Approximate Reasoning using Fuzzy Set Theory
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Lecture - 29
Fuzzy If-Then Rules

Hello and welcome to the second of the lectures in week 6 of this course titled Approximate Reasoning using Fuzzy Set Theory. The course offered over the NPTEL platform.

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In the last lecture, we had looked at different interpretations of fuzzy sets and also a few different perspectives or interpretations on fuzzy propositions themselves. In this lecture, as the next building block of a fuzzy inference system, we will look at Fuzzy If-Then Rules. To begin with we will look at classical if-then rules, from there we will move to fuzzy if-then rules. We will look at some existing classifications of these rules.

Note that, the different interpretations and perspectives that we have had on fuzzy propositions will make an appearance for sure when we are dealing with fuzzy if-then rules.

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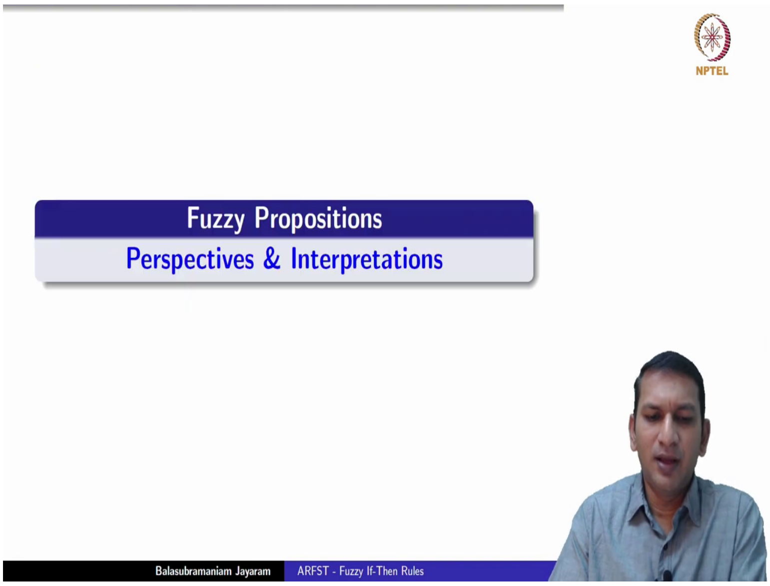

A quick recap of fuzzy propositions, perspectives and interpretation that we have seen in the last lecture.

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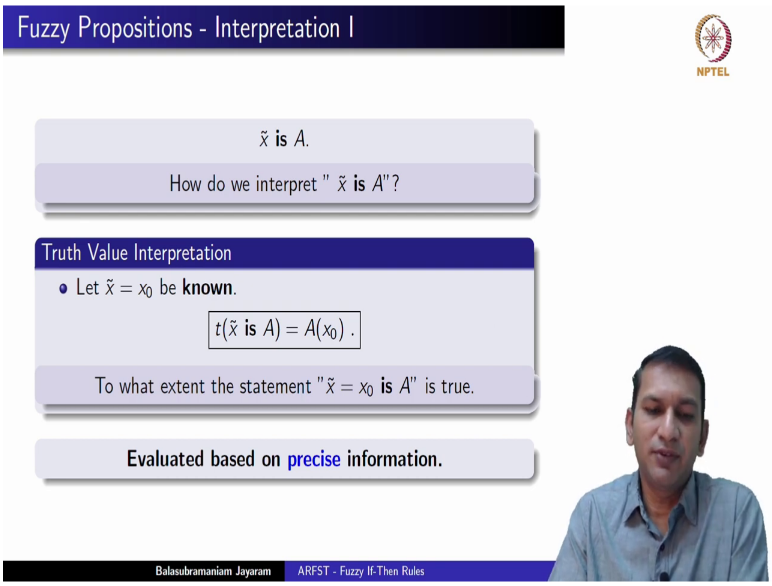

Now, we know a fuzzy proposition is of the form x tilde is A. The question was, how to interpret this? One interpretation is as a truth value determination; that means, if you are given a particular precise value for x tilde, then the membership value of this x naught in A becomes the truth value of the statement itself. So, we are expression we are expressing to what extent the statement x tilde is equal to x naught is A is true. It is evaluated based on precise information.

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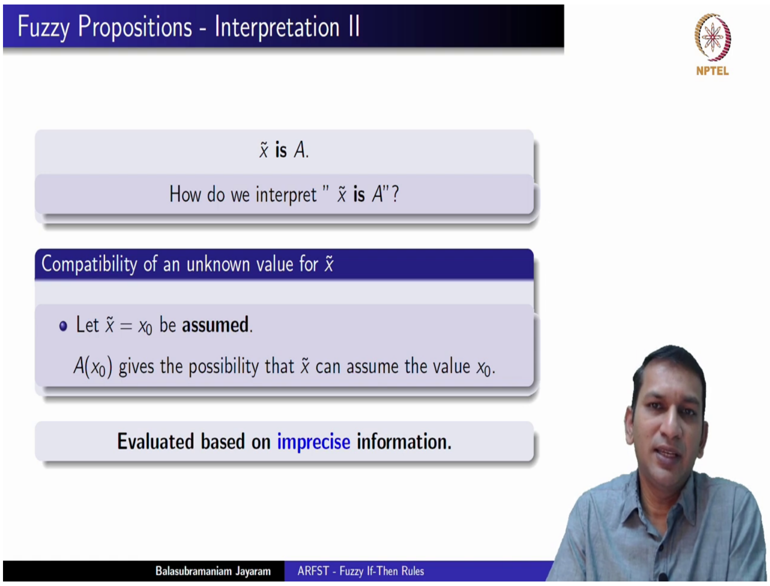

A second interpretation is to check how compatible an unknown value of x tilde is to the statement. Once again here x tilde is equal to x naught is not known, it is only assumed. And now the membership value gives the possibility that x tilde can assume the value x naught. Here we are evaluating based on imprecise information. And we are evaluating the possibility of x tilde as A, if x tilde is equal to x naught.

