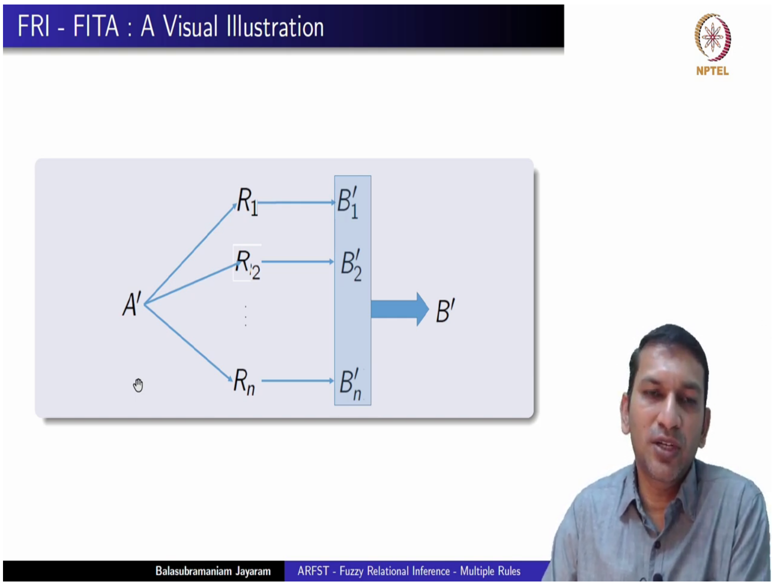

If you would like to look at it visually what we are doing in FITA is we are not aggregating all the rules instead we are taking A dash composing it with R 1 and getting a B 1 dash composing it with R 2 getting a B 2 dash so, on so, forth composing it with R n and getting a B n dash then we combine aggregate all these B i dashes to obtain a B dash. So, this is how these two strategies differ.

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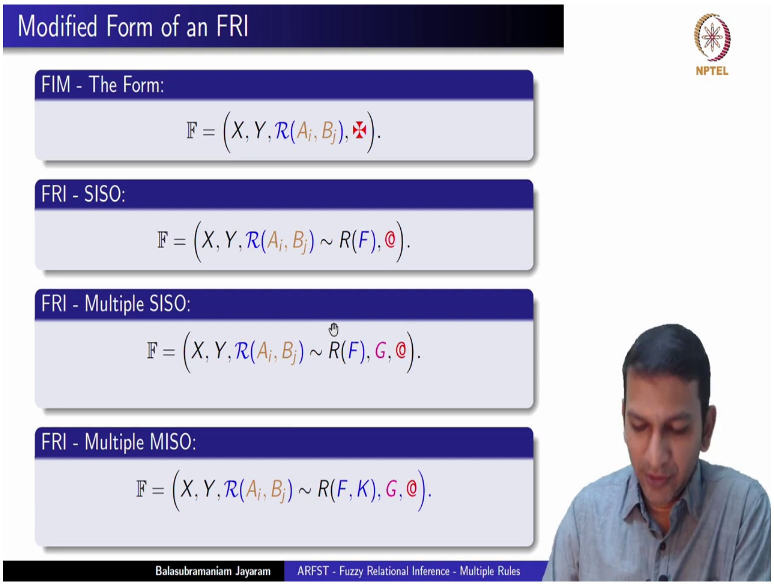

So, if we were to look at the modified form how you can capture it this is the general FIM form we have seen that for a single SISO rule case all we need is F which captures the relation and the composition in the case of multiple SISO not only do we need F to capture the relation, but we also need a G an aggregation function which either aggregates the rules or the local inputs B dashes Bi dashes and of course, we also need a computation.

So, if you are looking at multiple MISO rules of course, we need an F also the antecedent combiner the aggregation G and the composition. So, this essentially takes care of all possible scenarios.

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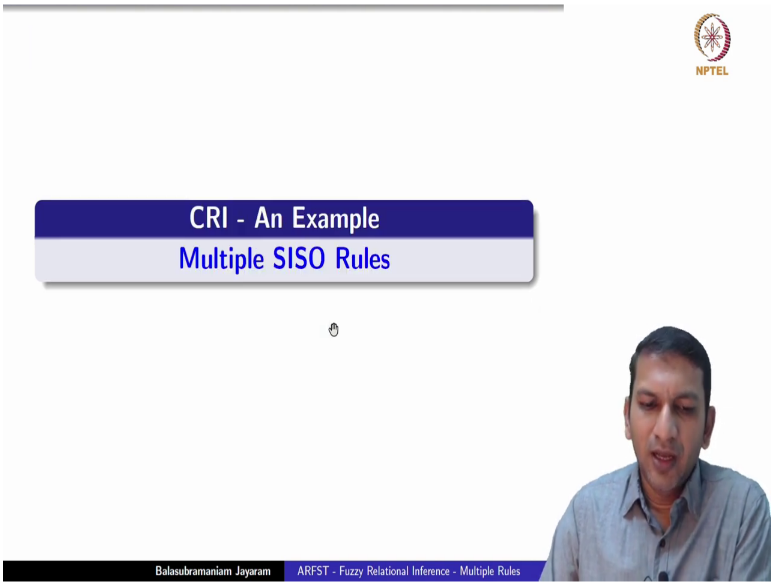

Let us look at a couple of examples let us start with CRI.

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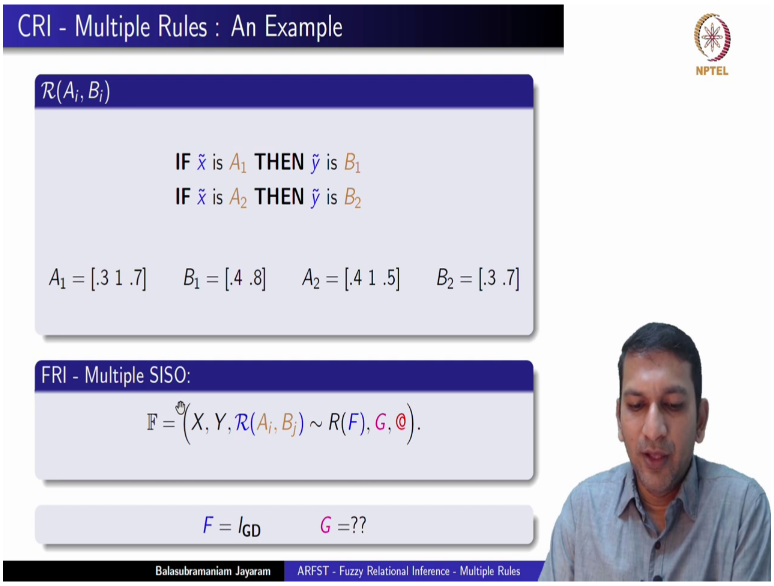

So, assume that we are given two rules and the A 1 A 2 B 1 B 2 are given as follows. So, once again we assume that x is discretized with three points and y by two points.

You might immediately recall this is something that we have already used earlier instead of A and B we are just now calling it as A 1 B 1 just. So, that we can make the calculations easier and now what we want is a multiple SISO rule is given to us and we need to handle this using a fuzzy relation inference.

Now, what are the things that we need? We need an F to relate the antecedent with the consequent we also need an aggregation operation G and of course, the composition.

