Approximate Reasoning using Fuzzy Set Theory
Prof. Balasubramaniam Jayaram
Department of Mathematics
Indian Institute of Technology, Hyderabad

Lecture - 30
Fuzzy Relational Inference

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Hello and welcome to the next of the lectures in this week 6 of the course titled Approximate Reasoning using Fuzzy Set Theory a course offered over the NPTEL platform. In this lecture, we will look at one particular type of fuzzy inference mechanism that of Fuzzy Relational Inference.

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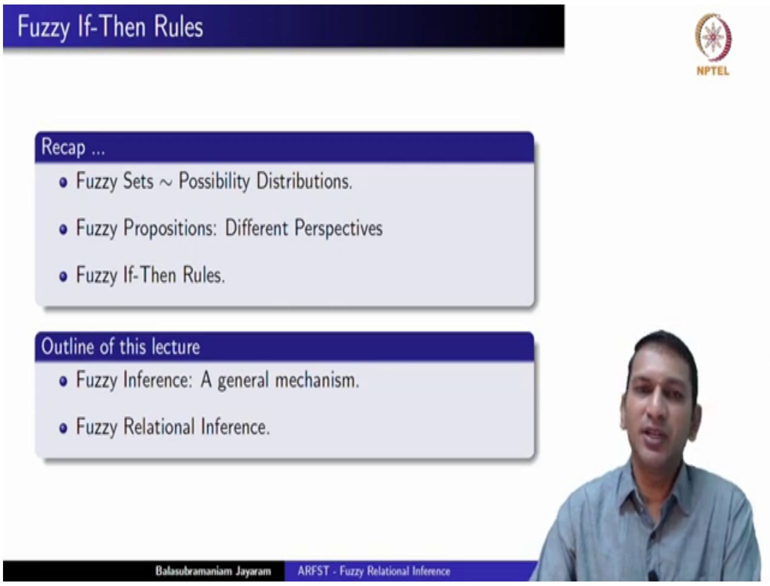

Leading up to this in the last two lectures, we have looked at a new perspective of fuzzy sets; we have looked at different interpretations of fuzzy proposition themselves and using these, we have moved ahead to discussing fuzzy if then rules.

As was already mentioned, these are the basic building blocks that will go into one particular important component of a fuzzy inference system what are they, that is what we will discuss in this lecture. We will begin by looking at a general mechanism, a general schematic for a fuzzy inference mechanism and move on to discuss one particular fuzzy inference mechanism that of fuzzy relational inference.

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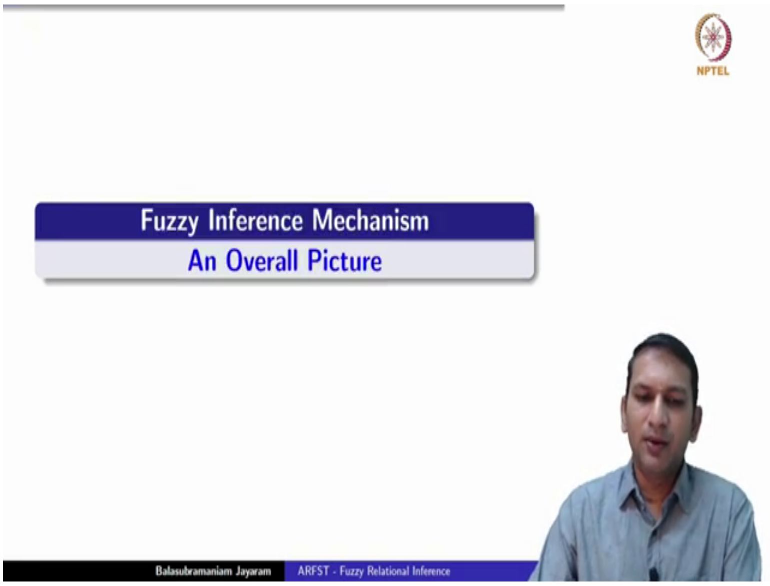

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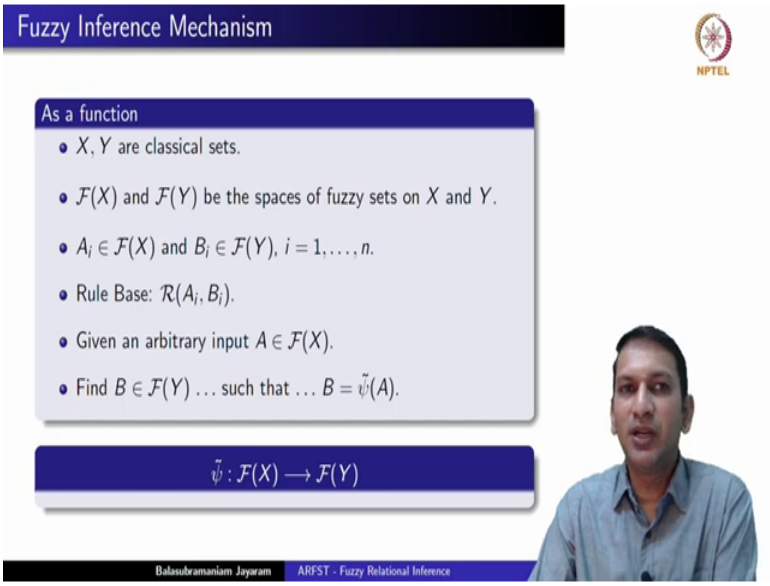

Well let us get an overall picture of what a fuzzy inference mechanism is. We would like to look at fuzzy inference mechanism itself as a function, means a fuzzy inference as a function. How so? So, we have these two classical sets X and Y, which are essentially the domain of the input and output. F(X) and F(Y) are the spaces of fuzzy sets that you can build on X and Y and it is from these that we are sourcing our antecedents and consequence of the fuzzy if then rules; A i’s and B i’s are actually fuzzy sets of X and Y.

It is using these we build this rule the set of rules, which we call the rule base, fuzzy if then rule base. And what we want to do now is given an input A, which is again a fuzzy set over X; we would like to find a B which is a fuzzy set over Y, such that B is somehow related to A. So, on the whole we can look at a fuzzy inference as being given a fuzzy set in X and obtaining a fuzzy set in Y.

So, essentially you can look at this as a mapping from F(X) to F(Y). However, there is not an arbitrary mapping we will have to make use of the ground truth which is given to us; the ground truth is in the form of knowledge of about particular system, which is encoded in the form of conditionals fuzzy if then rules.

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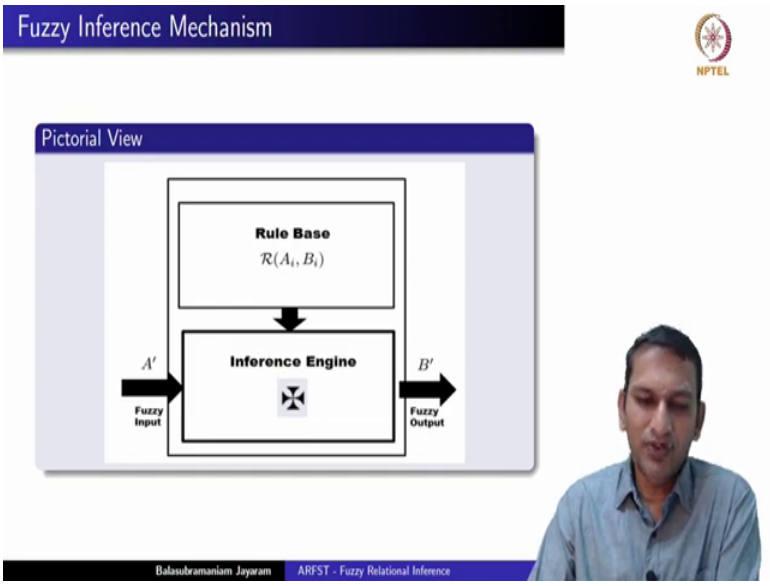

So, this is a pictorial view of the fuzzy inference mechanism itself, any fuzzy inference mechanism. Let us look into these two important components; the rule base and the inference engine.

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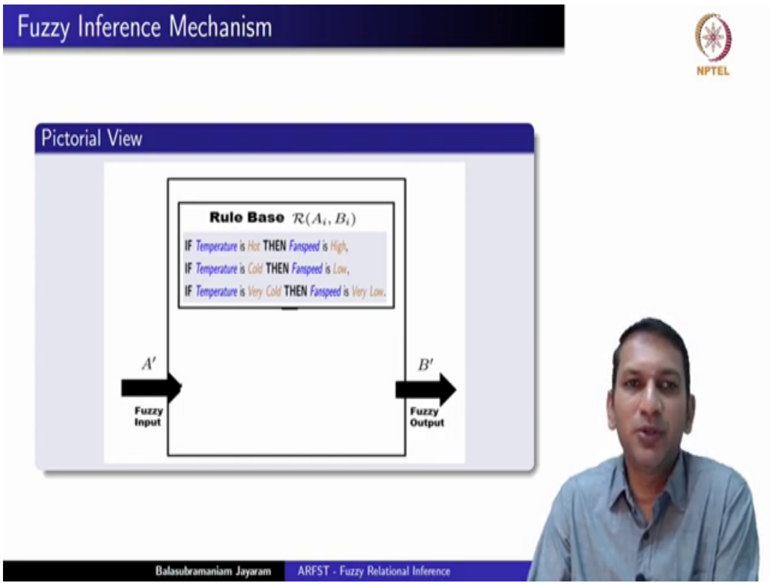

