Approximate Reasoning using Fuzzy Set Theory
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Lecture - 28
Fuzzy Propositions: Some Interpretations

Hello and welcome to the 6th week of lectures of this course titled Approximate Reasoning using Fuzzy Set Theory. A course offered over the NPTEL platform. In this week, we will look into fuzzy inference systems which are the mainstay of this course, specifically we hope to look into fuzzy relational inference.

However, even for a simple fuzzy inference system, there are many building blocks and unless we understand these components in some detail, it is difficult to build a useful fuzzy inference system. In that quest, we will look into Fuzzy Propositions in this lecture.

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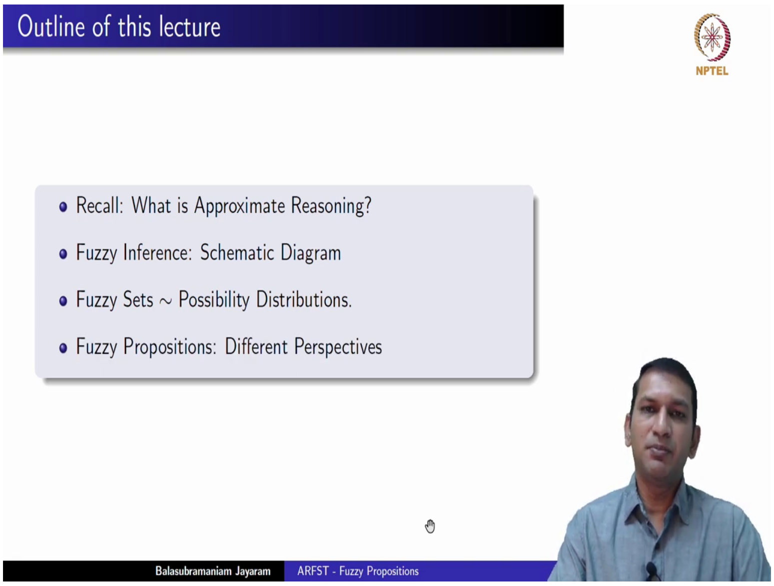

We will begin by recalling what is approximate reasoning? Approximate reasoning as we will understand and explore in this course. We will look at a schematic diagram of a fuzzy inference system. We will look at some different interpretations of fuzzy sets also and finally, we will look into fuzzy propositions from different perspectives.

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Well, what is approximate reasoning?

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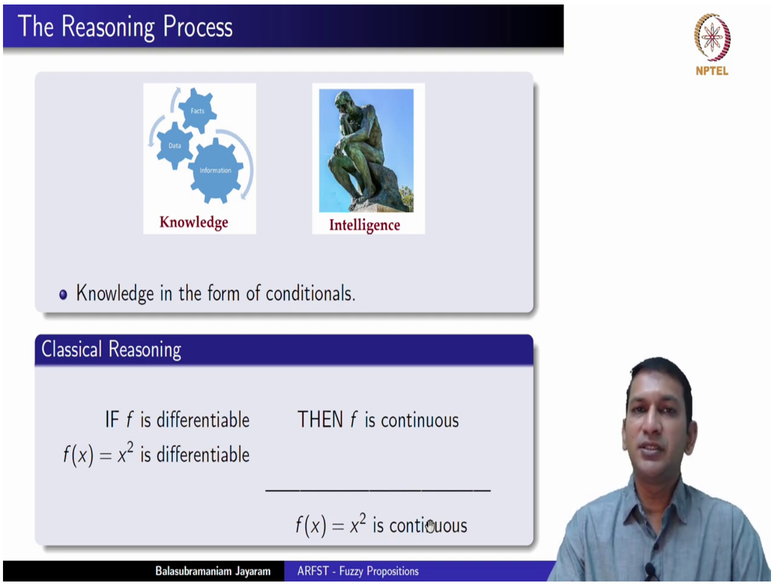

We have seen already in one of the earlier lectures that if you are looking at the reasoning process itself, you need two things. First the knowledge about the domain in which you are going to do the reasoning and also the intelligence to apply on this knowledge to reason.

Typically, this knowledge is given or represented in the form of conditions. If you look at classical reasoning, we might have the a piece of knowledge in the form of this conditional which by now must be extremely familiar to all of you. If f is differentiable, then f is continuous. Now, given this piece of knowledge and a function f(x) is equal to x square, we know it is differentiable, the function x square is differentiable.

Then, we ask the question given this piece of knowledge and expressed in the form of conditional and this new input, once again it is in the form of knowledge that f(x) is equal to x square is differentiable; what is it that we can infer from the general piece of knowledge that we have and a specific piece of information?

And, we see that from these two we can actually infer that f(x) is equal to x square is continuous. Now, what gives us the backing to do such an inference or reasoning? We know that there is a rule, mathematical principle called the Classical Modus Ponens which says that if A implies B and A then we actually can infer B.