Now let us assume the Godel implication for relating the antecedent with the consequent then the question now comes what should this aggregation G be.

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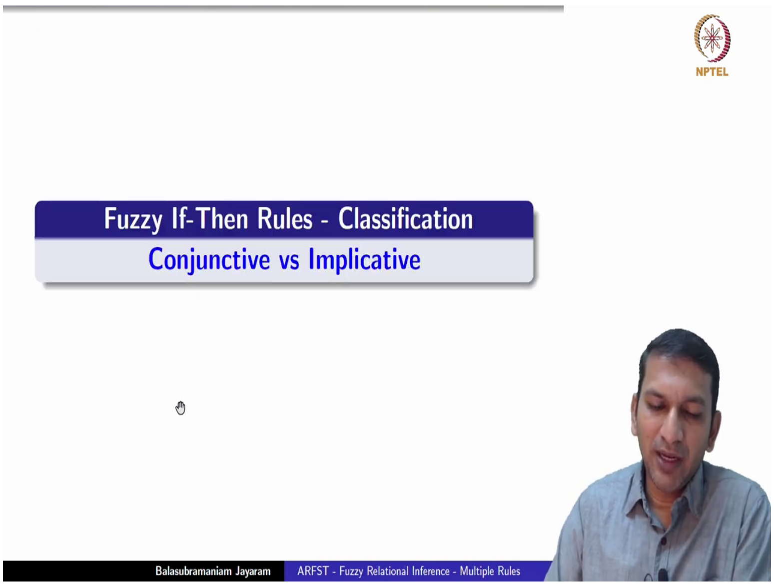

Well, this is where we will go back to one of the classifications that we have given on fuzzy if then rules.

If we recall, we discussed when a fuzzy if then rule is conjunctive or implicated.

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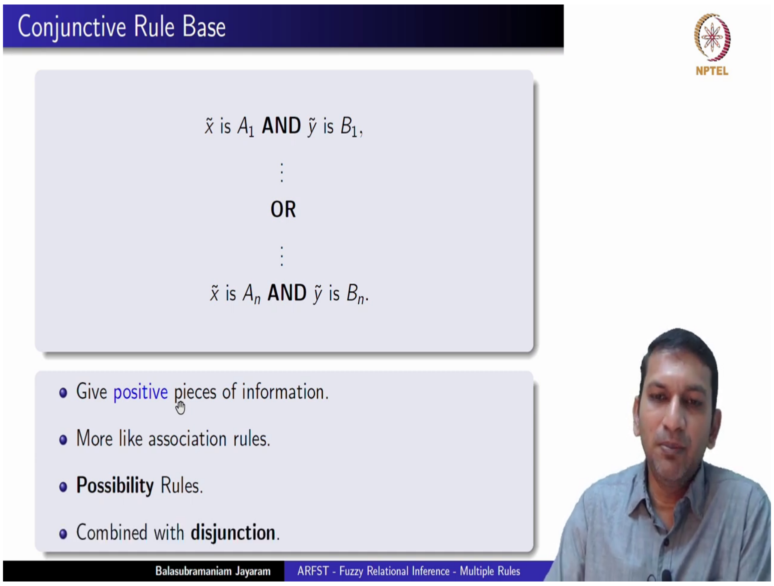

Even though we have written it in this form we could also represent it as if x is A then y is B. So, now, if the rules are conjunctive in nature; that means, they actually give positive pieces of information they are more like association rules and they emphasize on the different possibilities that you have.

So, in that sense these are called possibility rules and what we will do is, we combine them with a disjunction because any one of them is possible. So, we use a disjunction to combine them.

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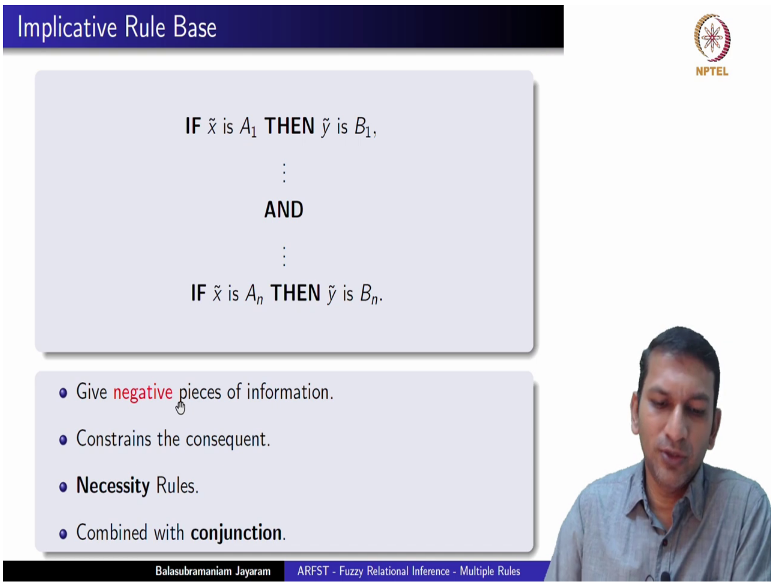

However, if it is an implicative rule then they actually give negative pieces of information they in fact, constrain the consequent if x is A then y necessarily has to be B. So, in that sense these are necessity rules and if you have pieces of knowledge each of which is constraining then you will have to ensure that all of them are valid in which sense we will have to use a conjunctive operator to combine all these rules.

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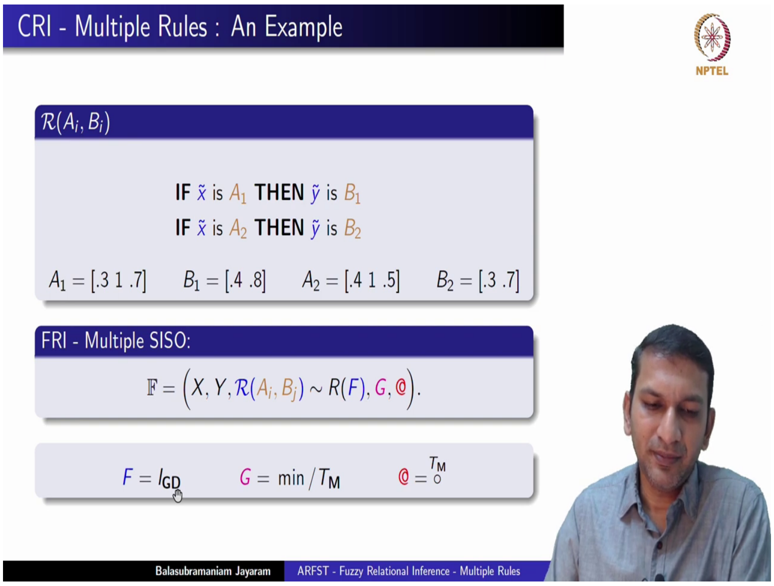

So, let us return to this scenario now we are in fact, using an implication to relate the antecedent and the consequent. So, it is only incumbent on us that we should consider a conjunction for G. Remember this is when we are actually going with what we know about the rules, but of course, theoretically nothing precludes you from using any other operation for G, but since we have some interpretation at hand let us try to stick to it and so, let us consider G to be a T norm and for ease of calculation let us take it to the minimum T norm T m.