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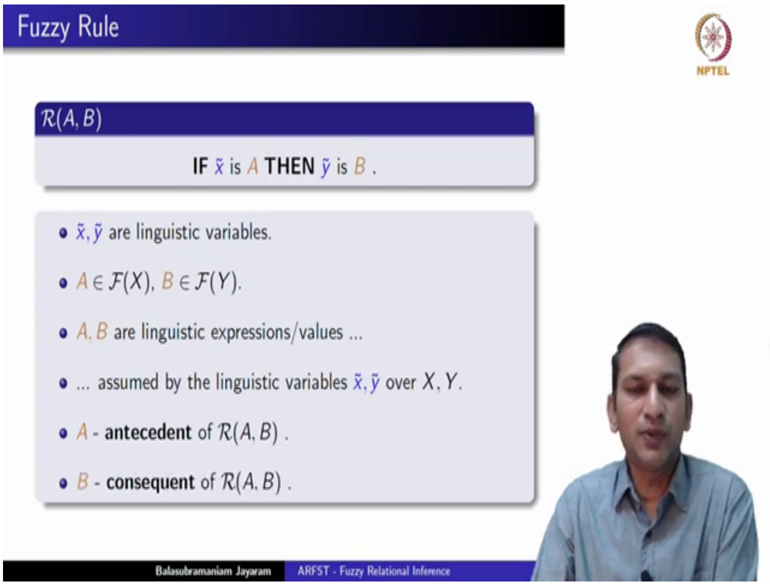

Now, a rule base as we have seen is a collection of fuzzy if then rules. Let us have a quick recap of how a fuzzy if then rule will look like; we know that a fuzzy if then rule connects two fuzzy propositions x tilde as A and y tilde as B. And how do we interpret them? One way to interpret them is x tilde and y tilde are linguistic variables and if A and B are fizzy sets over X and Y, the appropriate domain for x tilde and y tilde.

Then we look at A and B as linguistic expressions of values which are assumed by this linguistic variables x tilde y tilde over the respective spaces X and Y. Note that A is called the antecedent of this rule and B as the consequent of this rule, ok.

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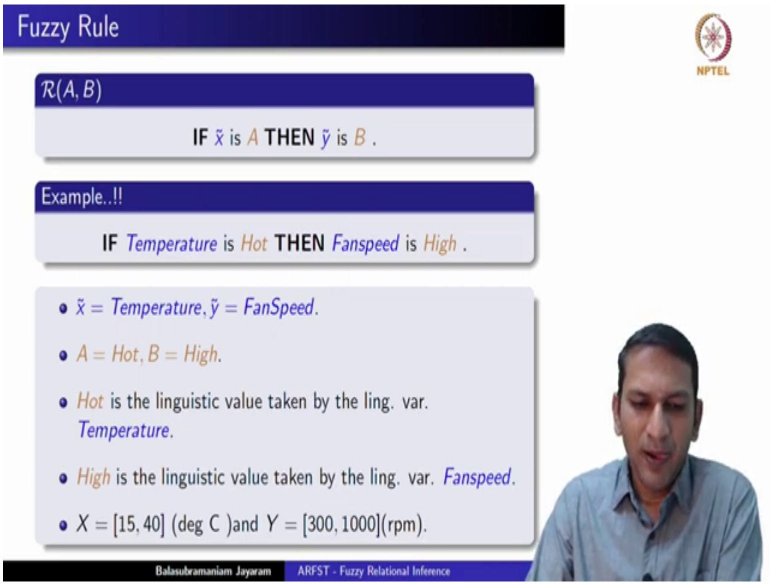

One example that we have seen earlier also; so we have this rule if temperature is hot, then fanspeed is high, then temperature and fanspeed are the linguistic variables, hot and high are the linguistic values, hot is the linguistic value taken by the linguistic variable temperature and so is high, which is taken by the fanspeed.

Note that the fuzzy sets themselves are defined over the domain X and Y. In this case you could choose domain X to be the interval 15 to 40 and Y to be from 300 to 1000 which indicates the value of the r p m. And in the case of X, it may denote the numbers which are in degrees integral.

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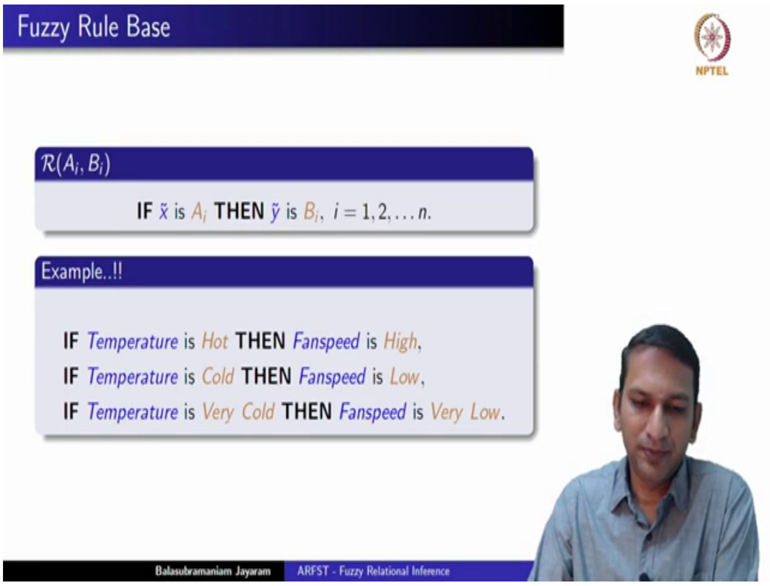

Well, just the single rule is not sufficient to capture the entire knowledge about the system. So, we need multiple such rules. So, this is one example of a set of rules you could have, if you are trying to say for example automatically control an air conditioner.

So, now, you see here, here once again temperature and fanspeed are linguistic variables; temperature can take the linguistic values hot, cold or very cold, similarly fanspeed which is a linguistic variable can take fuzzy sets defined on Y, which in our case is between 300 and 1000 and they could be labeled as high, low or very low.

So, it is from here we are getting this rule base. So, this main component which contains a knowledge about the system, we have dealt with and this was just only a quick recap. Let us be given of an input, which is in the form of A dash which is a fuzzy set. What we essentially want is to make use of this ground truth, this knowledge which is encoded in the form of fuzzy if then rules, make use of these and come up with a reasonable output B dash, which is fuzzy set on Y.

Now, this is like much like having a book which you know can solve your problem; the problem at hand is A dash and you want a solution which is B dash. And you know that making use of this resource the knowledge that is contained in the book, you can solve this, you can get an appropriate output. But we need intelligence, we need something more to be able to make use of this knowledge and that is what is the inference engine.

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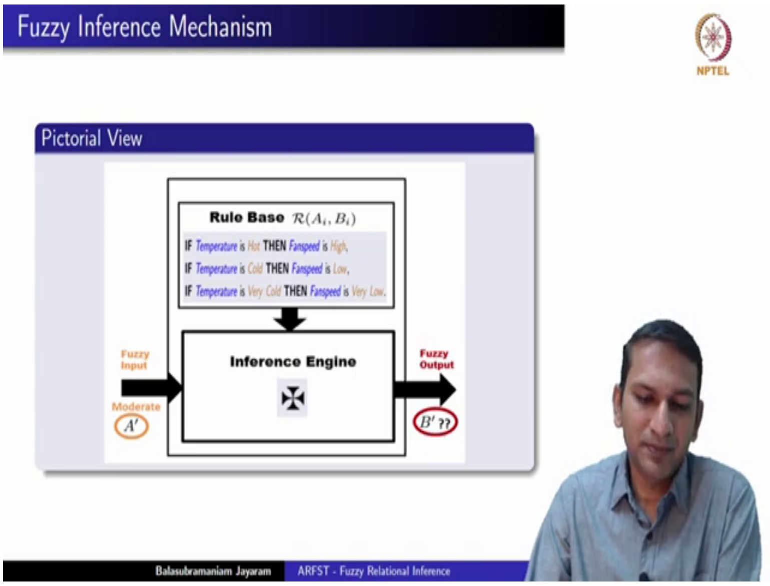

Of course, the inference engine takes the inputs or talks to the rule base, the knowledge base and looks at the input and based on this ground truth of knowledge, makes a reasonable conclusion B dash. For instance, given this rule base, if you are given the input that temperature is moderate, A dash is moderate; then we ask ourselves the question what should be the fanspeed, what should the fanspeed be?

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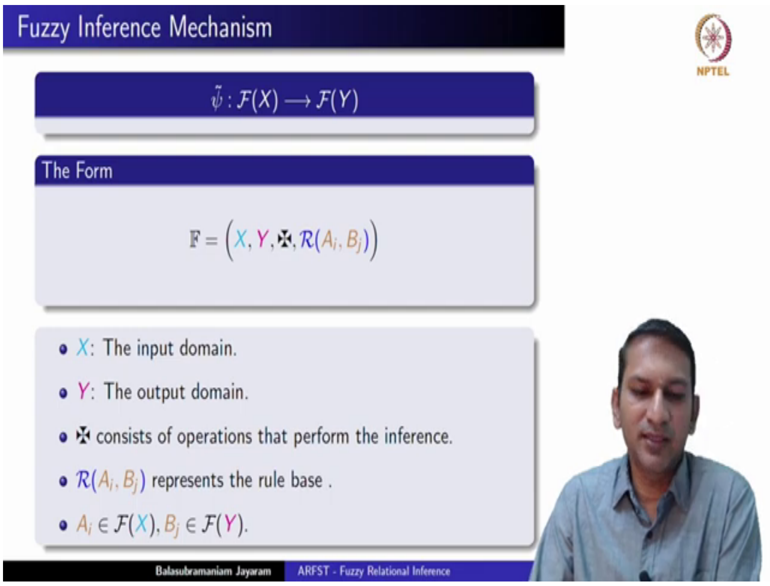

So, we said that a fuzzy inference can be looked at as a mapping from F(X) to F(Y), we could give represented by this quadruple. For the moment, we will fix this; as we go along, we will see that we will need more and more parameters coming into picture, this is very simple water down form of fuzzy inference that we are going to get introduced to.