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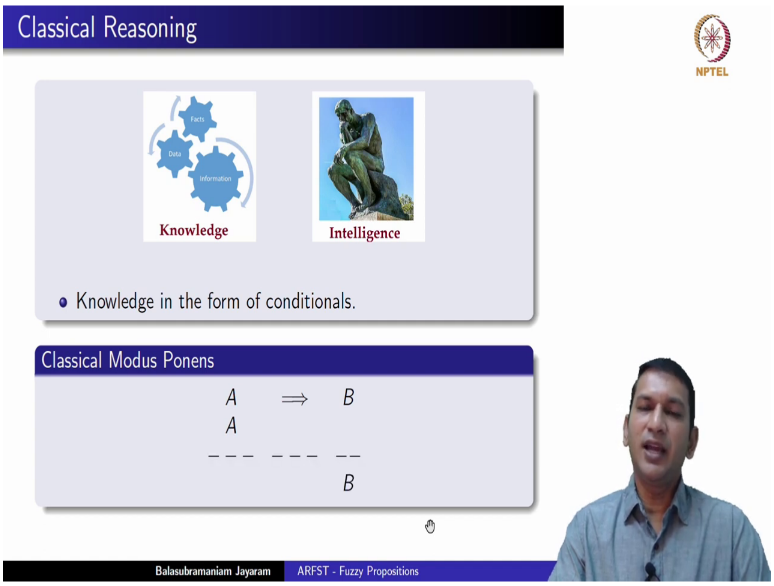

Now, think of the same piece of knowledge given to us in terms of the conditional, but now we have the function f(x) is equal to mod x.

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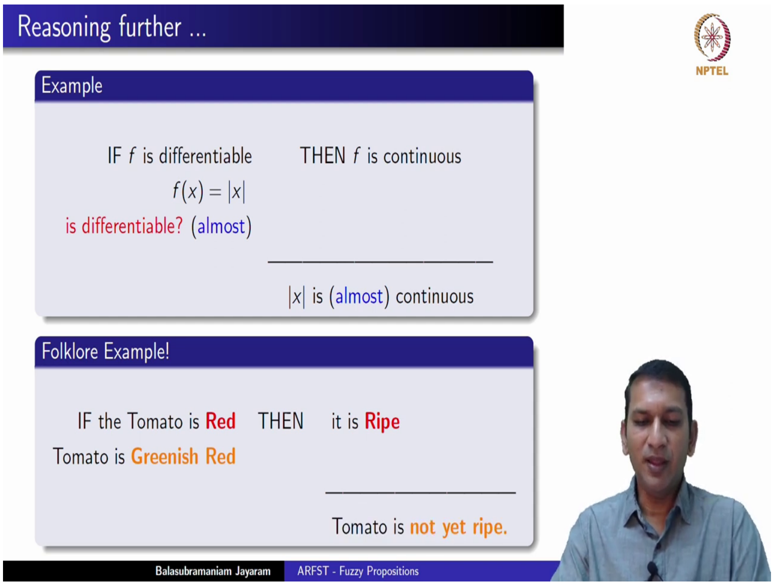

Now, if we ask the question is it differentiable? We all know that if we consider the domain of this function, the entire domain of this function which is r, then except at the origin everywhere else it is differentiable. So, in some sense colloquially speaking you could say it is almost differential.

The question is given this piece of knowledge and this new information, can we infer something? Well, classical modus ponens does not allow us to infer anything. Note that we should also point out something very important here. The given function f of x is equal to mod x, the new piece of information right now does not exactly fit into the space about which the antecedent is expressing.

If you recall in the case of implication, we said that if antecedent is false then the truth value of the conditional is always true. But, here note that we are not talking about the truth value of the given piece of knowledge, that is a given. Now, using that piece of knowledge and a new piece of information that has come into existence, we want to do some inference.

So, now in this case that is where we see that the piece of information that we have newly obtained does not exactly match the antecedent of the given room, which actually contains the knowledge that we are considering within this domain. However, if you look at what we called common sense reasoning, we would perhaps would like to come up with such a reasoning that mod x is almost continuous.

Of course, as was mentioned earlier also we know that mod x is actually continuous. But, given this piece of knowledge and this information what is it that, what best could we infer? Now, this is something that we could probably infer.

Now, let us go back to the folklore example that we have discussed in one of the earlier lectures. Just so, that we will fit this in the form of fuzzy if then rules. Now, with the knowledge of fuzzy set theory perhaps this rule will make more sense to you. We have this rule if the tomato is red, then it is ripe. Now, we pick up a tomato and it is greenish red and we nonchalantly make the inference that tomato is not yet ripe.