Of course, we are considering CRI which means the composition automatically becomes sup T composition and once again for ease of calculation let us consider the minimum T norm sup mean composition ok.

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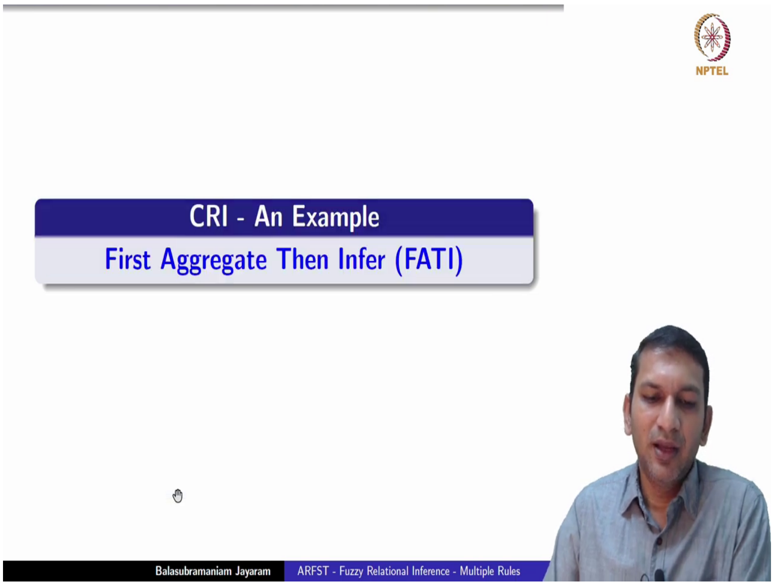

So, let us start with the calculations.

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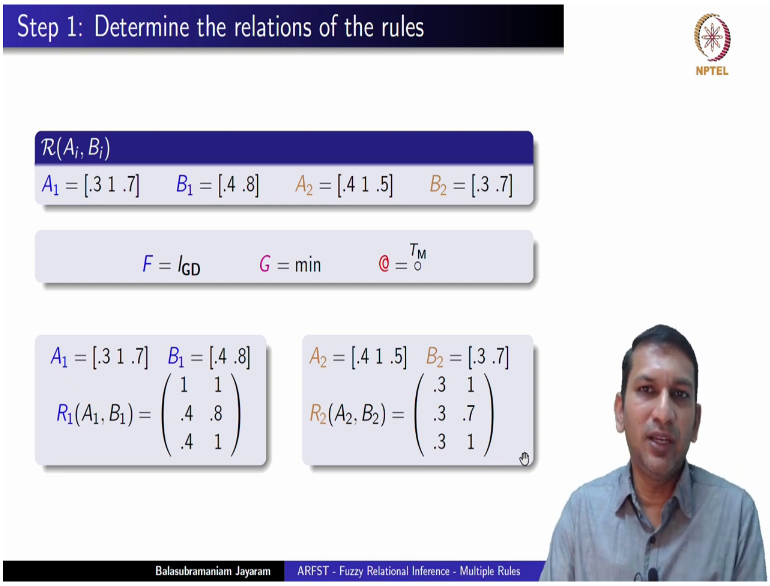

First, we need to determine the relations of the rules. Remember these are two rules given to us A 1 B 1 and A 2 B 2 and we use F to relate the antecedent with the consequent. So, now, given A 1 B 1 we obtain the corresponding relation this is exactly the same thing that we have done a few minutes ago. So, we know that this relation is right and similarly if you use A 2 B 2 and the Godel implication we would get this relation. So, we have two relations from two rules R 1 and R 2.

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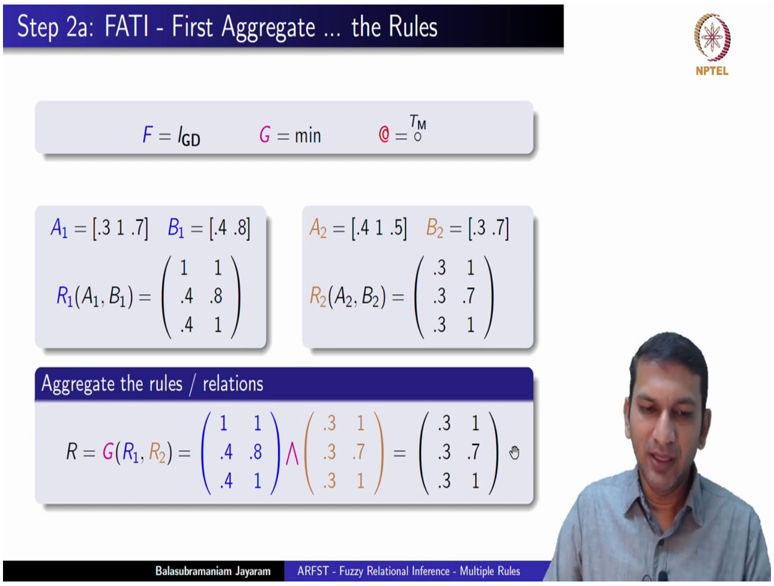

This is the first step in the second step of FATI we first aggregate the rules. So, now, these are the two rules the relations to relations representing the rules. So, we aggregate these relations with the aggregation operation G in our case G happens to be the minimum.

So, we take R 1 we take the minimum operation and we take R 2. Now this is even though they look like they are written in terms of matrices the operation min is in fact, being done component wise. So, if you take 1 and 0.3 minimum with 0.3 then it is 0.3, 1 and 1 is 1 0.4 and 0.3 is 0.3 0.8 and 0.7 is 0.7 and 0.4 and 0.3 is 0.3 1 and 1 is 1.

In fact, you can see that this entire matrix is smaller than this with respect to the usual component wise all that. So, the min of that is essentially going to turn out to be this. So, in the second step we have aggregated the relations that we have into our overall relation R.

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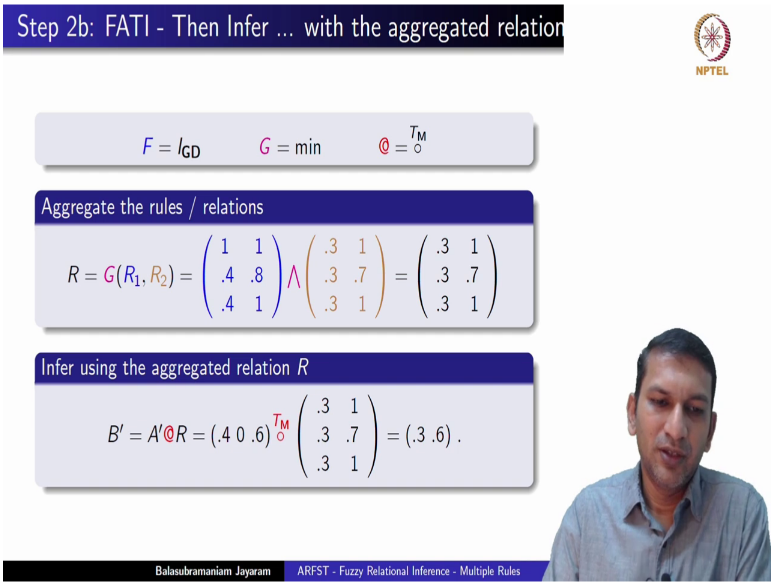

The third step is to infer with this aggregated relation we have this. So, B dash is A dash composed with R.