Here what do we have? The input domain X, the output domain Y and this symbol denotes operations that actually perform the inference and R of A i , B j there it represents the rule base, the set of fuzzy if then rules and note that A i's and B j's themselves are fuzzy sets defined over X and Y.

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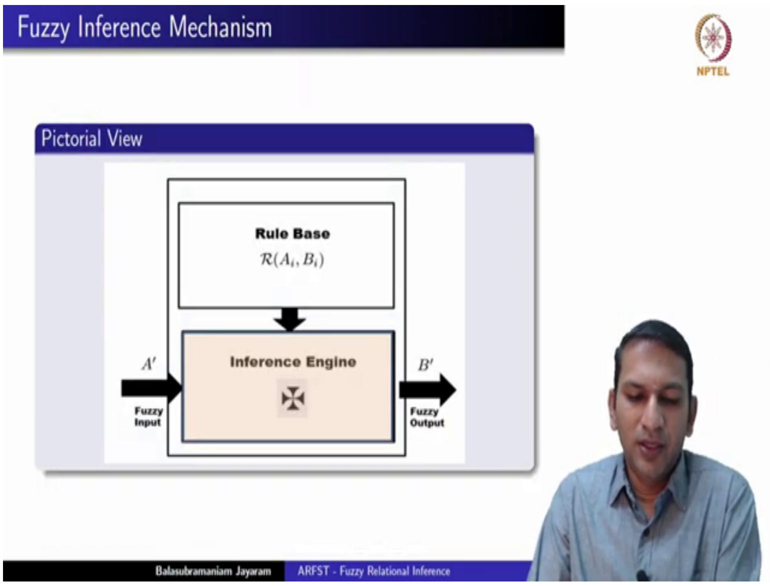

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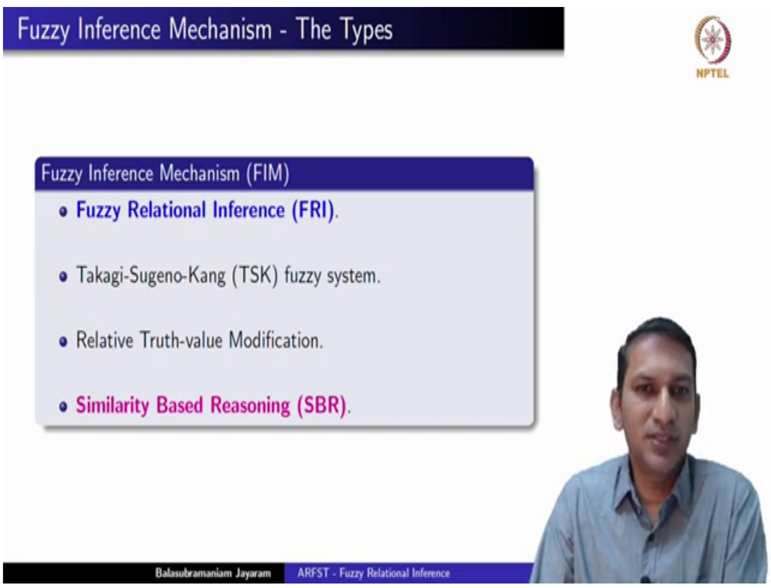

Now, let us look at inference engine in depth. If you look at this inference mechanism, there are many types; first is that of a fuzzy relational inference. So, for a given set of rules, you could use a fuzzy relational inference to make use of the rules, so that you can obtain reasonable outputs for given inputs; there is this Takagi Sugeno Kang system which is typically abbreviated as TSK fuzzy system.

This is relative truth value modification to obtain the output given an input making use of the rules. And we could also generalize some of these into what we call similarity based reasoning. In this lecture series, we will look at only two major categories of fuzzy inference mechanism; that of fuzzy relational inference and similarity based reasoning. Fuzzy relational inference is what we are going to deal with in this week and similarity based reasoning is something that we will discuss in the next week.

And you might note that TSK fuzzy system can also be looked at as one particular example of similarity based reasoning. In fact, some particular types of FRI is fuzzy relational inferences can also be looked at as similarity based reasoning.

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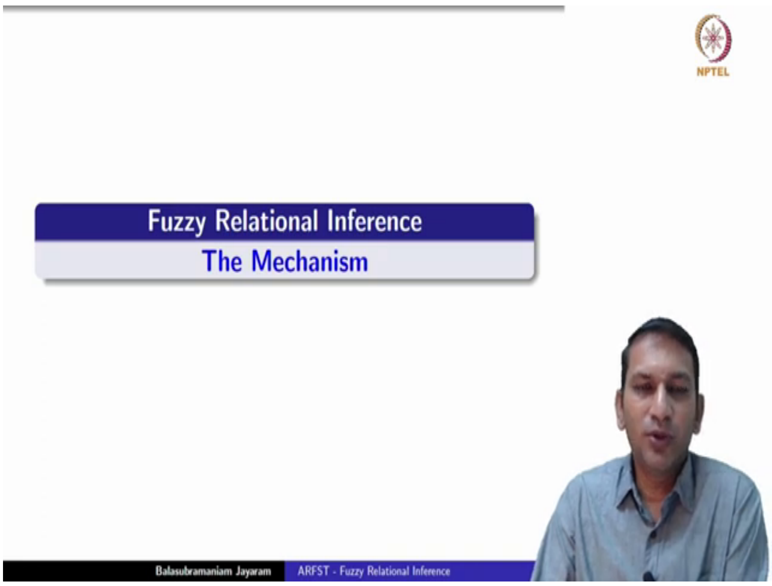

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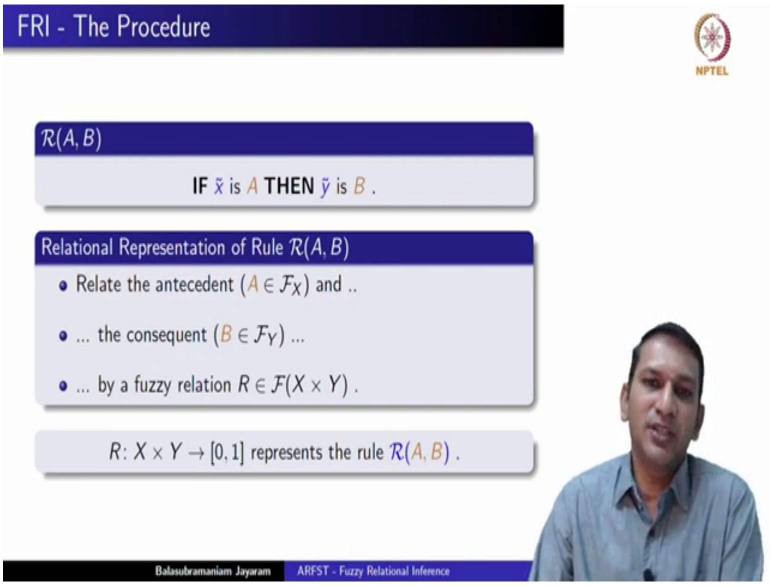

Well, let us move to looking at fuzzy relational inference. Let us look at the mechanism, how do we do this? We are given the rule. Firstly we are going to represent this rule in terms of a relation.

What do we mean by this? We would like to relate the antecedent A which is the fuzzy subset on x, which is a fuzzy set on x to the consequently which is a fuzzy set on y, by a fuzzy relation on F(X) cross Y, this is what we are going to do. Now, note that what we need actually is a relation on X cross Y, fuzzy relation on X cross Y that represents the rule R of A , B, this is what we want. So, we encode the rule as a relation, as a fuzzy relation.

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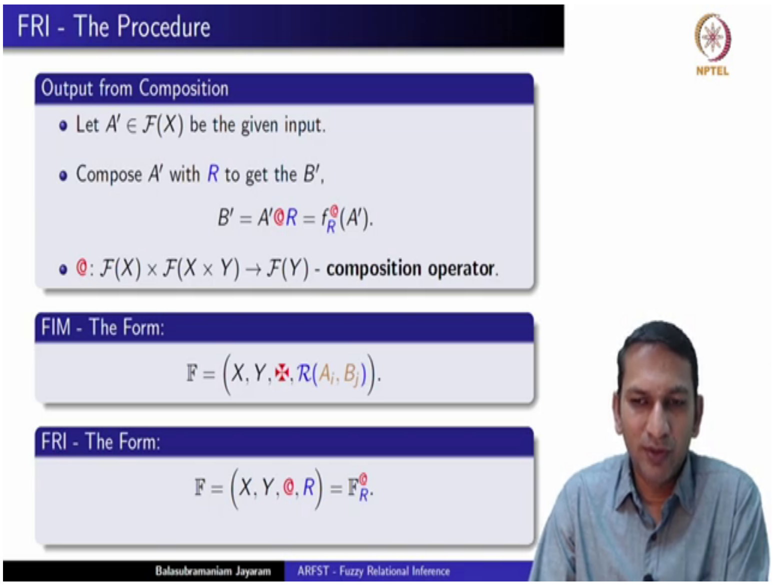

Next now we hold this knowledge in the form of relation and given an input how do we get the output? So, let us assume that A dash is a in fact, a fuzzy set on X; let us be the given input. What we do is, we have this A dash, we compose it with the rule, which is represented as a relation R to get the B dash; that means B dash is A dash composed with R, some kind of a composition operation is being made use of.