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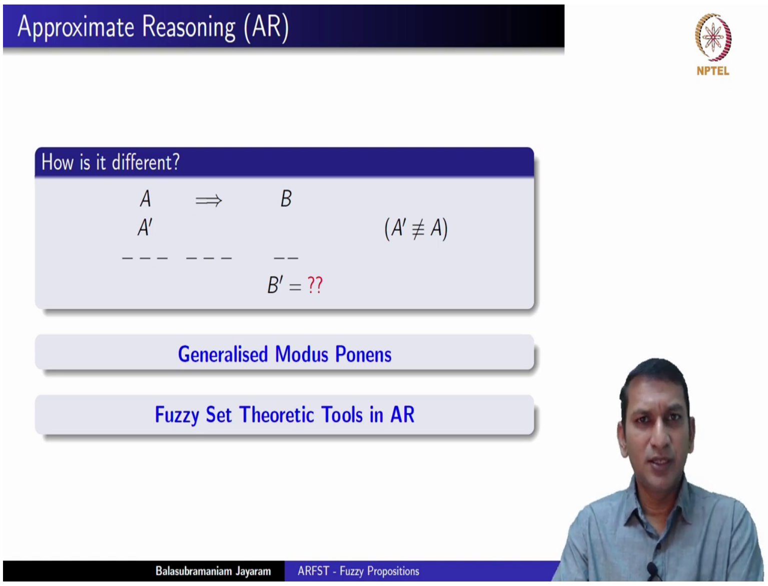

So, what is the difference between classical reasoning and approximate reasoning? Well, we still have a conditional in the form of A implies B. We are given an A dash which is typically not identically equal to A and we still would like to make some reasonable inference B dash. This is called generalised modus ponens and once again we will look at performing approximate reasoning in the framework of fuzzy set theory; that means, the A is A dashes, B is B dashes. All of them that we are going to discuss are fuzzy sets.

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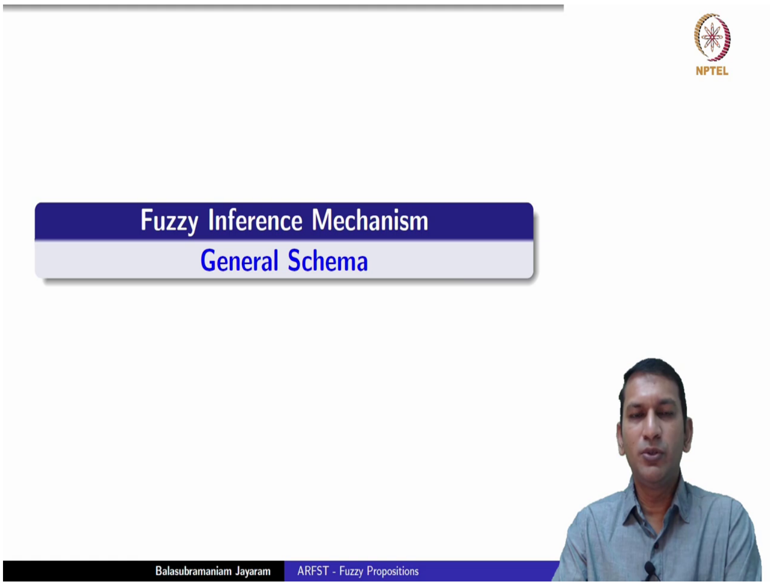

Well, with this quick recap, let us go to look at how a general schema of a fuzzy inference mechanism will look like.

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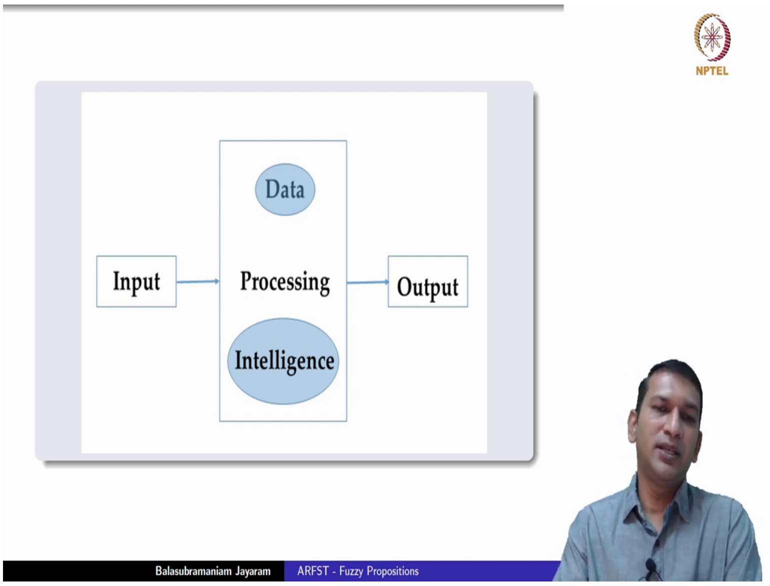

Now, we know that even when we are dealing with a computer or a calculator, this is essentially the setup. So, you we give some input. It does some processing and gives us an output. Now, to do this processing, it will depend on two essential components. These are the data and the intelligence. So, now data could be some supplementary data that is required for processing the input or it could also be data about the data which is meta data.