Now, let us use the same A dash that we have been considering 0.4 0 6 and the composition is given as sup min composition. So, this is what we will have and clearly when we take this and compose with this we have to look at taking min component wise and the max of them. So, and in if you would like to write this once again what we have is 0.4 0 6 composed with 0.3 0.3 0.3 1 0.7 1.

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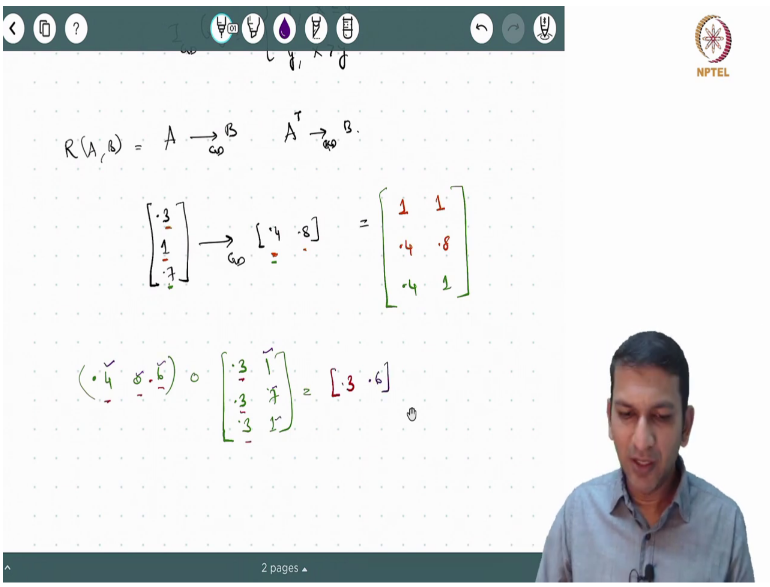

What essentially we are doing is we are going to look at the minimum between 0.4 0.3 which is 0.3 0 and 0.3 is 0.6 and 0.3 is 0.3. So, minimum is essentially 0.3 and once again let us consider with respect to the second. So, 0.4 and 1 it is 0.4, 0 and 0.7 it is 0.6 and 1 it is 0.6. So, the maximum is 0.6.

So, what we will get is 0.3 , 0.6. So, this is the inferred output what did we do? We considered both these relations aggregated them first and inferred using this relation. Let us also look at an example for FITA first infer then aggregate.

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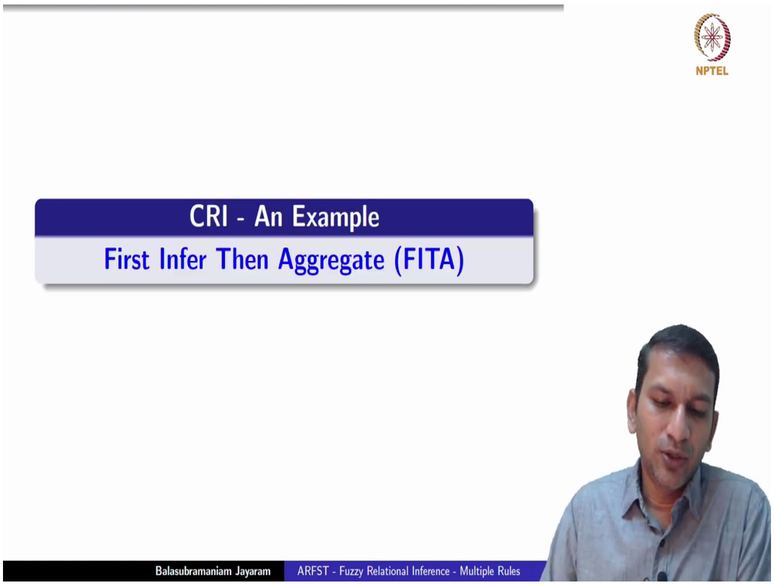

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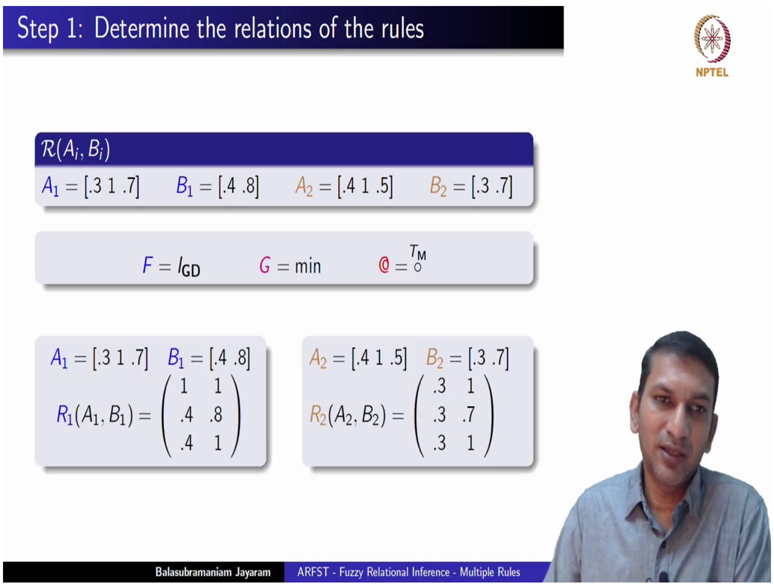

So, essentially, we are going to consider the same system which means we have the same relations representing the rules.

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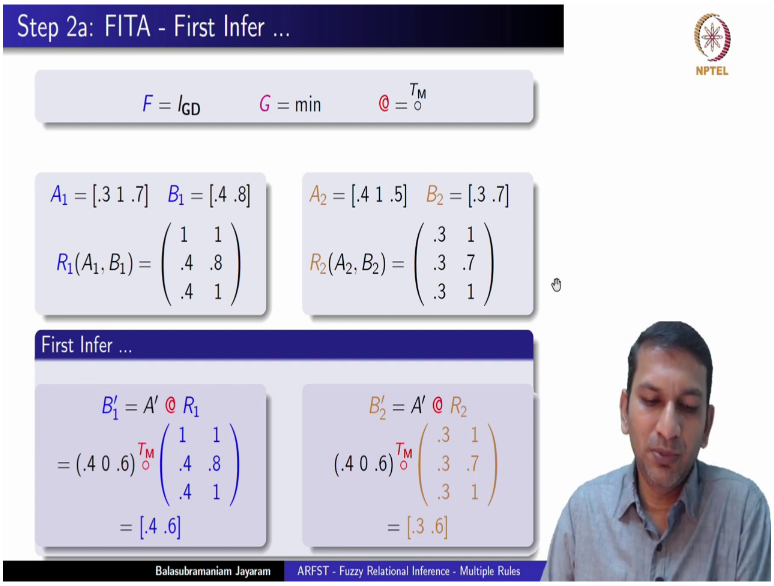

However now here we first infer; that means, given these two relations we independently infer the corresponding B 1 dash and B 2 dash.

So, B 1 dash is A dash composed with R 1 in this case A dash is this, this is the R 1. Once again if we perform the sup min composition this is the output you would get and similarly for B 2 dash we take A dash and compose it with this R 2 that is here and if we see what we would get is we would get 0.3 and 0.6 as the output. So, these are the individual local outputs B 1 dash and B 2 dash.