Since we said that the fuzzy inference mechanism can be looked at as a mapping between fuzzy sets F(X) to F(Y); we could also denote it like this f R at the rate of at A dash. Look at this A dash is in fact a fuzzy set on X and what we are getting is a B dash, which is a fuzzy set on Y through a relation and a composition. This operation if you see actually takes an element from F(X) and an element from F(X) cross Y that is your fuzzy relation which captures the essence of the rule and what it outputs is a fuzzy set on Y.

So, in that sense you can look at it as a composition operator. Well, we said this is the general form of any fuzzy inference mechanism, we can represent it by quadruple. How will an F R I look like, the fuzzy relation inference look like? So, if we were to write this, so X and Y remains same. What we need for doing the inference are these two; the composition and R which actually captures the rule base in terms of a relation.

So, we could write this also and we will often write it like this in a short term notation with this bold F with subscript R and the composition on top. So, as we go along we will see even;

by the end of this lecture we will see that, we have many many options on how to build this relation and also which composition operator to use.

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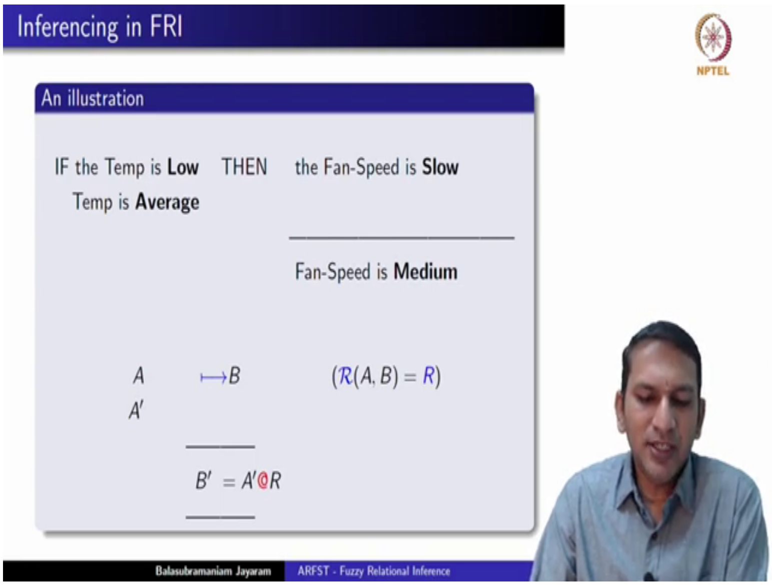

How do we infer in FRI, let us have an illustrative example. Consider this rule; if the temperature is low, then the fanspeed is low. Now, we are given an input temperature is average, now we want to say something about the fanspeed. Let us for the moment assume that we want the output to be fanspeed is medium. So, this is the kind of inference we are trying to do. Now, what are we given? A implies B or A and B the fuzzy rule; if X tilde is A, then Y tilde is B.

Now, we want to actually capture this in the form of a relation this rule and given an A dash we want a B dash, this is what we want. In fuzzy relational inference, how do we do this? First we convert capture the rule in terms of a relation. So, R of A, B is essentially becomes an R the fuzzy relation and B dash is obtained by composing A dash with R.

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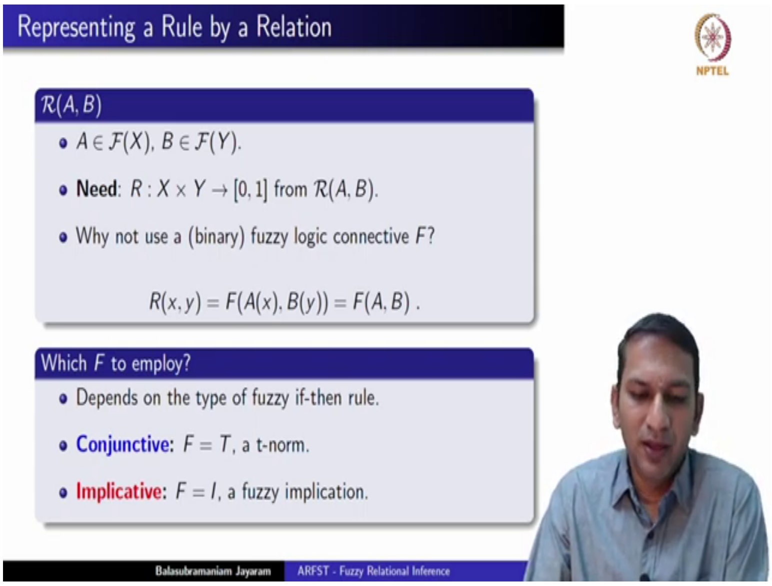

Let us look at how do we represent a rule by a relation. Note, we denote the rule by R of A, B, where A is the fuzzy set on X and B is the fuzzy set on Y. What do we need? We need to build a fuzzy relation on X cross Y, on the underlying domains X and Y, on the Cartesian product of X cross Y from this given rule. Now, immediately the thought should arise why not use a binary fuzzy logic connective F?

So, you have a fuzzy set on X, you have a fuzzy set on Y; we want to construct a fuzzy set on X cross Y, essentially a fuzzy relation, binary fuzzy relation can be looked at as a fuzzy set on the corresponding Cartesian product. So, this is what we will try to use. So, R of x, y can be obtained from some fuzzy logic connective F evaluated at the membership values of x in A and y in B.

In the shorthand notation, we will also moving going forward write it as F of A , B; this is something that we have seen earlier too when we write min of A , B, essentially we are taking minimum of A x , B x, product of A , B is A of x dot B of x or y, depending on where the elements are coming from.

So, in this case we need to note that, x is coming from the domain X and y is coming from Y and they need not be equal, the domains x and y need not be equal. However, you should you know notice here that F is only a fuzzy logic connective; so all it is expecting is elements coming from [0,1] interval. So, now, A of x is a membership value, B of y is a membership value; so it does not matter if X is not equal to Y, still we will be able to aggregate or apply this operation F on these two values which are essentially numbers from the [0,1] interval, ok.

Now, the question is which F to employ? Well, it depends on the type of fuzzy if then rule. Now, we have seen that one classification of a fuzzy if then rule is either it is conjunctive or implicated, depending on what kind of information it conveys; is it positive information or negative information. In the case of conjunctive type rule, we know that it gives you positive information; that means essentially examples positive examples. So, in that sense, both of them can occur, A and B can occur that is what it says.

So, hence it is typical to use a conjunction and in our case a t norm. If it is implicated; that means necessity kind of rules, which precludes some possibilities, then obviously implication is the choice that you can think of. So, the information contained in the rule decides for you what kind of an operation that you can use, which fuzzy logic connective you can use to actually build the relation from the corresponding group.

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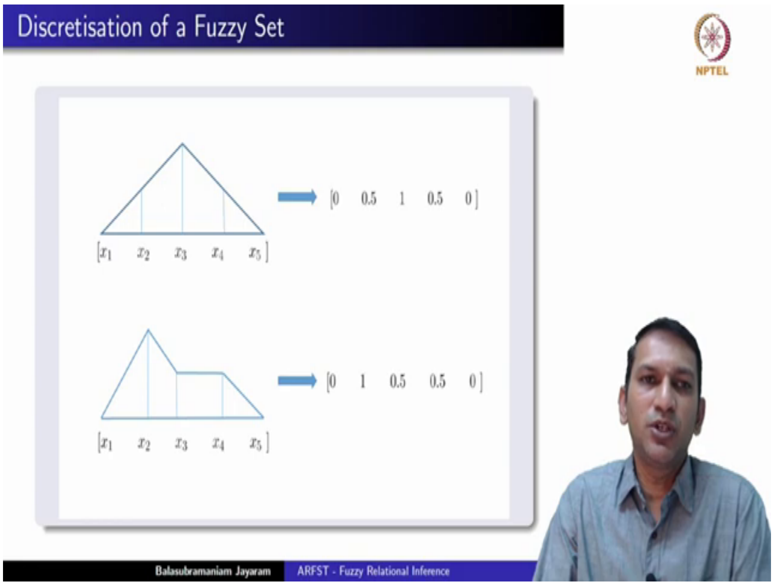

Let us look at some examples. Before that recall that typically when you implement in a system, we will have to discretize whatever we are doing. So, even if your fuzzy set looks likes this as a triangular fuzzy set, we often need to discretize it. So, if you discretize it with five points, all you are doing is you are looking at the values, the membership values at these points and converting it into vector.

So, fuzzy set looks like this and these points if you want to discretize, this is the vector that you get. If your fuzzy set looks like this and at this point if you are discretizing, this is the vector you would get, vector of membership value. So, essentially instead of looking at fuzzy set as a graph; now we can look at it as just a vector with values varying from 0 to 1, right.