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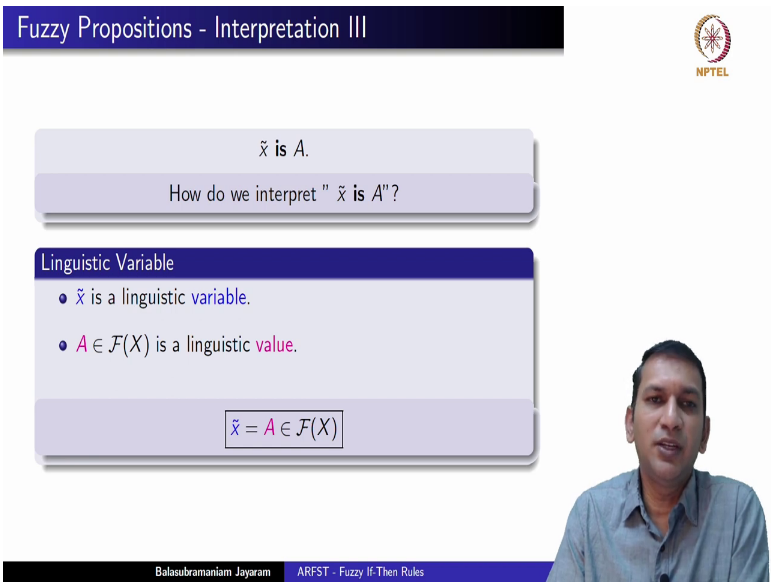

A third interpretation is as x tilde being a linguistic variable. The A which is a fuzzy set on x is called the linguistic value that x tilde can assume. So, in essence it is like a variable which assumes a value and as was already mentioned in the last lecture we need to be consistent in the interpretation of the linguistic variable which is essentially shown in the appropriate domain that you choose for A.

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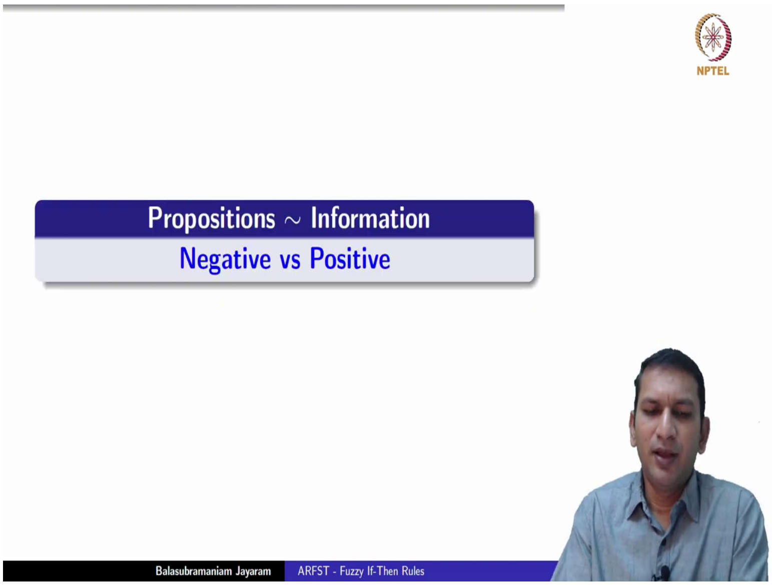

Well, propositions also as you know they are assertions, so they also give information. So, now, this information can be either negative or positive.

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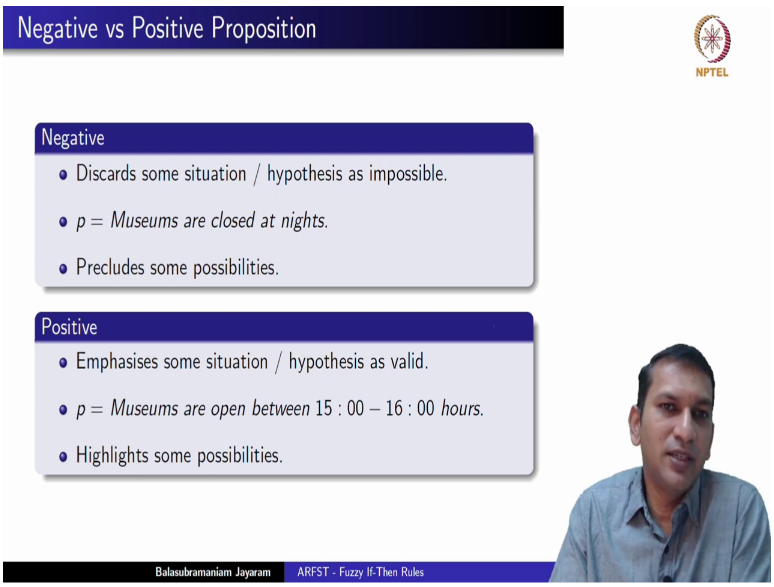

What do you mean by this? A negative piece of information is something that discards some in situation or hypothesis as impossible. For instance, consider this proposition. Museums are closed at nights. So, it clearly precludes some possibilities, that museums are not open. But note that this proposition p, museums are closed to nights does not say anything about museums during the day. It does not mean museums are open throughout the day, they can also be closed.

Perhaps, especially during lunch hours or on some special days. So, negative information, it precludes some possibilities, but when the condition is not valid, it does not mean that it is automatically true. In the case of positive piece of information, that information emphasises some situation or hypothesis that is valid. For instance, you could say museums are open between 15 to 16 hours between 3 and 4 in the afternoon.

Now, this only highlights some possibilities. If something does not match this, then this proposition does not say anything more about it. It is essentially unknown. So, if you ask the question outside of this hour, is it open or closed, it does not answer this question. So, it only picks up positive examples for a particular situation that is considered. So, it emphasises a situation as valid.