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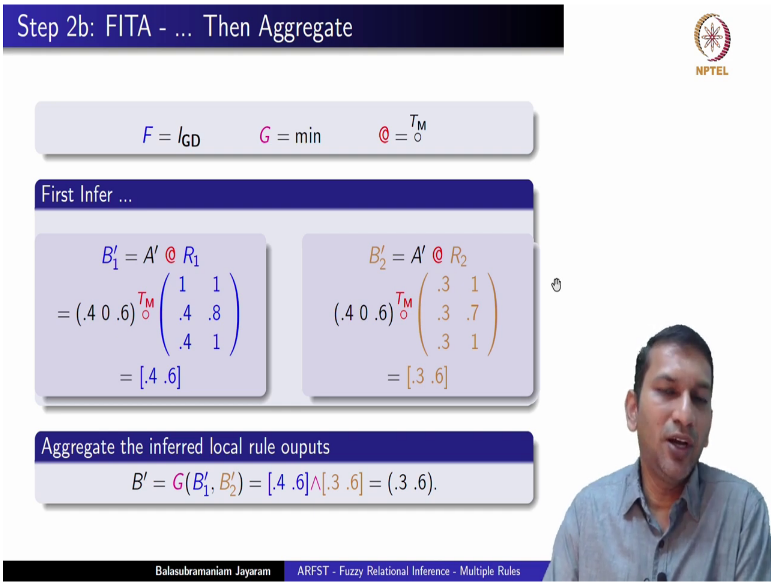

In the second step of FITA what we are going to do is, we are going to aggregate the inferred local rule outputs to aggregate again we use the min operation. So, taking these two doing the component wise min what we get is 0.3 0.6.

So, this is essentially how we do the inferencing with respect to CRI whether we apply the FATI inference strategy or first infer then aggregate inference strategy.

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Let us also look at an example in the case of BKS.

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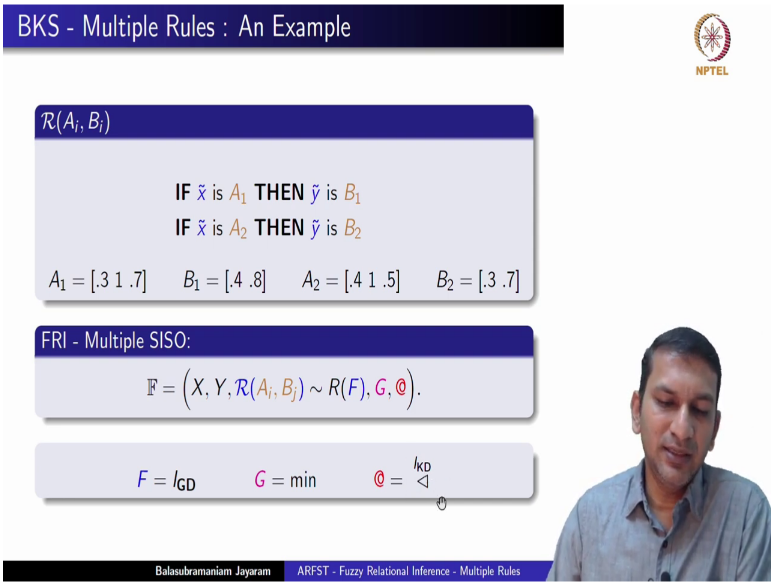

So, once again here we consider the same system we consider the Godel implication for relating the antecedent with the consequent min because it is implicative type. So, staying true to the interpretation we want to use the conjunction operation which we have chosen to be the min T norm and obviously, we are discussing BKS. So, this becomes inf i composition and in this case we have chosen the Kleene dienes implication.

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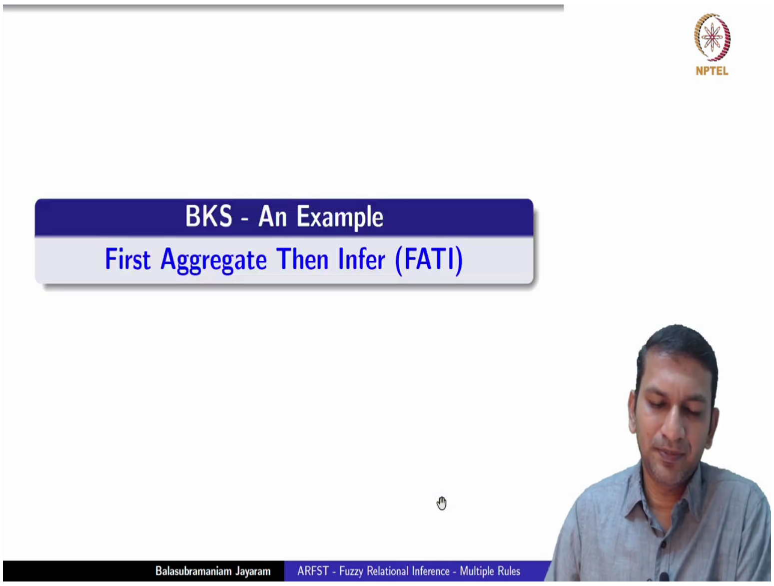

Let us look at how to do FATI in this.

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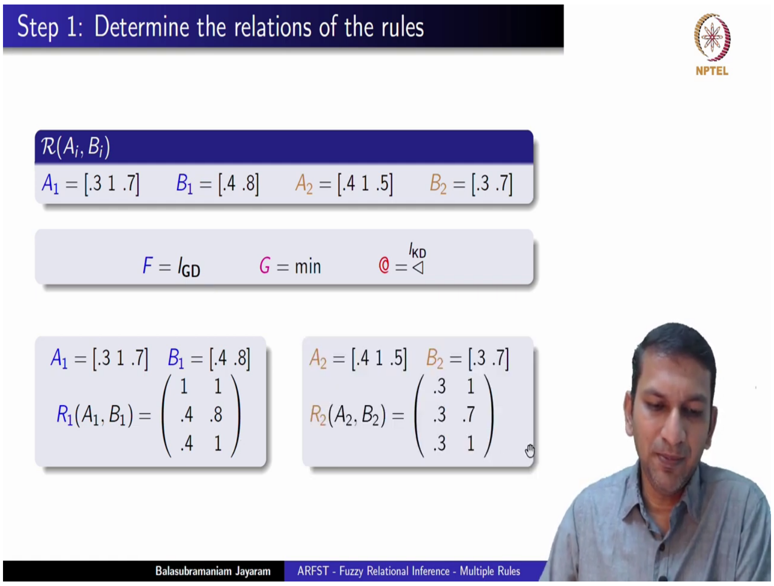

So, now, it is the same system ais and bis are not changed and we are obtaining the relations from the rules using the Godel implication. So, hence these relational matrices also do not change they remain the same.