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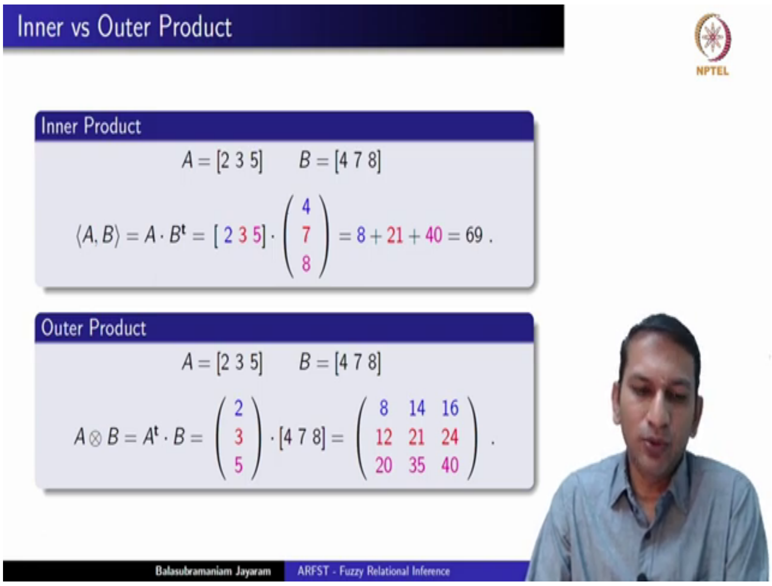

In linear algebra or basic matrix theory; we would have heard of inner products, perhaps some of you are aware of outer products too. So, what is an inner product? Typically we are given two vectors of similar dimension; now to talk about inner product of A and B means, essentially you are doing this A into the transpose of B, it is essentially like matrix multiplication row into column. So, all we are doing here is sum of products.

So, if you do the inner product of A, B; then what you would get is 2 into 4 plus 3 into 7 plus 5 into 8 and you would say it gives you the value 16. So, the inner product of two vectors essentially gives you a scale; but there is also something called an outer product. For the same two vectors, how does the outer product look like?

So, if you use this symbol to denote the outer product; what we do is, we take the transpose of it and multiply it to that of B. So, now, it is column into row. Now, what do we obtain? We in fact obtain a matrix here; essentially what we are doing is, we are multiplying 2 to all the elements of B. So, 2 into 4, 2 into 7 and 2 into 8 and we write it in the first row; similarly multiply 4, 7 and 8 with 3 and put it in the second row and multiply B with 5 and put it in the third row.

So, outer product gives you a matrix typically, whereas the inner product of two vectors only gives you a scalar. Now, why this? Because we are going to apply a similar kind of operation as that of an outer product to obtain a relation from a given rule.

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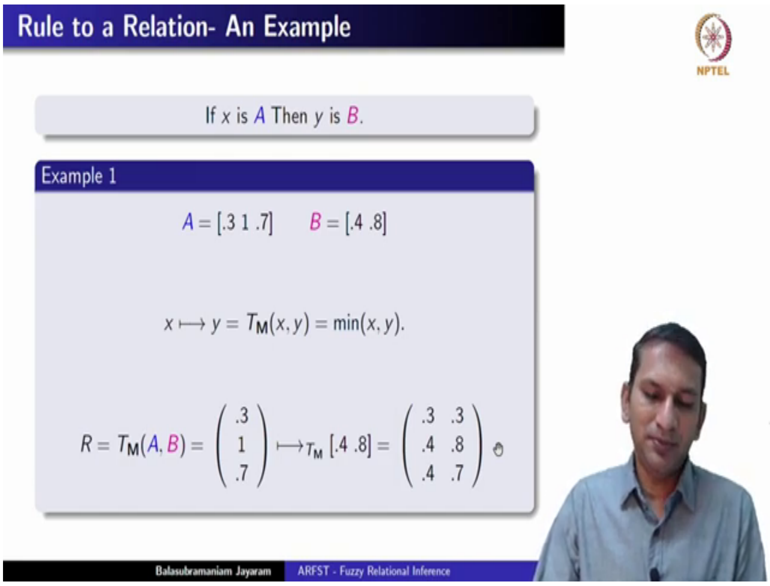

Let us look at an example. So, this is the rule that we have; let A, B given by this fuzzy set. So, once again we have discretized it and just three points and we have taken it, just for convenience say. And let us say that the set Y, the domain Y is discretized at only two points and we have this fuzzy set B, 0.4 and 0.8 are membership values that y 1 and y 2 take in this concept that is being represented by B.

Now, what is the fuzzy logic connective that we would use to capture this rule into a relation? So, let us use this symbol to relate this these two fuzzy proposition, the antecedent and the consequent. For the moment let us assume that we are using simply the t norm; of course that means, you would have to assume that what is being captured by this rule is conjunctive in nature, that means it is giving you positive examples.

So, let us not worry about the interpretation for the moment, let us look at only the calculations part of it. So, let us use the minimum t norm to construct a fuzzy relation from this given group. So, R is constructed from A and B using minimum; that means if you take this as I said it is an outer product. So, you take A transpose and B as such and instead of applying product, we are applying the minimum t norm.

Now, what you would get is this matrix; it is clear how we are getting this. So, we are taking this 0.3 and multiplying with this row which is the B vector; but instead of applying the product, we are applying the minimum t norm. So, minimum of 0.3 and 0.4 is 0.3, 0.3 and 0.8 is 0.3; similarly 1 and minimum of 0.4 is 0.4, 1 minimum of 1 and 0.8 is 0.8, 0.7, 0.4 will give you 0.4 and 0.7, 0.8 will give you 0.7. So, this is how we have constructed the relation from this rule using minimum t norm.

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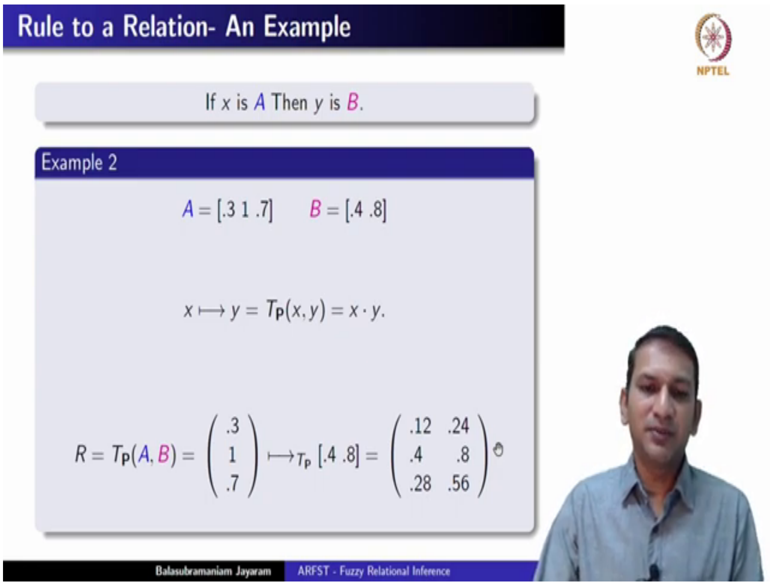

Now, for the same example, if you were to use the product t norm to construct the relation; this is what you would have and it is very clear because 3 into 4 is 12. So, you get 0.12 and so on and so forth. Now, this is when we are using a t norm to construct the relation.

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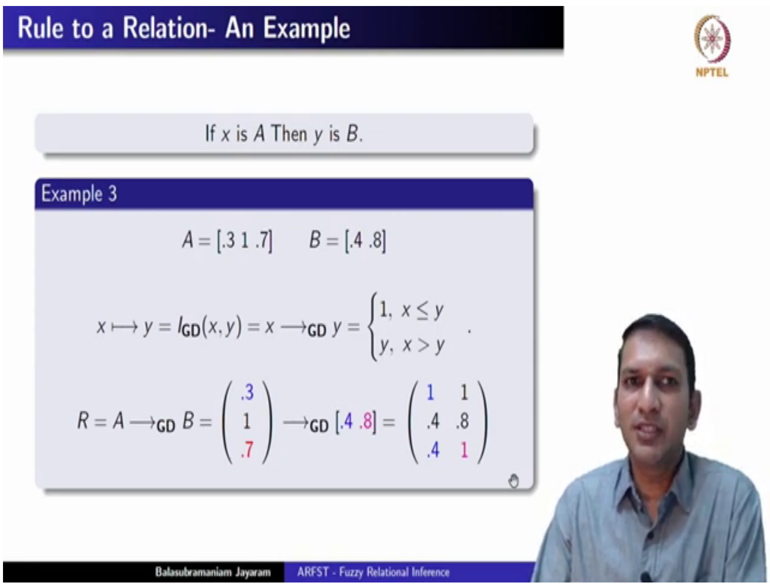

We could also use a fuzzy implication; let us construct the relation using the Godel implication. You may recall Godel implication says that when x is less than or equal to y it is 1, otherwise it is y. Of course, it is an R implication obtained from the minimum t norm. So, which means you know that minimum t norm is left continuous, in fact it is continuing. So, the implication will have ordering property and that is what is let me expressed here; that it is 1 when x is less than or equal to y and otherwise it is y.

So, now, let us construct the relation from this rule using an implication. So, once again we follow the similar methodology as the outer product. So, what we do is, we take this 0.3 here and 0.4 here; instead of just using the product, we are going to apply the Godel implication. So, 0.3 is less than or equal to 0.4, so you will get a 1; 0.3 is less than or equal to 0.8, you get a 1; 1 is left neutral element of Godel implications.