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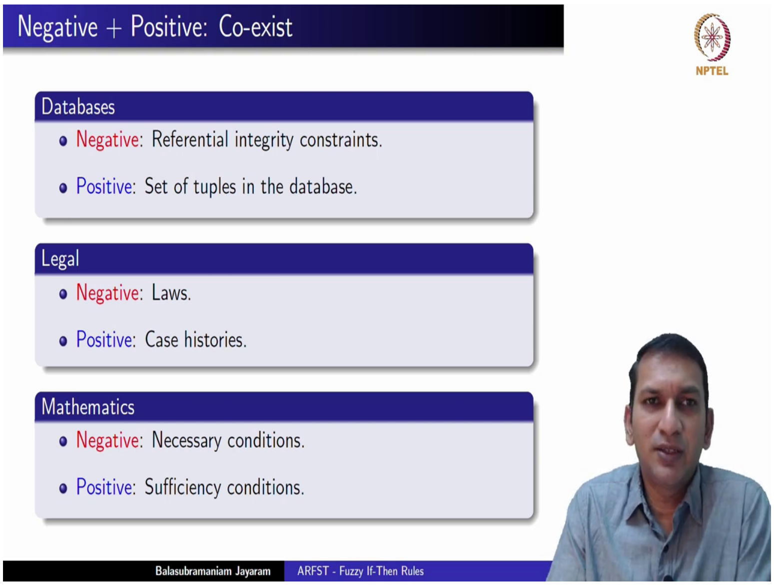

Now, often both pieces or both types of information coexist. For instance, if you look at databases. Then the negative information is encoded in the form of referential integrity constraints. And the positive information is essentially the set of tuples essence, the set of tuples in the database they tell you that yeah these pairs can occur together or these tuples occur together. So, that is a positive kind of information.

In the legal framework, the negative piece of information can be thought of in terms of some laws which prohibit you from doing something or essentially lay out what would be the punishment if such a thing were committed. The positive ones are the case histories where given such a situation what kind of a judgment was pronounced.

In maths itself, negative pieces of information or perhaps necessary conditions which tell you that without this cannot happen and positive pieces of information are sufficiency conditions. There can also be other types. For instance, we could have, we could say that a function is monotonic and continuous. We could also have a function which is monotonic and not continuous.

So, these are pairs of function or function which has these pairs of properties, monotonicity and continuity, monotonicity and discontinuity. So, these are positive examples of pairs of such function that you could give and they become positive information.

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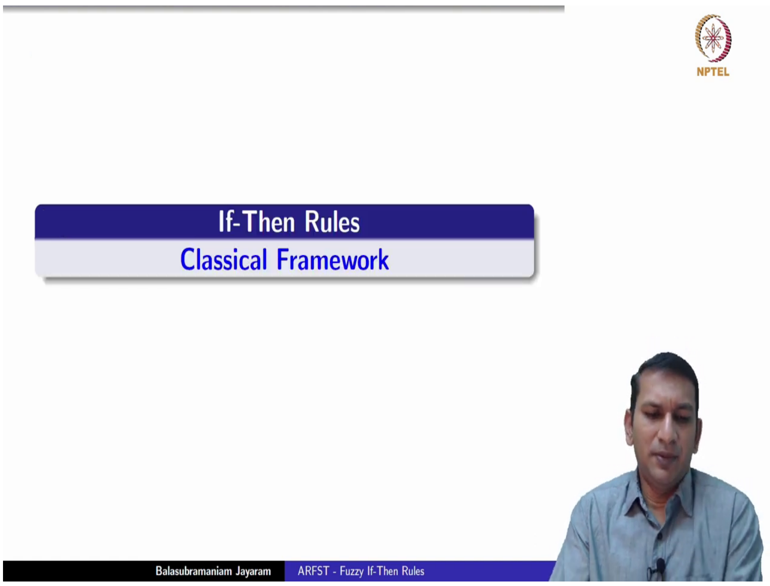

Well, let us move to If-Then Rules themselves. Let us begin with looking at if-then rules in the classical framework.

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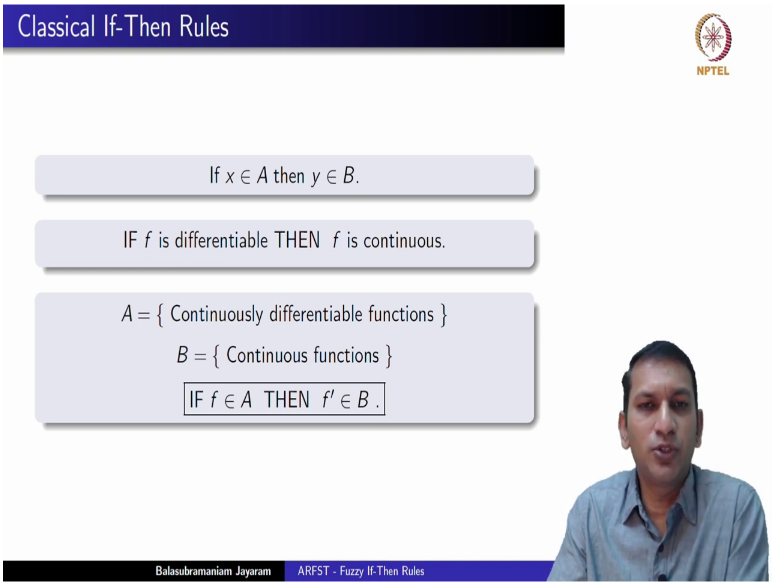

This is typically how an if-then rule in the classical framework will look like. Note that the antecedent A and the consequent B both are in fact classical sets. So, we say if x belongs to A, then y belongs to B. Now, familiar example, if f is differentiable THEN f is continuous. So, f belongs to set up for differentiable functions, THEN f belongs to the set up for continuous functions. You might say that here the x is essentially the same; that means, A is subset of B.

Yes, that is what this implication on this condition and this piece of knowledge tells us, but if you want to construct examples you could do this like a (Refer Time: 07:50) as follows A, and for A consider the set of all continuously differentiable functions and B to be just continuous functions. Now, we can assert this if f belongs to a then f dash belongs to B. That means, if f is continuously differentiable, clearly its derivative is continuous.

Well, now let us try to look at what is this piece of knowledge tell us, if x belongs to A, then y belongs to B. Let us try to look at it.