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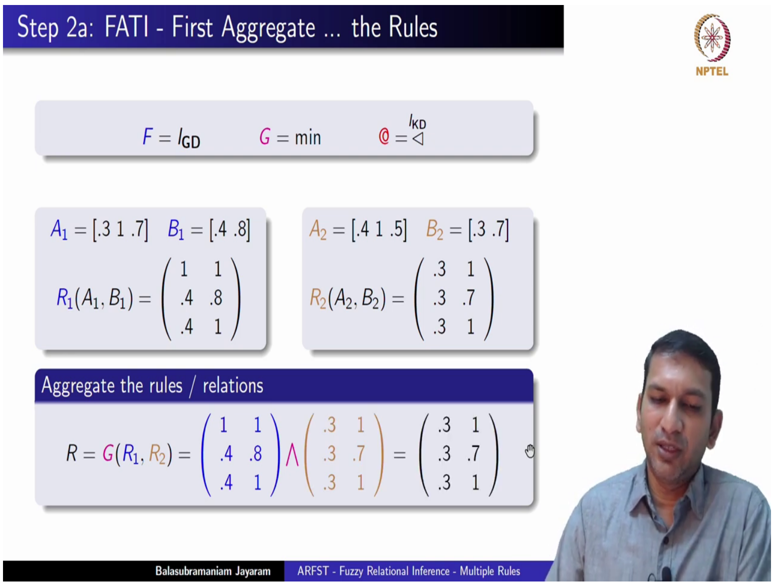

Now, in this case we first aggregate. Once again we are using the same aggregation operation min. So, nothing changes in this step either.

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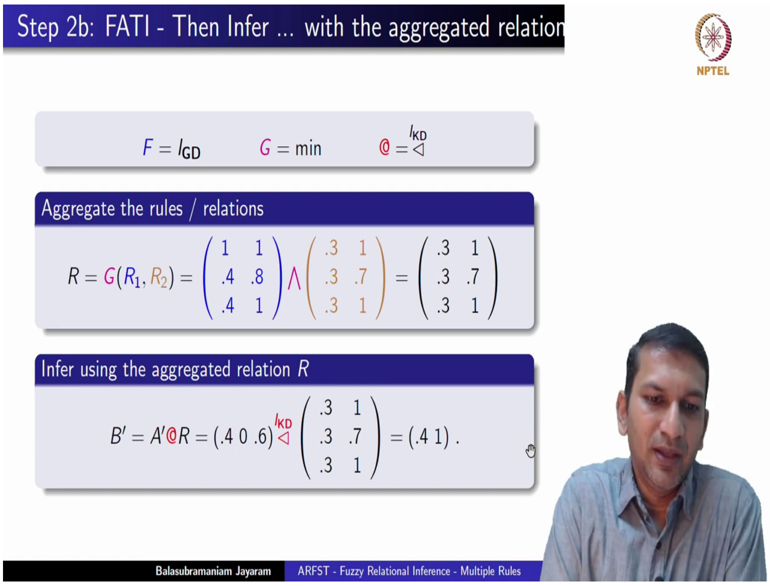

Now things start to change. Now we need to infer with this aggregated relation and now inferring with this relation means composition comes into picture and we are using BKS inference which means the inf i composition comes into place. So, B dash is obtained as A dash composed with R in this case it is essentially taking A dash and using inf i composition with i being the Kleene dienes implication.

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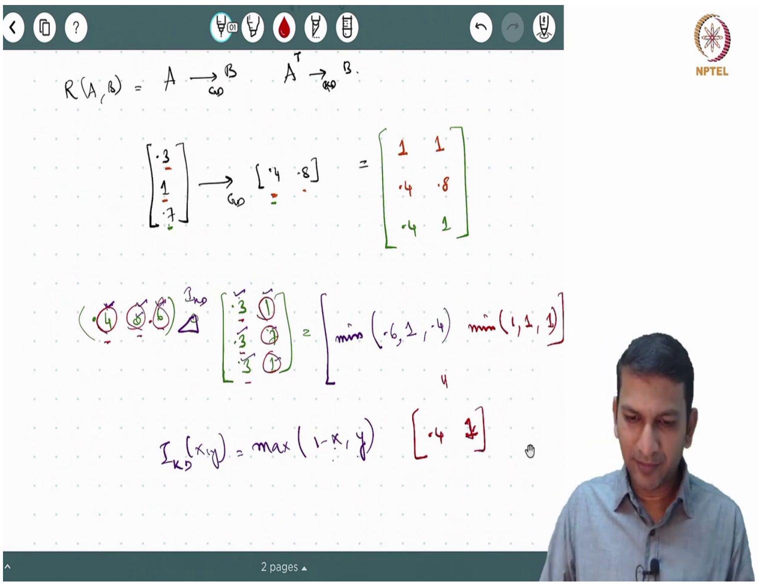

So, once again let us look at this here it is essentially the same matrix. So, I will need erase this and (Refer Time: 23:50) and they influence IND. So, how will it look like. So, now, the two components that we have here let us first take 0.4 and 0.3. So, now, we are using minimum. So, inf i composition I mean i composition because it is a discrete few of finite number of elements are only there and we are using Kleene dienes implication. Please recall what Kleene dienes implication is it is an sn implication where the T co norm is max and the negation is 1 minus x.

So, when we consider 0.4 and 0.3 it is 1 minus 0.4 , 3 max 0.3 max of them. So, 0.6 of 0.3 it is 0.6. If we consider 0 and 0.3 since it is an implication it will be 1 and if we consider 0.6 and 0.3 it is min of maximum of 1 minus 0.6 , 0.3 which is maximum of 0.4 , 0.3 and so, it is 0.4.

Similarly, if we consider 0.4 and 1 neither we will talk about min minimum we know that if y is 1 it is 1 0 implies min term it is 1 and once again 0.6 and 1 if you take is 1. So, now, this is what we are looking at this is equal to if we take the min of them it is 0.4 and 1. So, you see here this is the output we get using B KS and the FATI strategy.

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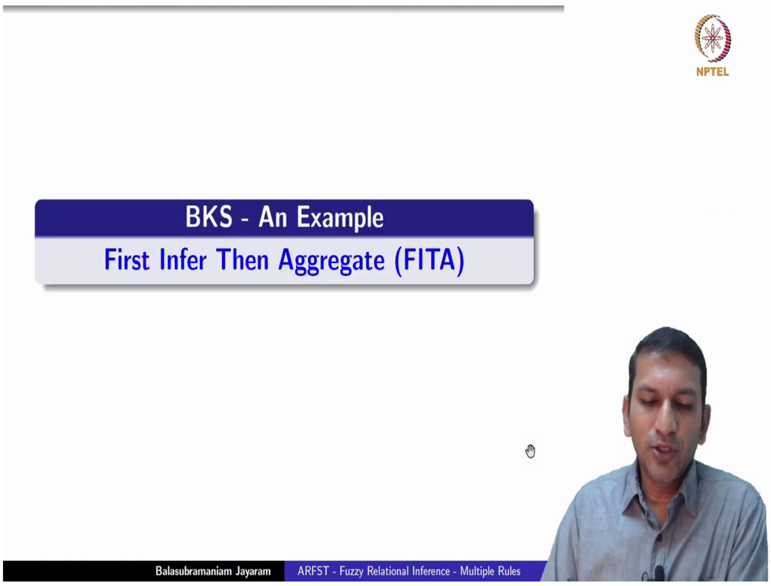

Let us also apply on the same system BKS with FITA strategy.