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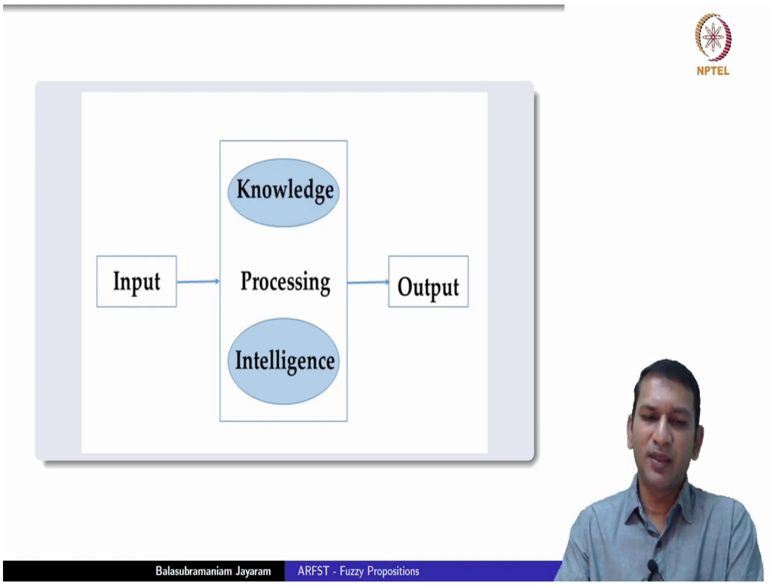

So, in that sense you could say it is also the knowledge. Now, processing means we need to process input using these inclusions, but of course, the intelligence has to talk to the knowledge. It has to interface with the knowledge so, as to obtain some meaningful outcomes.

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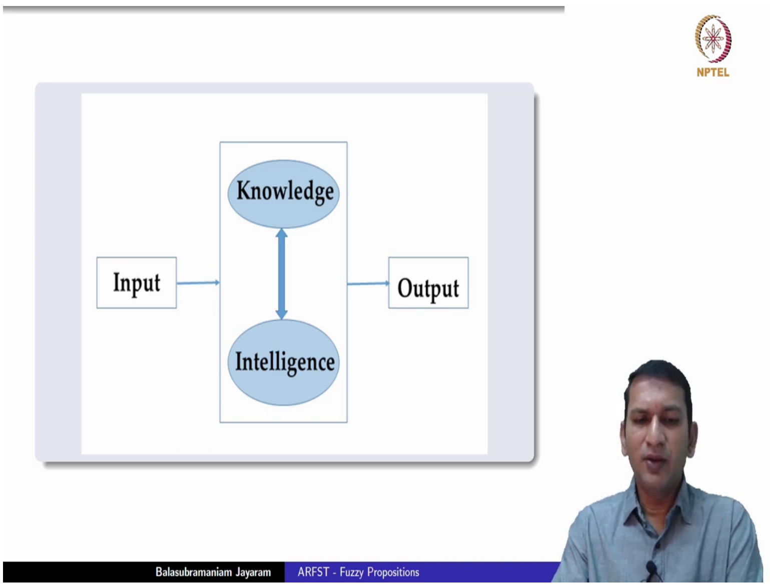

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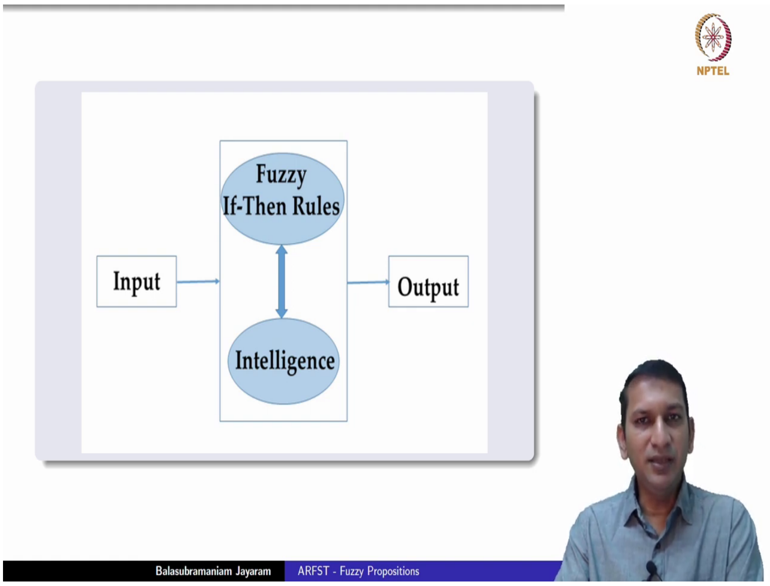

Now, in our case the same thing happens. We are being given an input and we want an output and in the place of knowledge we have actually captured this knowledge in the form of fuzzy if then rules. And, we are going to apply intelligence on this. Intelligence is in the form of an inference scheme.