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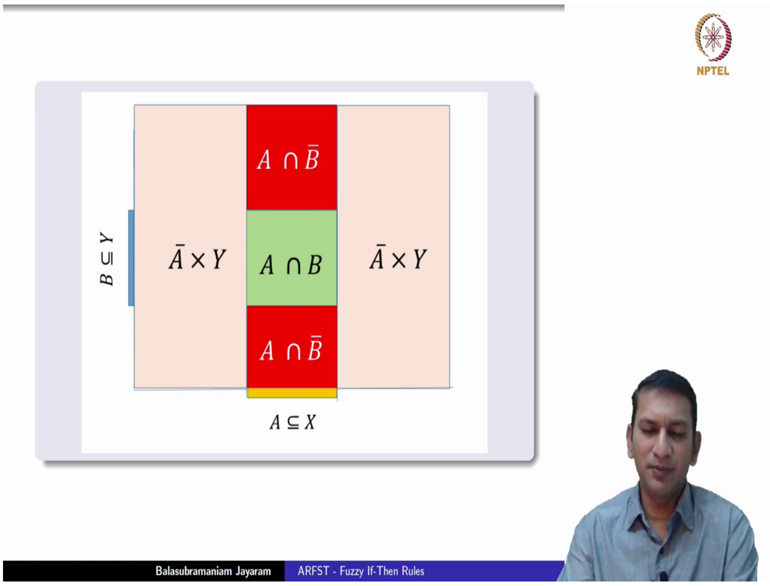

So, A is a subset of X classical subset of X, B is a classical subset of Y. Now, if you look at this domain X cross Y, we could perhaps partition it as follows. This is the part where this A intersection B; that means, any pair u , v coming from here to be u will belong to A and v will belong to Y.

These are the pairs, this region, the red region indicates the region where the pairs u , v are such that u belongs to A, but v does not belong to B, that is v comes from the B complement. And these are essentially A bar cross Y; that means, these are ordered pairs where u does not belong to A. So, u belongs to A complement. Now, if you look at the pairs belonging to the green region A intersection B, essentially, they are the positive examples.

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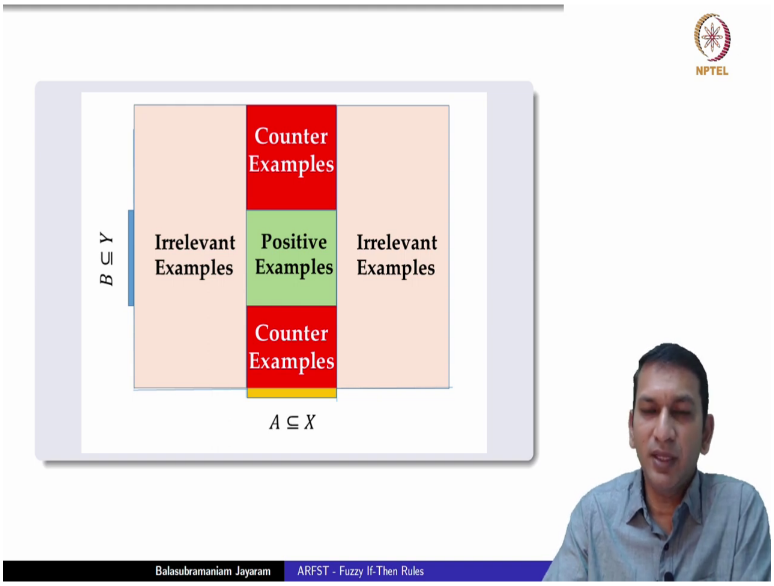

They tell you if u X is u, then Y can be B. The ones in the red region ordered pairs, they are the counter examples. So, they tell you there is an X which is equal to u or u coming from A, but look at it v is not in B. So, these are the counter examples. What about A bar cross Y? Essentially these are irrelevant examples because the moment u does not belong to A we do not know. They do not either enforce or contradict the given conditioner, the given piece of knowledge.

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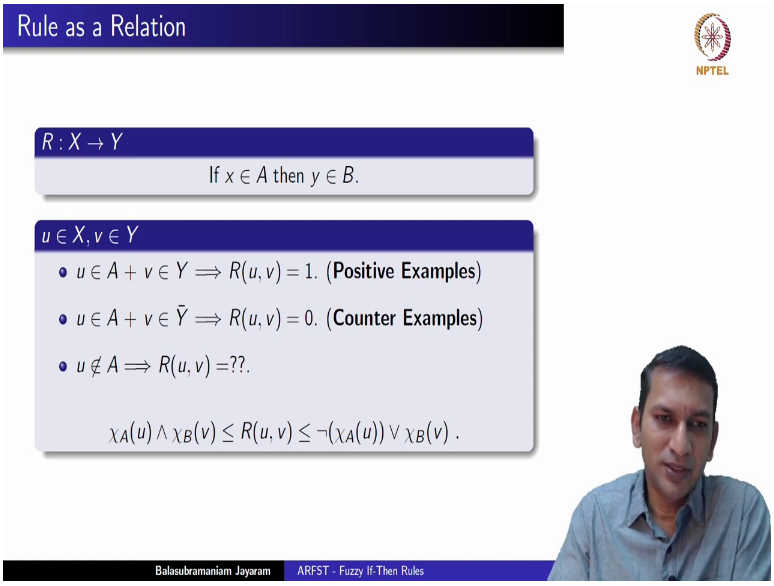

So, when you see it from this point of view, you can think of representing this rule by relation, x element of A then y element of B, then you wonder given a pair of elements u from X and v from Y, then you wonder about how this relation should be construct. It is clear that if u belongs to A and v belongs to Y, for this ordered pair R of u, v is equal to 1 because that is what this relation tells you, this condition tells you.

Similarly, so these are the positive examples. Similarly, if u belongs to A and v belongs to Y complement. So, if you take this pair, if this such a pair happens, then R at that point is actually 0. These are the counter examples. Now, what happens when u does not belong to A? Then essentially R of u, v is undetermined, these are these irrelevant examples. But from this scenario what we can summarize is this, that R of u, v, it is perhaps lower bounded by the conjunction of these two characteristic functions. And its upper bounded by this value.

Essentially, what we have seen, that if we know that u does not belong to A, then you could look at it like this, it is essentially coming to the material implication. So, R of u, v is bounded below by this conjunctive operation and above by the what we now know R implicative implication operations. Well, how does this help us?

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Let us move to the Fuzzy Set Theoretic Framework.