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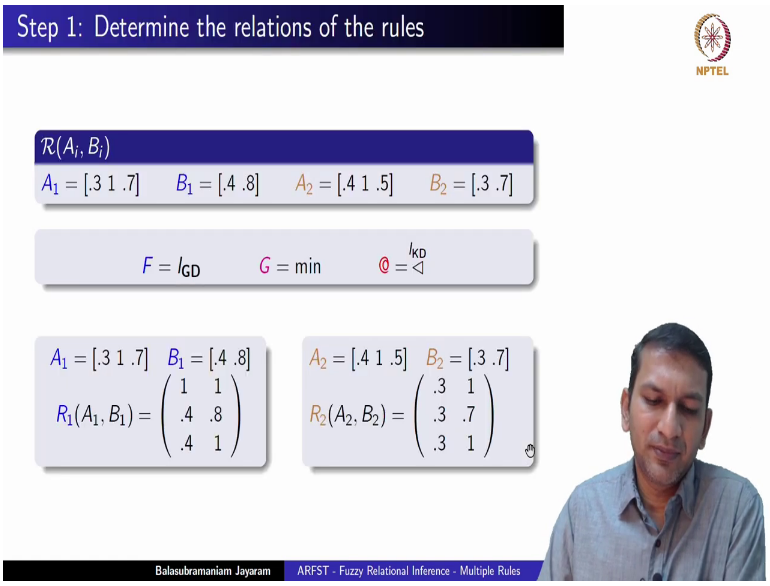

Once again the relations do not change.

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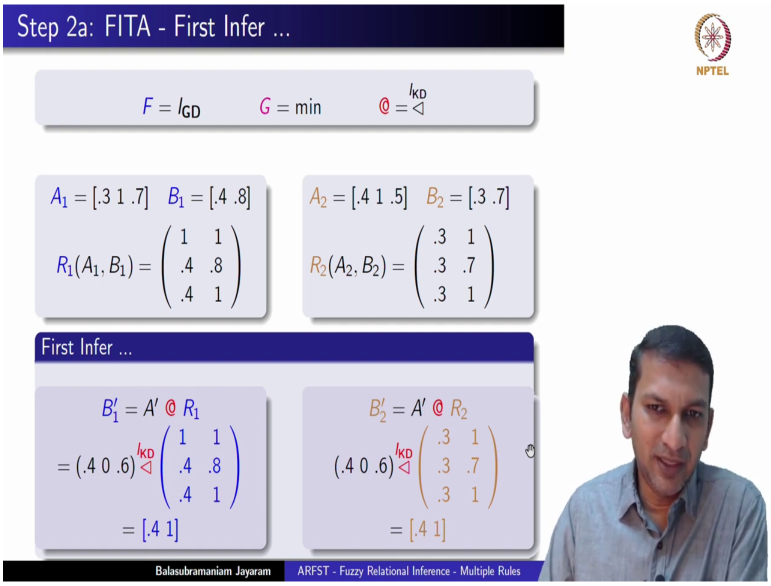

But now the first step itself will change because using these relations we need to infer B 1 dash and B 2 dash using the inf i composition.

So, these are R 1 and R 2. So, B 1 dash is A dash composed with R 1 using inf i where i is the Kleene dienes implication just as we have done now if you apply this what you would get is this as the output the vector 0.4 1 and if you apply A dash on it mean composite with R 2 once again you get the vector 0.4 1. So, now, we have inferred independently these are the local outputs.

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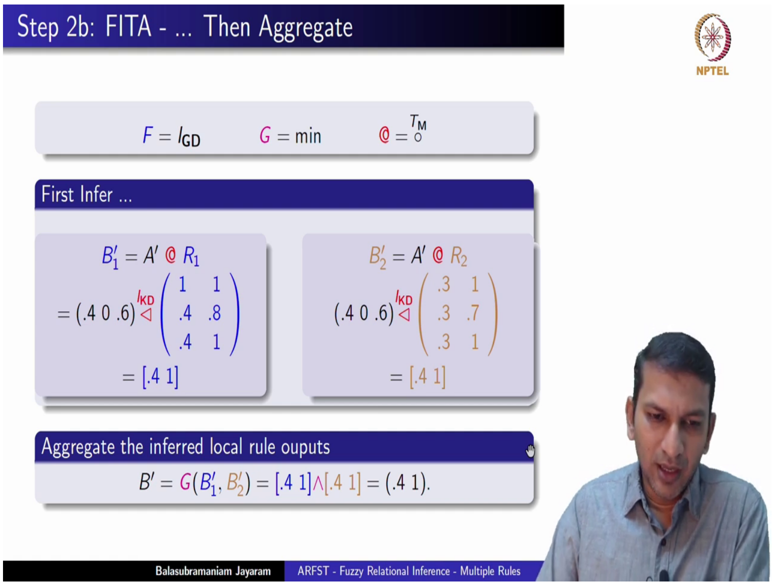

Now, we need to aggregate these local outputs using the aggregation operation which in this case is the min operation here both the vectors are identical. So, you get this as the output. So, this is all you would do you would handle BKS inference when you have multiple SISO rules either using FITA or FATI.

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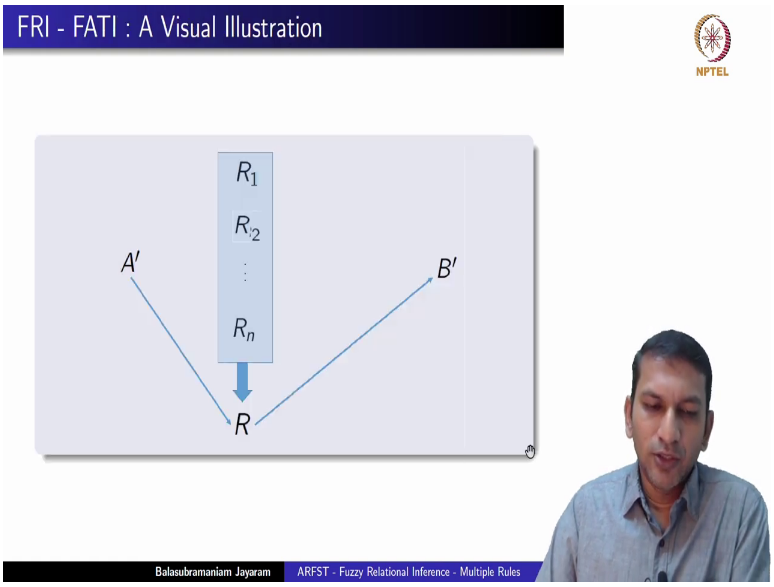

So, once again if you are looking at FATI all you do is get individual rules aggregate them first and use that aggregated overall global rule global relation to infer from the given A dash by composition.