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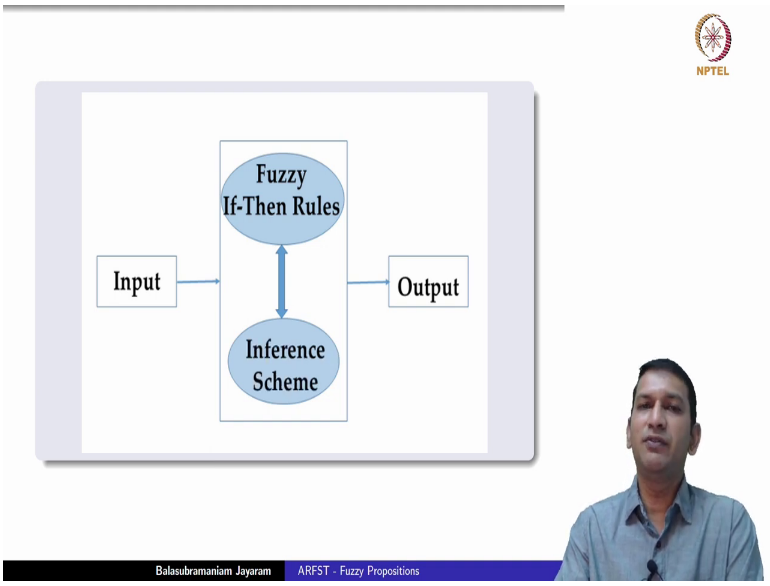

So, these are the two important components, when you are talking about any inference scheme. The actual algorithm itself how do you infer, also how the knowledge is stored. So, that given an input based on the knowledge this inference algorithm can act on it and give us a meaningful output.

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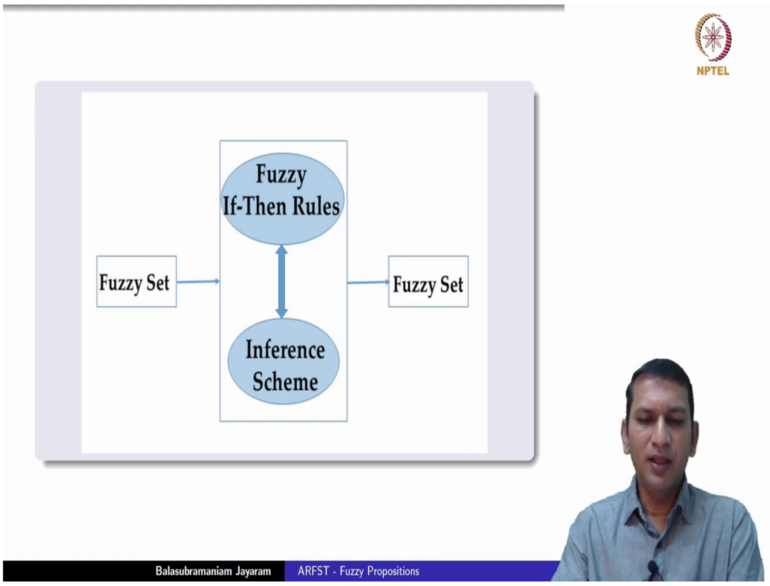

Here the input and the output typically will be fuzzy sets, but we can also take them as points from classical sets. We will come to it when we deal with fuzzy inference systems, perhaps in the 3rd or 4th lecture of this week.

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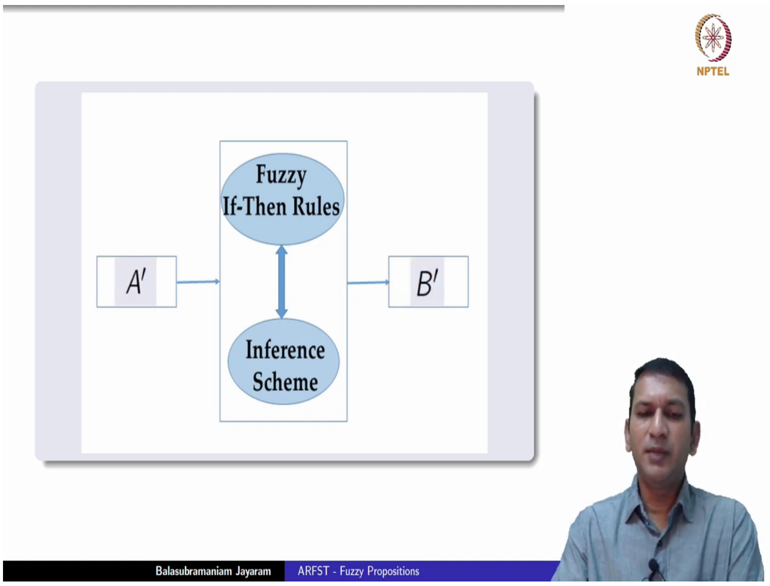

So, typically we given A dash and we are expecting B dash.