So, 1 implies 0.4 is 0.4, 1 implies 0.8 is 0.8. Now, when it comes to 0.7 and 0.4, you see that 0.7 is not less than or equal to 0.4; so then y appears here which is essentially 0.4. Now, 0.7 is less than or equal to 0.8, which means we put 1 here. So, this is how you construct the fuzzy relation from the given group, where A and B are antecedents and if they are given in terms of these two vectors, ok.

So, we have seen a couple of examples of how to translate a fuzzy rule into a fuzzy relation.

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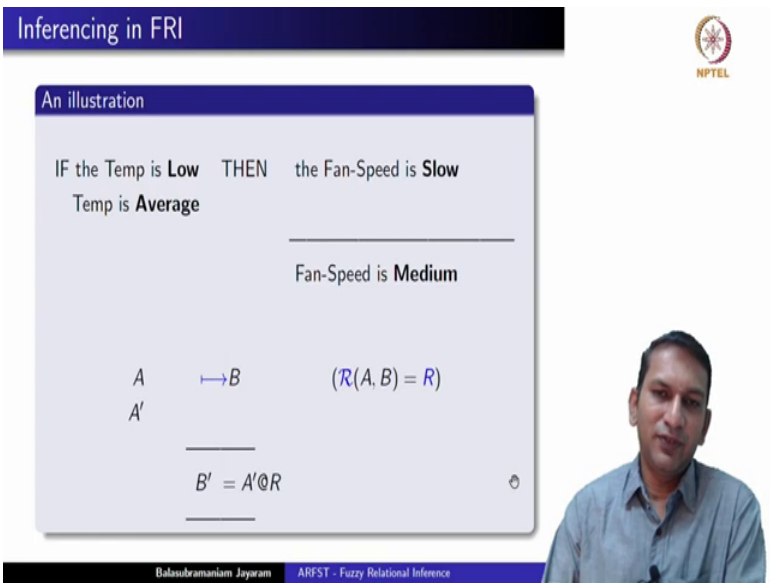

Now, what we have seen is, in the process of inferring using an FRI fuzzy relational inference; there are two steps, one to convert the given rule into a relation, that is what we have seen now. Now, let us look at the next part, which is applying a composition operation for a given input with this constructed fuzzy relation.

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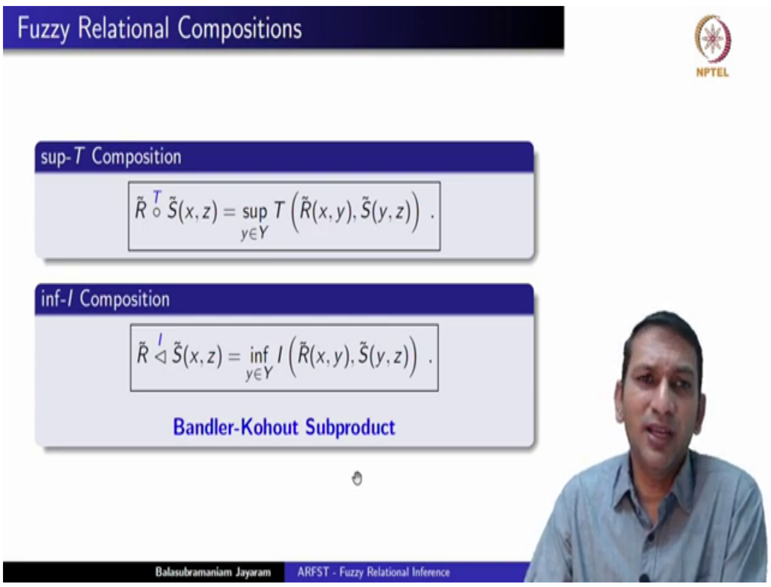

We know that there are two relational compositions that we have been exposed to; the first of them is the sup T composition, where you are taking supremum or y and you are combining those corresponding relational values using a t norm t. The other one is the inf I composition, where instead of a t norm, you substitute with an implication and instead of the supremum operation, you are actually using the infimum operation. Clearly if the sets y here that we have indicated; if they are finite, then you would essentially the supremum will become maximum and infimum will become minimum.

Note also that we had called this, this was this composition itself was proposed by Bandler and Kohout in one of their works and it is also called the Bandler Kohout subproduct, ok. Now, we want to apply, we are wanting to explore whether these two compositions could help us in an FRI; but note that here we actually have relations which are from x cross y to y cross z. So, these are the two ones that we are I mean composing to get a relation from a relation on x, y, z.

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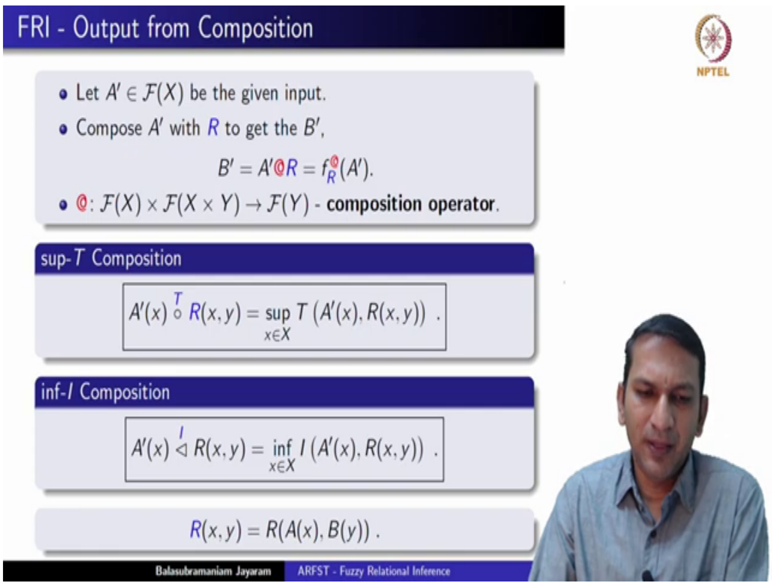

Now, we are given an input which is coming from F(X), so A dash is fuzzy set on X. We want to compose this A dash with R to get the B dash. And in fact if you look at this composition operator, what we call the composition operator at the rate of symbol used to denote it; it takes an element from F(X), another element from F(X) cross Y, because its fuzzy relation on X cross Y and gives us fuzzy set on Y.

Now, how do we do this? Simply we take A dash as the vector and R as the matrix; from matrix multiplication, we know that we could use the same operation to multiply a vector to a matrix. In fact, all we are doing in the matrix multiplication is row into column, which is essentially the inner product. So, what we have done first is, we have used the outer product kind of a structure to obtain the relation.; then now we are actually applying the inner product kind of a structure to obtain the output.

So, A dash which is fuzzy set on x is composed with R of x, y using the any one of these composition operators. So, if you use sup T composition, this is how it will look like; if you use inf I composition, this is how it will look like. Note that R of x, y is originally obtained from the given rule, which relates the antecedent coming from F(X) to a consequent coming from F(Y).

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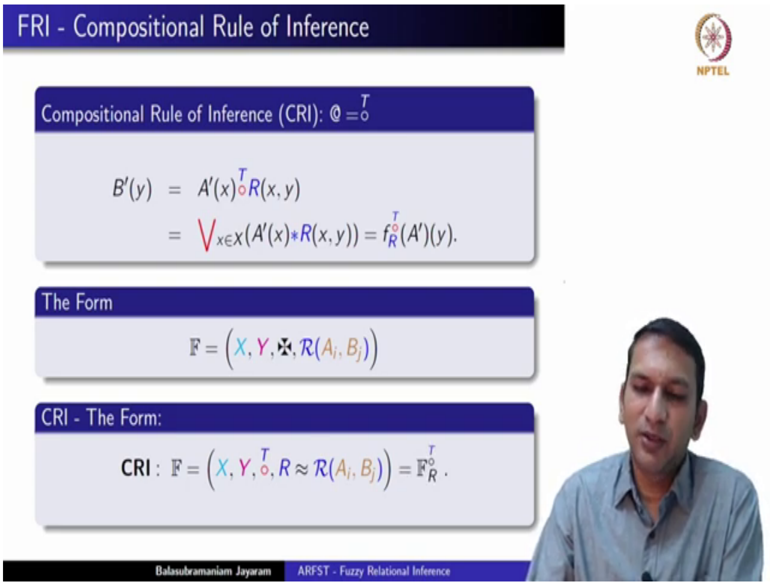

Now, if you are using the sup T composition in the in an FRI, then it is called the compositional rule of inference. So, if you want an output, B dash which is a fuzzy set on y. So, the membership value of any y in B dash is given like this B dash of y is essentially composing A dash with R and it looks like this. So, as you can see here, it is a function which maps A dash to B dash and what is important in an FRI is that, it is an R, you need a relation and also composition. So, this is a general form of an a FRI.

Now, if you restrict it to a CRI, we still have X and Y, the domains of input and output; then the composition is given in terms of sup T and R we do not specify, we have seen that R can be obtained from either a t norm or an implication. We said it depends on the interpretation of the rule, whether it is conjunctive or an implicative rule; but in a sense you could use also some other fuzzy logic operation.

We will see some of these going forward, but for the moment somehow we are able to obtain a fuzzy relation from the given rule. And we will call it a CRI if you are applying the sup T composition. Once again this is the shorthand notation that we might use often.