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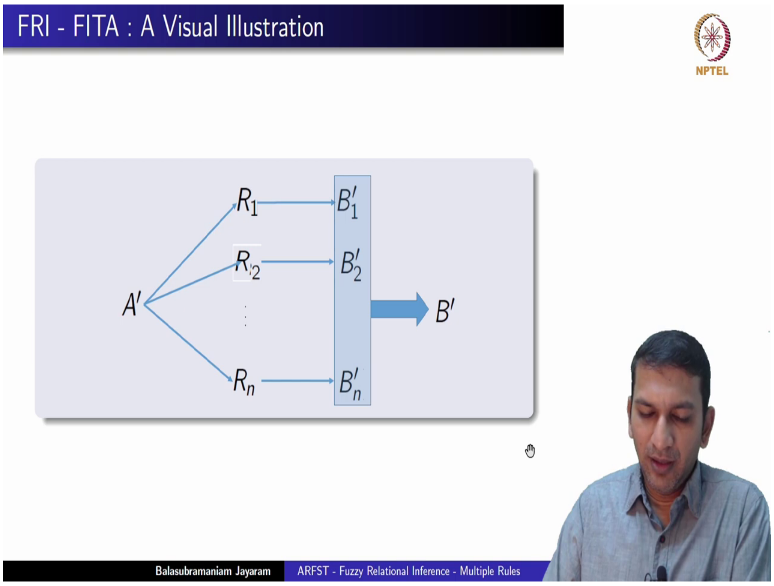

If you are using FITA then keep all these relations separately infer locally; that means, use R 1 and A dash to get a B 1 dash R 2 and A dash to get a B 2 dash so, on and so, forth till B n dash then aggregate these fuzzy sets B 1 dash B 2 dash till B n dash to obtain your B dash.

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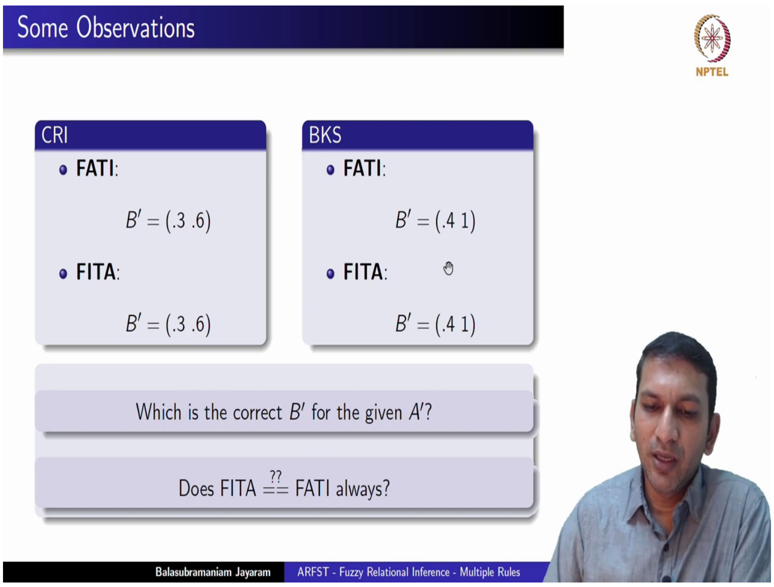

Well, some observations are worthy of making look at for this particular example where the Ais and Bis have remained the same and so, has A dash when we applied CRI with FATI inference strategy to obtain B dash was 0.3 0.6 when we applied FITA strategy once again it was 0.3 0.6.

On the other hand when we applied BKS on the same system with FATI we have obtained the output as 0.4 1 and with FITA again we obtained the output 0.4 1. It throws up many interesting questions and couple of them for you first question that you would ask is on the same system just by changing the composition we have got two different outputs two different B dash which is the correct one.

Another question that we could ask is just looking at it. So, it appears that in CRI whether you use FITA or FATI or in BKS whether you use FITA or FATI, the outputs seem identical, but is this magic will this magic work every time or is it just an anomaly or are there clear conditions under which FITA will be equal to 5? Now these are very interesting questions as was mentioned earlier in one of the lectures we will discuss these and to be able to discuss them and give a clear answer.

We would make use of some of the theoretical structures that we have built up we have seen earlier in the earlier weeks of this course.

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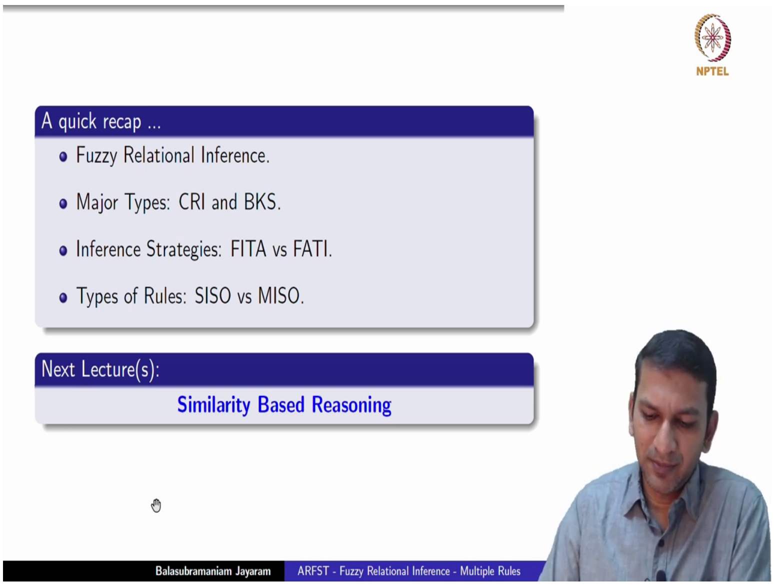

A quick recap of what we have dealt with in this whole week. We have looked at fuzzy relational inference in depth we have seen there are two major types the compositional rule of inference and the Bandler Kohout Subproduct. There are two inference strategies when we are considering multiple rules FITA first infer then aggregate or first aggregate then infer.

And there are two types of rules to consider single input single output or multiple input single output. What next? We will look at the other major type of fuzzy inference mechanism essentially the similarity based reasoning system.

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Once again a good resource for the topics covered in this lecture is the book of Driankov Hellendoorn and Reinfrank.

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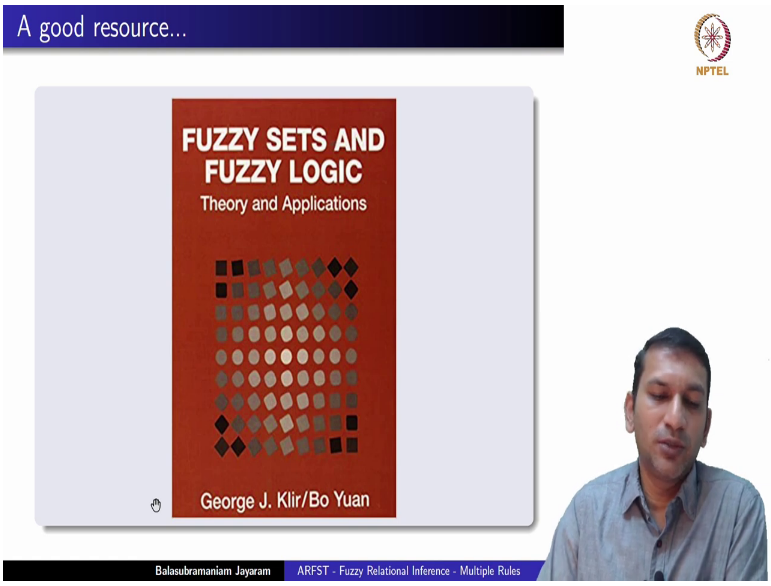

And of course, also the book of George Klir and Bo Yuan. Glad that you could join us for this lecture and hope to see you soon in the next lecture.

Thank you all.