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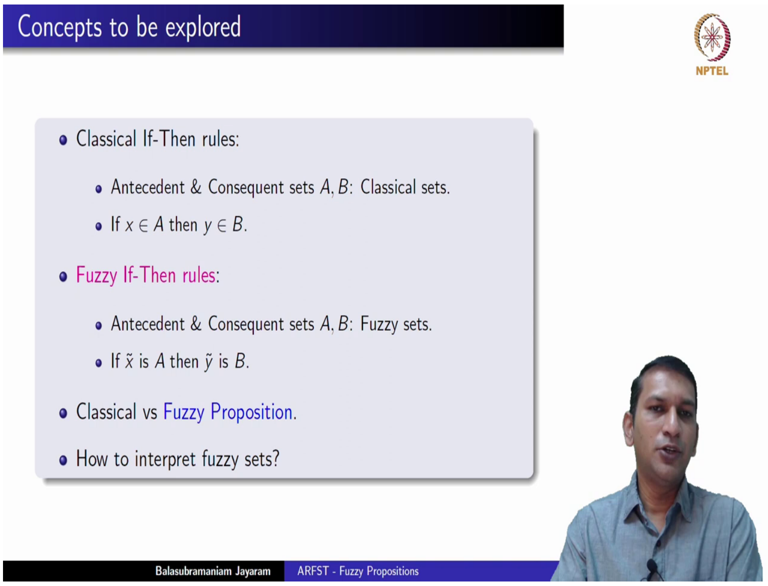

Now, to be able to understand this general schema, a few concepts need to be explored and understood. Firstly, we are talking about knowledge being captured as if then rules as fuzzy if then rules. To be able to understand this, we need to know how classical if then rules themselves look like and how are they interpreted.

By classical if then rules we mean rules where the antecedent and consequence sets A and B are actually classical sets, subsets of the corresponding domains. For instance, it might look like this if x belongs to A, then y belongs to B. From here we will move into fuzzy if then rules, where the antecedent and consequent sets are actually fuzzy sets over some appropriate domains. And, the rules themselves would look like this x x tilde is A, then y tilde is B.

So, if tomato is red, then it is ripe. But, towards understanding these things we need to understand also the individual components of the rules x tilde is A, y tilde is B. So, these we call as fuzzy propositions. So, to understand them we need to see how they are interpreted and what are the perspectives they offer us.

So, in that sense we also might want to look at fuzzy sets and try to interpret them also in a different way. So, in this lecture our main focus will be on understanding fuzzy propositions. In the next lecture, we will look at fuzzy if then rules and with these two basic building blocks. Then, we will be in a better position to discuss fuzzy inference schemes.

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Well, let us look at fuzzy sets themselves.

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The first perspective of fuzzy set is as a membership function. It talks about the belongingness of an element to a concept.

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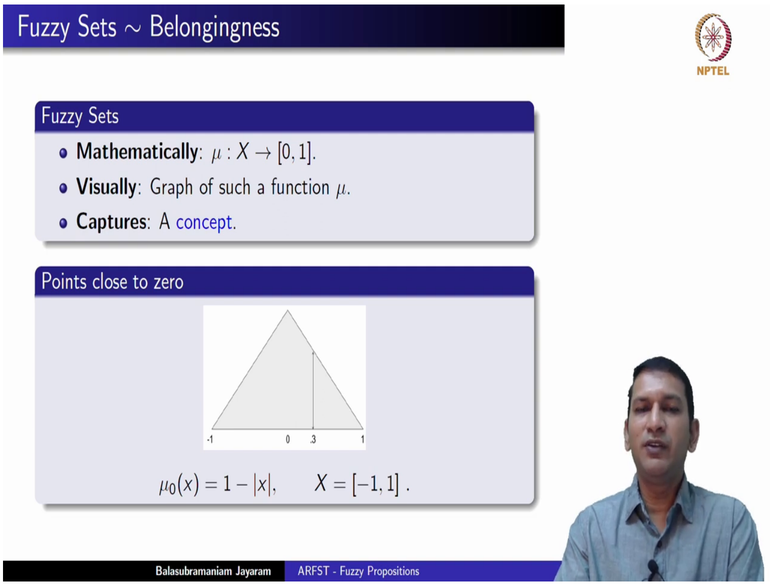

So, we know that mathematically a fuzzy set is a function from an underlying domain X to [0,1]. Visually, we indicated in terms of a graph of such a function and we know that it actually captures or represents a concept; hot, very hot, cool, high, medium, average, fast, slow, costly, cheap. These are the concepts that we try to represent using a fuzzy set over an appropriate domain X.

Now, let us look at one particular example which we have seen earlier too. We want to represent the concept points close to zero. Now, let us limit the domain to be just the minus 1 1 interval and then this could be one way of representing this concept. So, now, as you move away from 0; obviously, the membership value will start to taper down. And, in this case they taper down in a linear fashion. And, now if you take the value 0.3, then you are essentially asking to what extent 0.3 is close to zero. This is well known.

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So, this is of how often fuzzy set is thought of, but we can also offer a different perspective of fuzzy sets as a possibility distribution. What do we mean by this?