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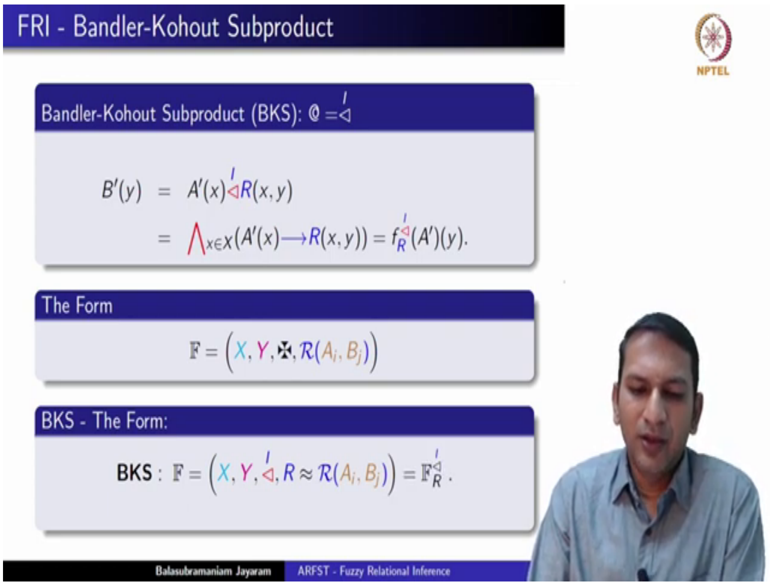

If instead if you use the inf I composition, which is the Bandler Kohout Subproduct composition; then it is called the Bandler Kohout Subproduct inference or typically shortened as BKS inference. So, this is how you obtain the B dash of y. Once again the general form of an inf I is like this in the case of BKS; we are once again specifying only the composition, but not how you obtain the fuzzy relation from the rule.

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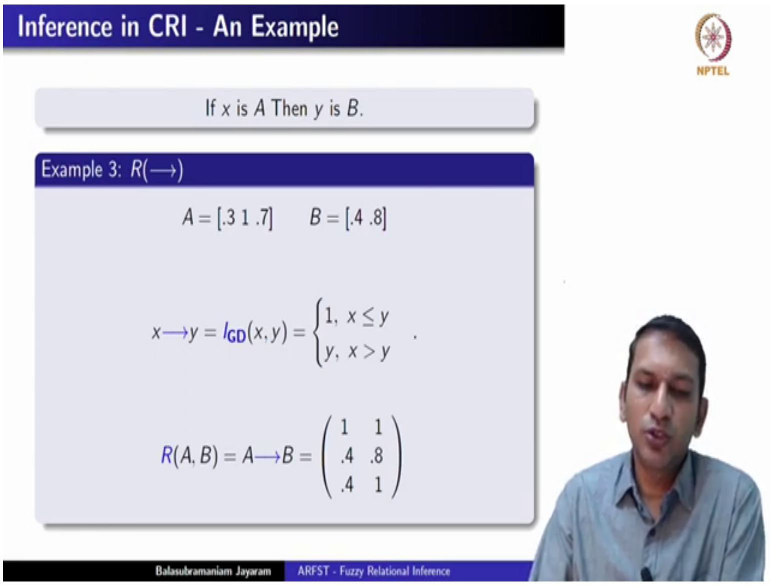

Now, let us look at revisit some of these examples, where we have constructed the relation apply either CRI or BKS and see what the output is. In example 3, in all the examples the A and B they have remained the same; A dash also has remained the same, what has changed is the operation that we have used to construct the relation. Since CRI means sup T composition; let us take the example 3, wherein we have constructed the relation using an implication just for variety.

So, now we use the Godel implication and if you recall; we got this as the fuzzy relational matrix representing fuzzy rule A implies B or if x is A, then y is B, where A and B are given by these vectors. Now, what we need to do is, given an A dash; we need to compose it with this R using sup T composition, that is what makes it CRI.

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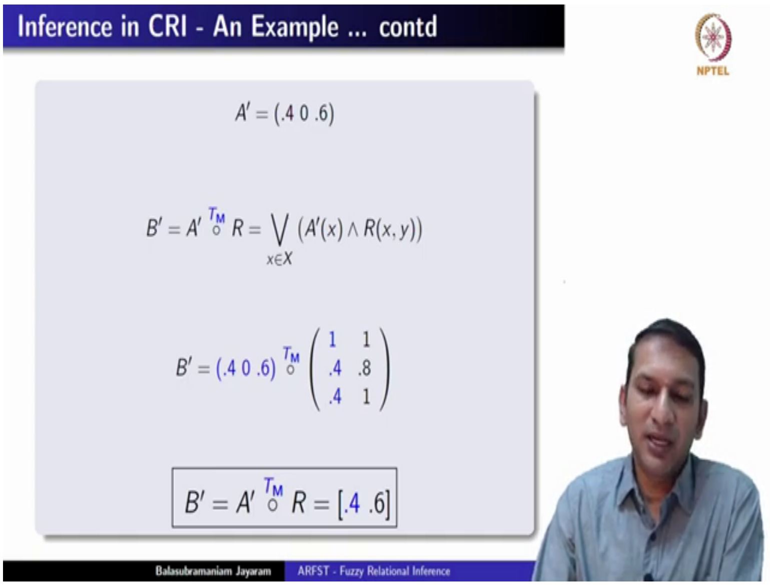

So, let us take A dash to be this vector 0.4 0 0.6. And what we want is a B dash. And now in sup T, we need to fix the T; that is for the moment for ease of illustration, consider this T to be the minimum t norm.

So, that means essentially you are looking at sup min composition. Now, B dashes you are we take this A dash and compose it with this R. And you will see that this is the output that we get, ok. How did we do this? Look at this, we are taking this row 0.4 0s 0.6 and using the column 1, 0.4, 0.4; in matrix multiplication, inner product we would actually use sum of products, but here we are using max of min.

So, if you look at it here minimum of 0.4 and 1 is 0.4, 0 and 0.4 is 0, 0.6 and 0.4 is 0.4. So, we need to take maximum among these obtained ones; we obtain 0.4, 0 or 0.4, so the maximum is 0.4. Once again you see here minimum of 0.4 and 1 is 0.4, 0 and 0.8 is 0, 0.6 and 1 is 0.6. So, maximum of 0.4, 0 and 0.6 is 0.6 and that is how we have obtained this output.

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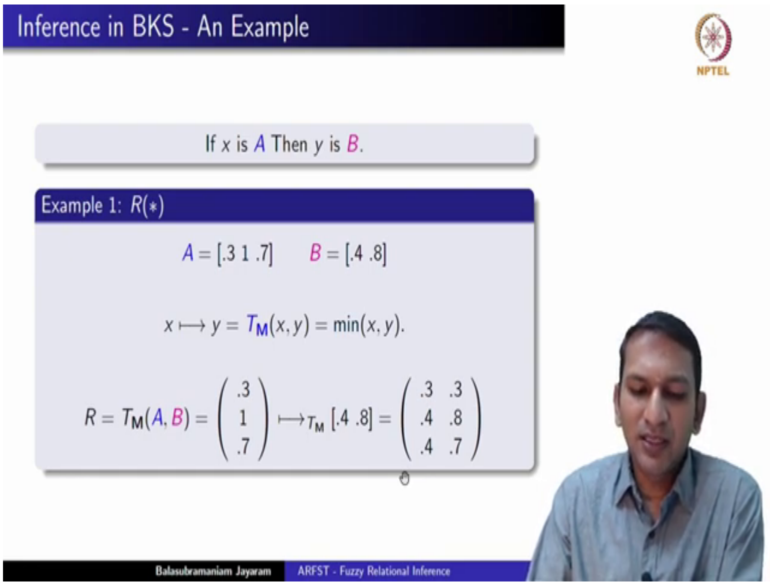

Let us also look at an example inference using BKS. For this let us consider the example 1; that is once again the A's and B's they have remained the same, only that we have considered a t norm to construct the relation and we have seen that this is the R that we obtained a little while earlier.

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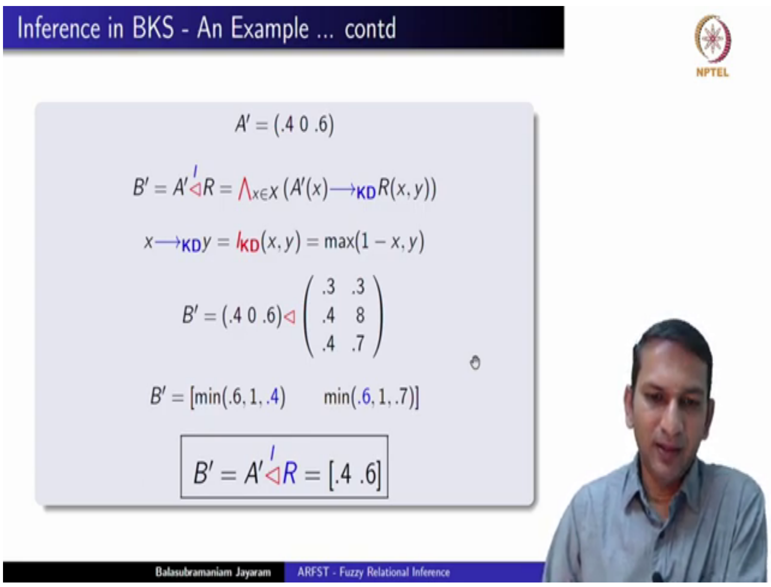

Now, for the same A dash, let us apply the BKS inference. In BKS we need to choose the implication; let us choose the Kleene Dienes implication in this case. How is Kleene Dienes implication given? You may recall it is an SN implication obtained from maximum T co norm and the usual classical negation 1 minus x. So, this is how it will look like.