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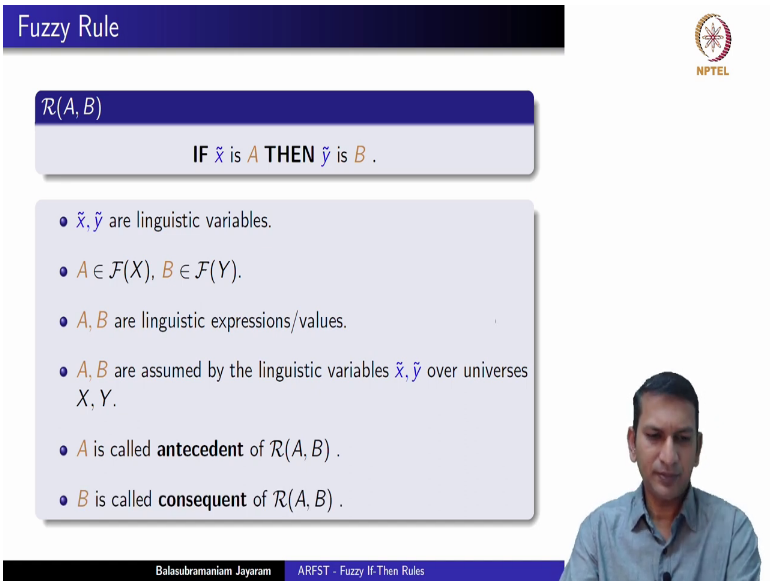

So, a rule here would look like this. IF x tilde is A THEN y tilde is B, clearly x tilde is A is a fuzzy proposition, y tilde is B is a fuzzy proposition, typically we in we denote such rules by R of A , B, both relating A and B. It is clear, it is essential to keep only A and B in the notation because x tilde and y tilde they are actually quite clear from the context.

Now, how do we interpret them? Clearly, as they are x tilde is A and y tilde is B are fuzzy propositions. Third interpretation can be used to look at it as x tilde and y tilde are linguistic variables, A belongs to F(X), its fuzzy set on X, B is fuzzy set on Y. And A and B are essentially the linguistic expressions or values that x tilde and y tilde can assume. Note that x tilde and y tilde themselves are linguistic variables. We restrict them to assume only fuzzy sets which are defined on X and Y, respectively.

Now, A is called the antecedent of R of A, B. So, as you can see x tilde does not play so much of a role except in the notation there. So, if you are denoting the rule by R of A, B, then we relay we refer to A as the antecedent and B as the consequent.

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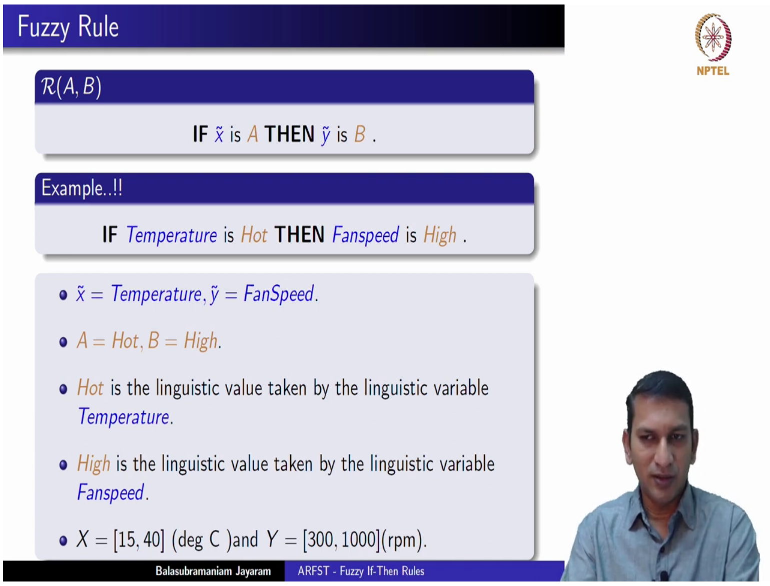

Now, let us look at an example. If Temperature is Hot THEN Fanspeed is High. Clearly, x tilde here is temperature, y tilde here is FanSpeed. And A and B are essentially Hot and High. These are the linguistic values taken by the linguistic variables Temperature and Fanspeed. Now, the underlying domain X itself can be thought of as an interval. Here we have simply assumed it to be 15 to 40 degree centigrade.

And y relates to the fan speeds that can be considered. So, in terms of rpm perhaps we could consider the interval 300 to 1000. So, clearly, you see here, the third interpretation of fuzzy propositions as linguistic variables assuming some linguistic values kicks in.

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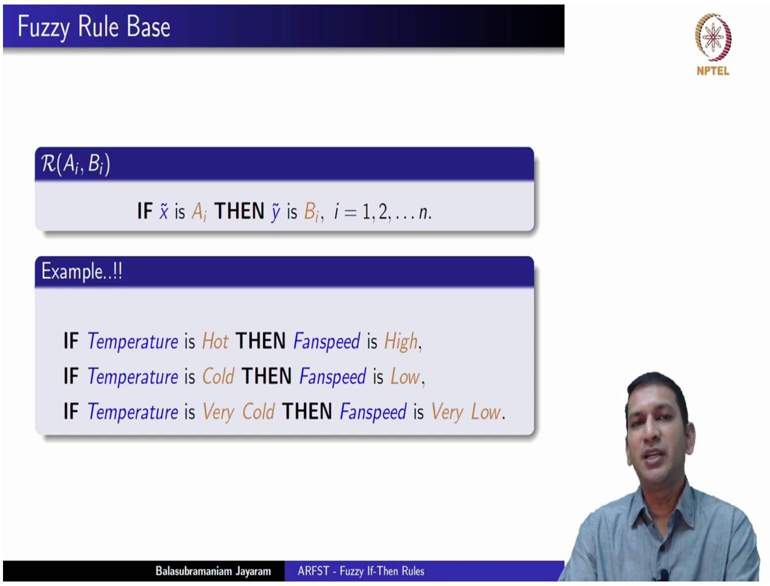

Now, a single rule will not contain all the knowledge about particular domain. We need many of them. So, we call that a fuzzy rule base. One example here. So, now, you see here why we do not perhaps this gives a justification as to why we are not including x tilde and y tilde when we are denoting a rule. Because once the domain is fixed, the context is fixed, we know that x tilde and y tilde are also fixed.

For instance here, x tilde as Temperature you can see that and y tilde as Fanspeed. And now the A i's and B i's they keep changing. Perhaps A 1 is hot, A 2 is Cold and A 3 is Very Cold. B 1 is high, B 2 is Low and B 3 is Very Low. These are the linguistic values assumed by x tilde and these are constraint because in the context these have to be somehow related to Temperature, and Fanspeed, and essentially then they are constrained by the appropriate domain that we have chosen.