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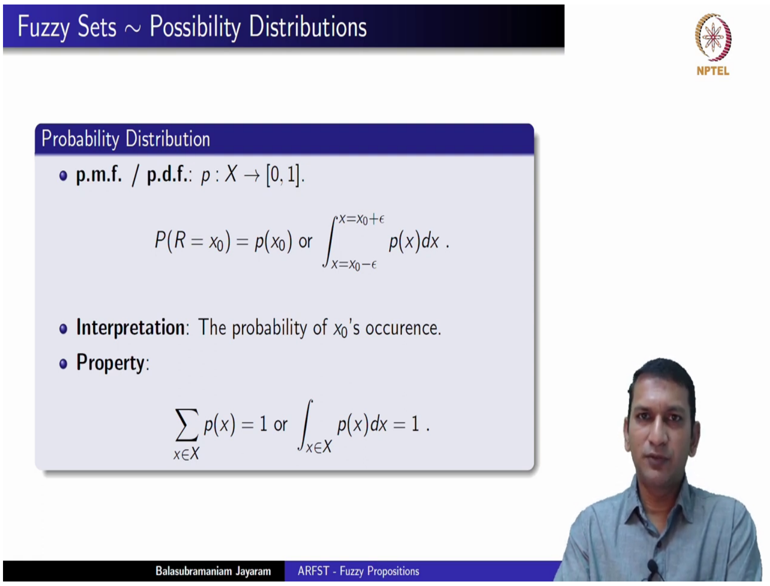

Let us begin with a probability distribution. We know that if you are given a random variable r, associated with that is either a pmf probability mass function or a probability density function.

Now, if you take a value x naught from the domain x and ask this question what is the probability that R could be z x naught? Then, this probability is essentially p of x naught, if it is a pmf, if the domain is discrete or if you have a continuous distribution probability density function.

Then, this is how we calculate the probability around that point x naught. How is it interpreted? It is interpreted as the probability of x naughts occurrence. Now, when we are talking about pmf or pdf, note also that this property of it being normalized is available to us. Let us stay with this probability distribution for a few minutes more.

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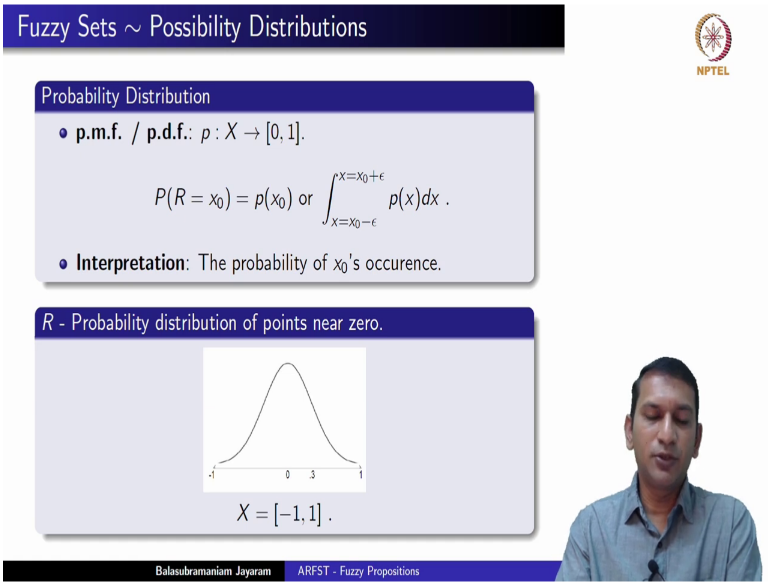

Let us construct a probability distribution of points near zero; that means, come up with a pmf or a pdf about how the probability of points near zero could be distributed. One way to do it is kind of use a Gaussian membership function and if you are band limiting the domain, then you could also truncate the gaussian.

So, let us once again fix the domain to be minus 1 and 1 and that is where we are actually sampling the points 1. And, we want to sample points in such a way that they are actually being represented at they are close to zero. So, now let us think of this tapered or truncated Gaussian. Now, this is one way to look at this distribution, the corresponding pdf could be given like this.

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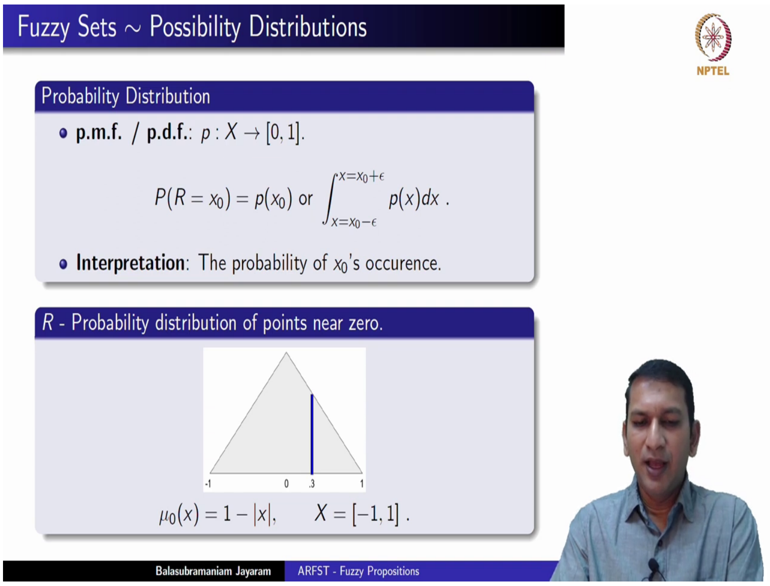

But, we could also consider the earlier fuzzy set the trapeze, the triangular fuzzy set that we considered on this domain also as a pdf. Because, the area under the curve is 1 and so, this could also be considered as a possible pdf. So, this gives a different distribution of the points that are close to zero. There is when you sample from this distribution, the probability with which you get 0.3 is different from how you would get with some other distribution.