Now, B dashes composing A dash with this matrix R that we have obtained. Note that B dashes we are taking this row into this column; but instead of taking the product sum of products, we are going to take min of implication, this is the implication that we are going to use. Now, let us look at 0.4 and 0.3. What is I K D of 0.4 , 0.3? We need to substitute 0.4 for x and 0.3 for y.

So, maximum of 1 minus 0.4 , 0.3, which is maximum of 0.6 , 0.3, so it is 0.6. Now, 0, I K D of 0 , 0.4, we know that for an implication if x is 0, it is 1. So, 0, 0.4 give us 1 and now we need to consider I K D of 0.6, 0.4. Once again substituting in this formula, we see that it has to be maximum of 0.4 or 0.4, so it is 0.4.

If you take this A dash and then compose it with the second column here; for 0.4 and 0.3, you would get 0.6, when it is 0, it will get 1. When it is 0.6 and 0.7, note that it does not have the ordering property, so it is not going to be 1. So, when you substitute 0.6 here; what you get is maximum of 1 minus 0.6 , 0.7, which is essentially looking at maximum of 0.4 and 0.7 and you get 0.7.

But the thing is we need to pick the minimum among these; that means here it is 0.4 and here it is 0.6. So, B dash turns out to be the fuzzy set 0.4 , 0.6.

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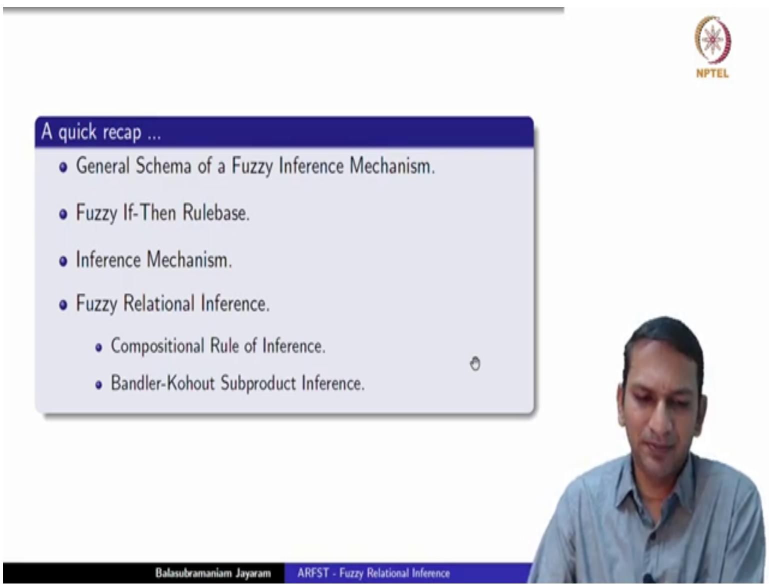

Well, this is how we have done the inference with two of the most important ways of obtaining inference using outputs using fuzzy relational inference that of CRI and BKS.A quick recap of what we have seen in this lecture, we began with the general schema of a fuzzy inference mechanism; we looked at the fuzzy if then rule base itself, which becomes one of the two main components of a fuzzy inference system. Then we looked at the inference mechanism itself, not very deep; we looked at one particular inference mechanism that of fuzzy relational inference.

Of course, we gave an overall idea of a inference, fuzzy inference essentially; how an inference engine will look like, I said we have just only started to look at the surface, we have not gone deep into it. There are many more things that will come into picture; what comes will be presented you in a few moments, which we will take up in the next few lectures. So, we looked at fuzzy relational inference.

In this again we looked at two kinds of fuzzy relational inference, which essentially differ based on the composition operation being used. If you use the sup T composition, we call it the compositional rule of inference, which we will abbreviate as CRI and if you use the inf I composition, you land in what is called the Bandler Kohout Subproduct inference or in short BKS inference.

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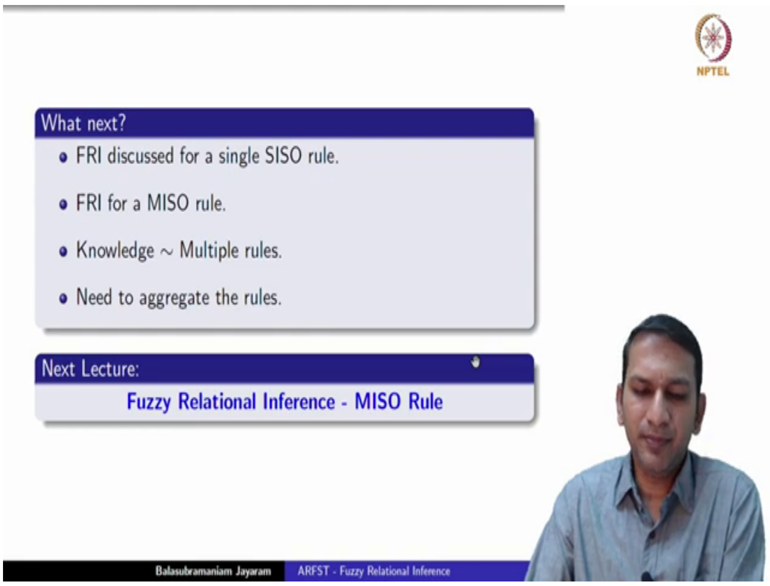

Now, what next? As you will see in this lecture, we have only discussed about obtaining outputs using fuzzy relational inference; but with a single rule and that to a single input, single output rule. So, the next step is to discuss how do you obtain an output if you are given a single multiple input, single output. Of course, we know that knowledge cannot be encoded in just a single rule, we need multiple rules to present to us the different scenarios.

And in this case again we need to look at how to infer given an input making use of all the knowledge that is available to us in the form of a set of rules of this rule base. It could be a SISO rule base or a MISO rule base; which means essentially we also need to discuss aggregating these rules. These are some things that we will see going forward. In the next lecture of course, we will limit ourselves to looking at fuzzy relational inference using a single MISO rule.

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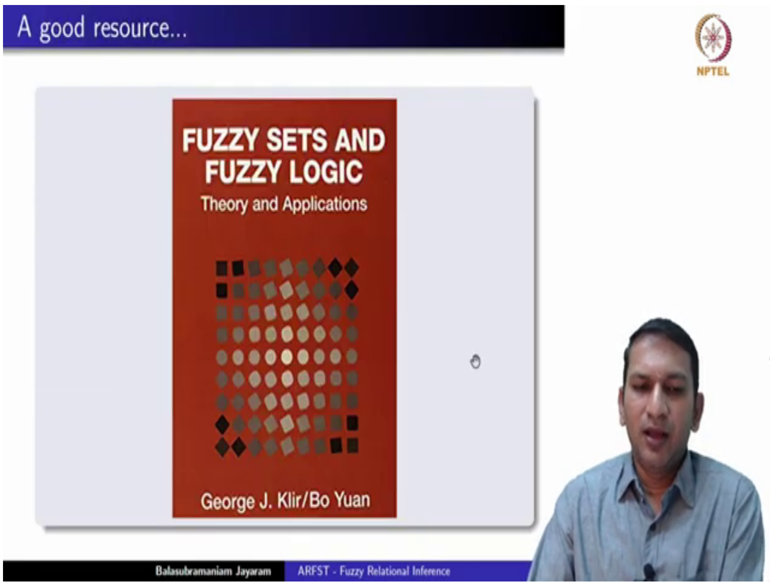

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A good resource for the topics covered in this lecture, once again is the book of George Klir and Bo Yuan; you could also look into this book of Drainkov, Hellendoorn and Reinfrank, which is a very interesting, very introductory text and it deals with fuzzy relational inference in particular.

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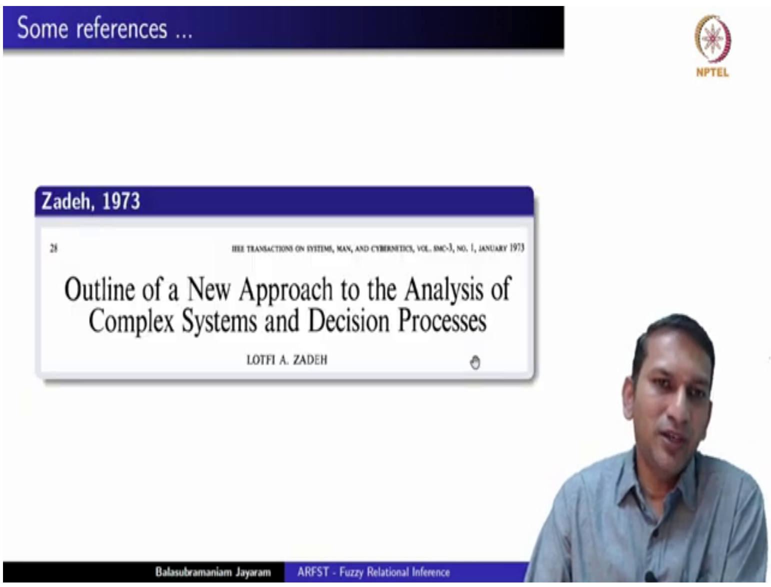

Some very important references, some seminal papers, this compositional rule of inference was introduced by Zadeh himself way back in 1973 in one of his earlier papers; title outline of a new approach to the analysis of complex systems and decision purpose processes. So, you will see that this was offered as a solution, while trying to analyze complex systems.

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Even though we call it the BKS inference, it was in fact proposed by Pedrycz almost a decade after CRI has been in existence. In the next lecture, we will deal with fuzzy relational inference as applied to a single MISO rule, multi input, single output rule. Glad you could join us for this lecture and looking forward to meeting you again in the next lecture.

Thank you again.