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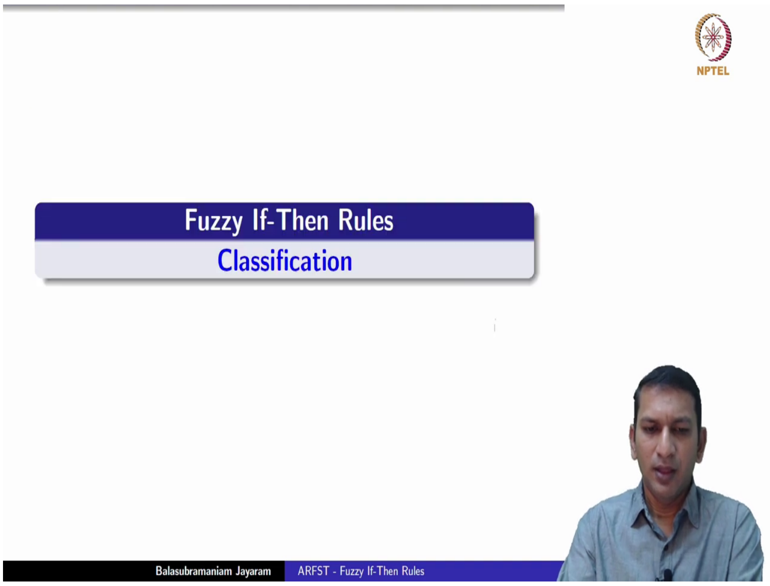

Fuzzy If-Then Rules themselves can be classified in different ways.

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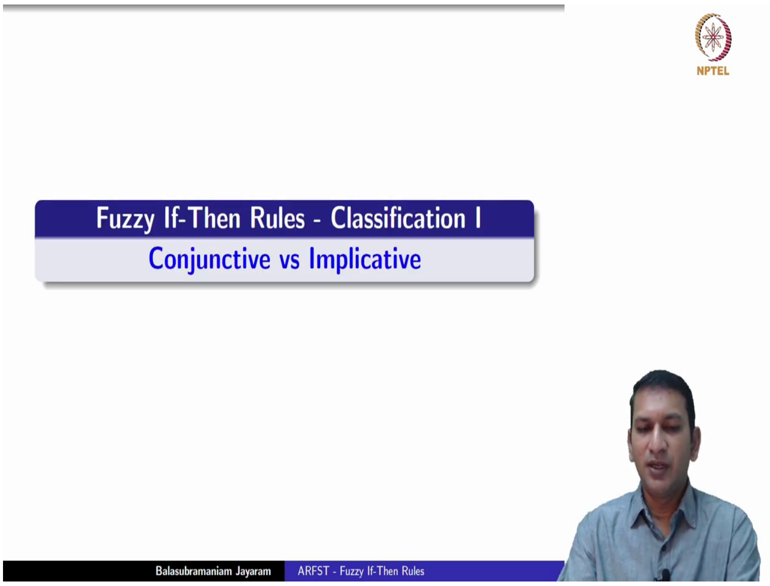

The first way is in terms of conjunctive or implicative. So, this was also clear a few slides earlier where we looked at the classical if-then rule, if you capture them as a relation R of u, v we know that it is bounded below by conjunctive interpretation and above by implicated interpretation.

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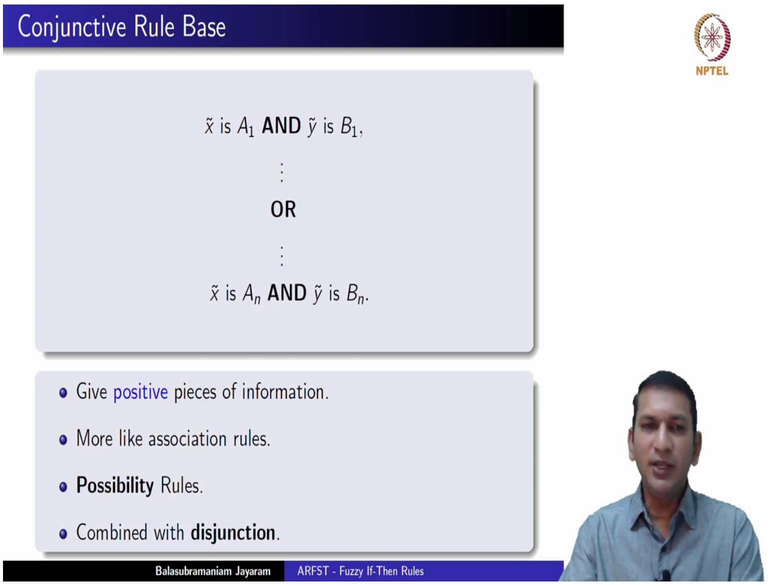

So, now, if you are given if x element of A AND then y element of B, it could also be represented like this x tilde as A 1 AND y tilde as B 1, so on and so forth. It is a rule base, it is conjunctive. In that sense; that means, this rule these rules give positive pieces of information. They are more like association rules.

So, you give examples of function which are monotonic and continuous, also functions which are monotonic and non-continuous. So, they are association rules. In that sense, they are possibility rules. They tell you these possibilities exist because there are examples, so from examples you are generalizing such possibilities do exist.

And since, these are possibilities, when you are given many such pieces of information, you actually combine them with a disjunctions, any one of them could be true. You do not expect all of them to be true because these are just possibilities.

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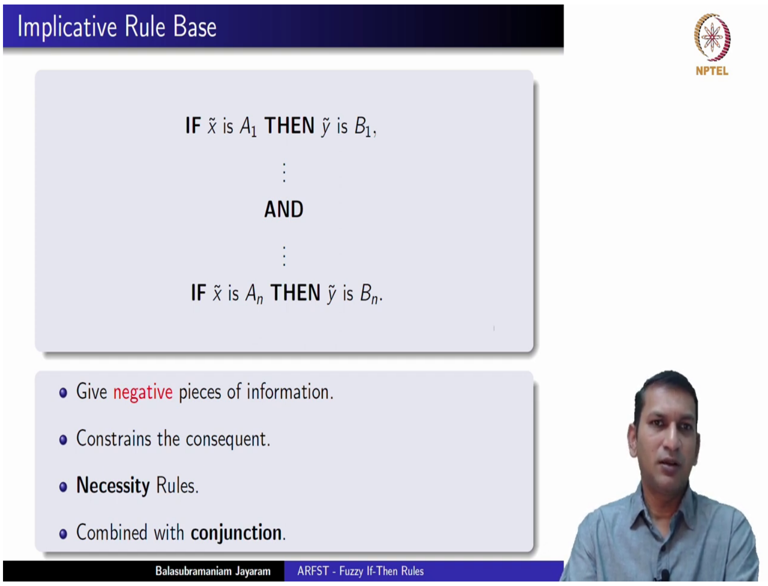

In the case of implicative rule base, each rule is thought of as essentially constrained; that means, it is giving us negative information if x is A 1, THEN it y necessarily has to be in B 1, much like if it is differentiable is continuous, if it is not continuous it cannot be differentiable. So, essentially it is constrains in the consequent based on the antecedent. In that sense, you could call this as necessity rules. It is necessary. If this happen, then this is necessary.

Finally, since these are each piece of knowledge is constrained, it is like a constrain, then we expect that if the all these pieces of knowledge are given, then we expect that all of them are satisfied and hence we combine them using a conjunction.

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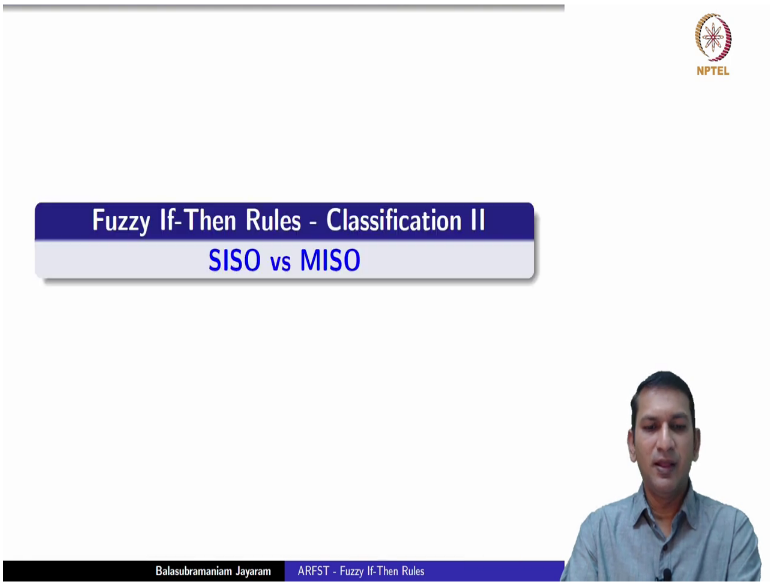

A second way to classify fuzzy if-then rules is in terms of its dimensionality of the input.

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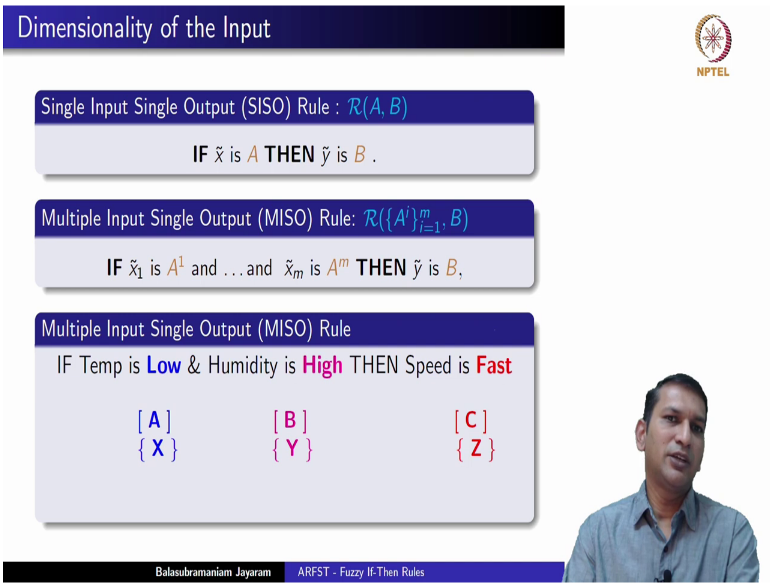

What do we mean by this? Look at this rule, we say IF x tilde as A, THEN y tilde as B. So, A is a fuzzy set defined on domain X, which is just only one. So, x tilde is the only input and y tilde is the only output or the only antecedent and the consequent. You could also have multiple input, single output rules; that means, if x 1 tilde is A 1 and x 2 tilde is A 2, so on and so forth and x till x m tilde as A m then y tilde as B. So, essentially you see here there are m different inputs. So, the dimensionality of the input is m.

Let us look at an example. We could have a rule of the form, if temperature is Low and humidity is High, then speed is Fast. Essentially, we are looking at temperature and fan speed. So, you see here this low is a fuzzy set A, high is B, and fast is C. Now, this A itself is defined on some domain X, B on some domain Y and C would be on completely different domain C Z.

So, based on the dimensionality of the input or the number of antecedents in the rule, we can classify as fuzzy if-then rules. Of course, there are also multiple input, multiple output rules, and the antecedents are many and the consequence are many. But as can be easily understood, these can always be broken down into systems, where each of the systems actually consists of only MISO rules, multiple input single output rules.

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Finally, a third classification can also be applied on the set of fuzzy if-then rules dependent on the nature of the consequent. And you will see that such rules essentially distinguish between two kinds of fuzzy inference systems that we will discuss in the near future.

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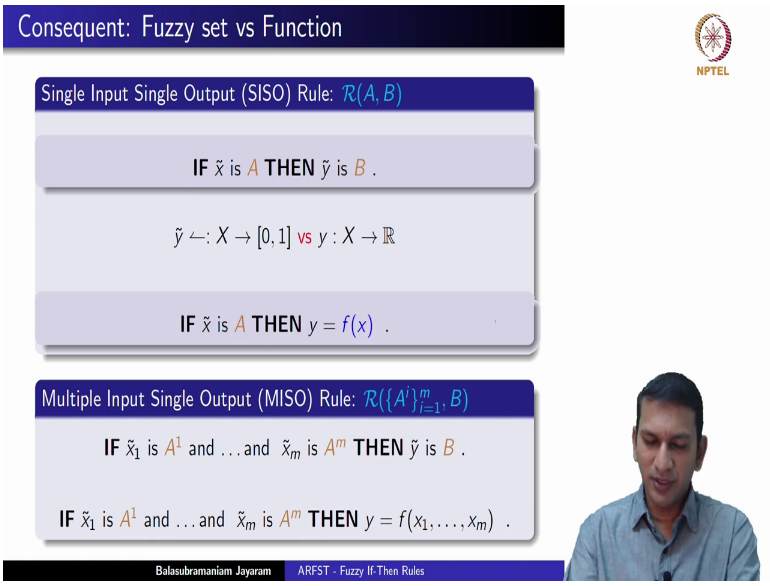

So, it depends on whether the consequent is a fuzzy set or a function. Look at this so, it is a single input single output rule. IF x tilde as A THEN y tilde as B. So, essentially y tilde is a linguistic variable which can assume a linguistic value, which can assume a fuzzy set defined on y in this case for X, whatever the case may be. Contrast this with if you just take y to be a function from X to R.

So, if your rule looks like this, IF x tilde as A THEN y is equal to f(x), y is essentially a function of x. Then, typically these are called TSK type fuzzy rules, you will see them perhaps in the next week of lectures when we deal with similarity based reasoning. So, this is another way to distinguish types of rules. Another, yet another way to classify. And this could happen also in the multiple input single output case.

This is the normal thing that we have seen where y tilde is B; that means, y tilde is a linguistic variable assuming a fuzzy set. But you could also have TSK type fuzzy rules here in case of MISO, multiple input case, where y now is a function of x 1 to x m; that means, values coming from the Cartesian product of his input domains.

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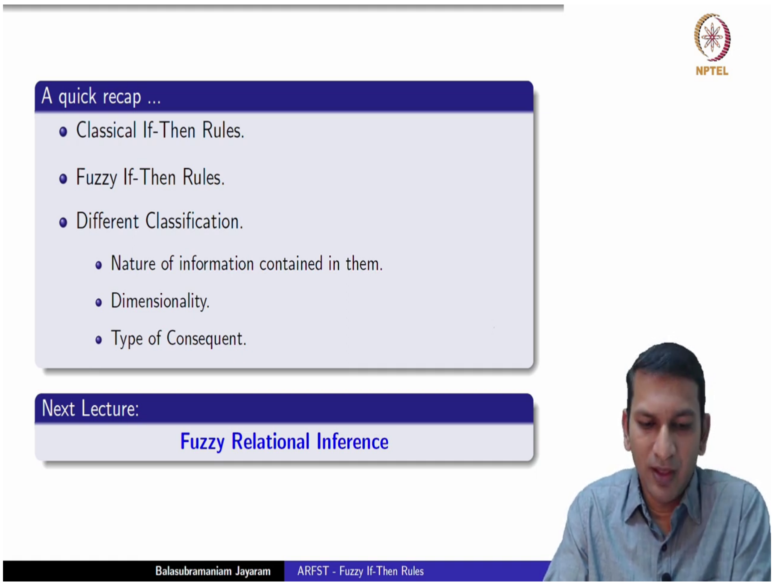

Well, in this lecture we have predominantly looked at fuzzy if-then rules. We began by looking into classical if-then rules, we try to understand them what are the positive, counter or irrelevant examples and how to actually generalize from them. We have seen fuzzy if-then rules are nothing but putting together fuzzy proportions relating them, we can relate them in different ways and that is what led to different types of classification.

If you relate them based on the nature of information contained in them, whereas, negative or positive, then you obtain conjunctive or implicative type rules. They can also be classified based on the dimensionality of inputs essentially a number of antecedents you have either single input or multiple input.

Finally, they can also be classified based on the type of consequent they have, either on the consequent side you have fuzzy set or a function of the input. So, if you have fuzzy set, these are called Mamdani type fuzzy rules and if you have a function they are called TSK type fuzzy rules. This is something that we will see going forward.

The next lecture we will get into the real mainstay of our course which is the fuzzy inference system. We will start by looking at general components and general schema of fuzzy inference system. And look at, as we know, it has these two components the knowledge which is contained in terms of fuzzy if-then rules which we have discussed so far to some sufficient extent.

The other one is the intelligence which is essentially the inference scheme. And one particular inference scheme is what we will see in the next lecture that of fuzzy relational inference. Where, a rule is converted into relation and then given an input we perform inferencing operation using that input and the relation that represents the rule.

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Those of you are keen to know more about these topics, there is some huge or vast amount of literature dealing with this especially types of if-then rules. We have just picked up a few of them. There is a work by Dubois and others which discusses this.

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There is also another work in the same year which discusses or gives us the positioning of fuzzy rules when it comes to reasoning with them.

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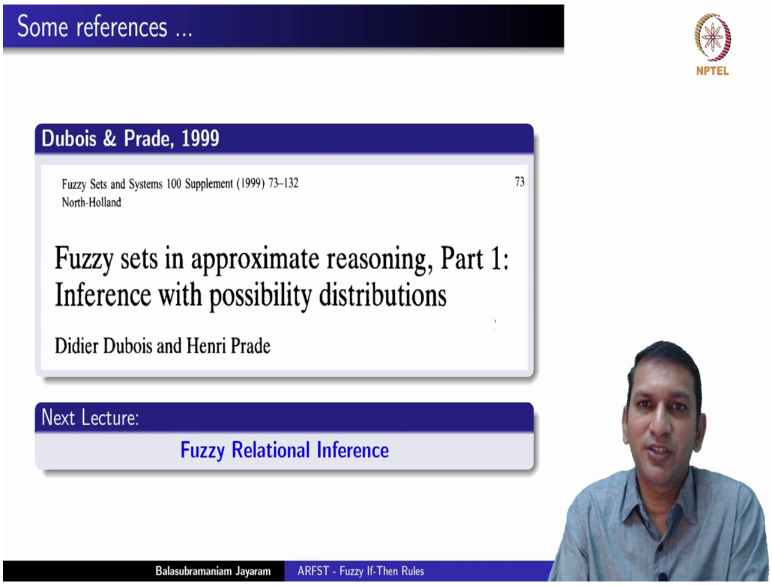

This is an excellent survey article, still perhaps very relevant, even though it appeared almost 25 years back dealing with fuzzy sets and approximate reason. As I said, in the next lecture, we will deal with Fuzzy Relational Influences. Glad, you could join us for this lecture, and looking forward to seeing you again in the next lecture.

Thank you again.