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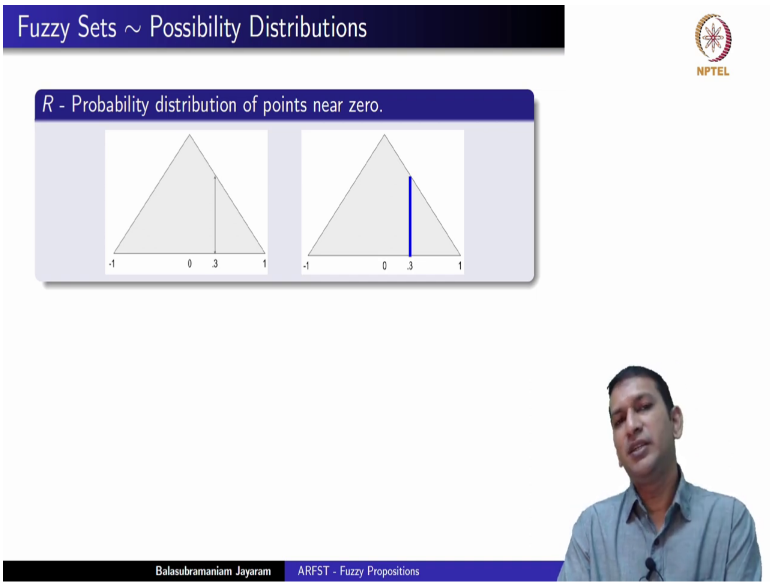

One point that we need to immediately notice this. Look at this, we are considering the same function. These are two graphs of the same function, but on the left hand side, on the right hand side we are interpreting them differently. On the left figure we are interpreting it as a fuzzy set, on the right side figure we are represent interpreting it as a probability density function.

Now, if you look at 0.3 both these values, what are we asking here with 0.3? When it is represented as a fuzzy set, when it is considered as a fuzzy set we are only asking the question to what extent does 0.3 belong to this fuzzy set? But, now when it represents a pdf, what is it that we are asking?

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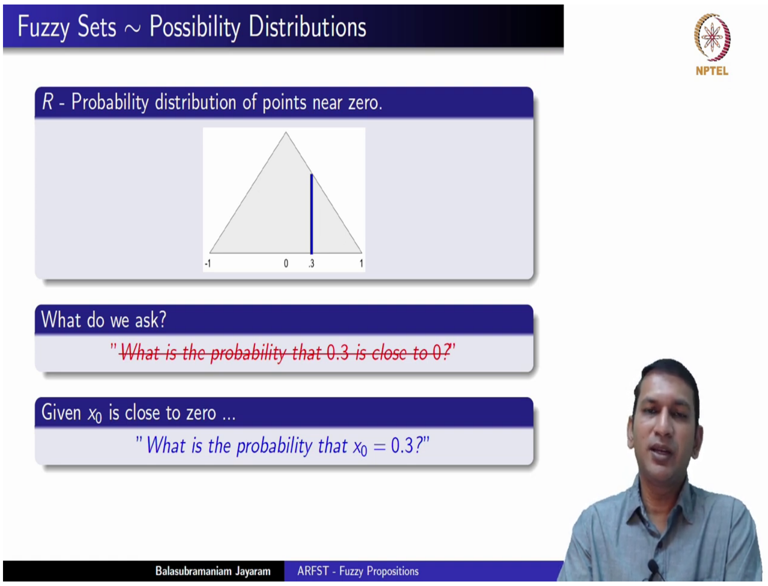

Well, note that probability of R is equal to 0.3, because it is a pdf, has to be taken around a small interval around 0.3 and the area under that is assigned as the probability. Now, interesting thing is we need to understand what is it that we are asking? What are we asking? Are you a are we asking this question, what is the probability that 0.3 is close to 0?

No, this is not the question we are asking. What we are asking here is given x naught is close to zero, we have sampled a point and we know that that point comes from distribution and it is from the distribution R which is actually capturing the concept of points being close to zero. We are asking the question given x naught is close to zero, what is the probability that x naught is equal to 0.3? As you see as we go closer to zero, the probability of sampling points closer to zero are quite high because of this distribution. So, this is the question that we are asking in when we have when you are in the framework or the setting of probability. But, now instead of looking at this as a probability distribution, let us go back to looking at it as a fuzzy set.

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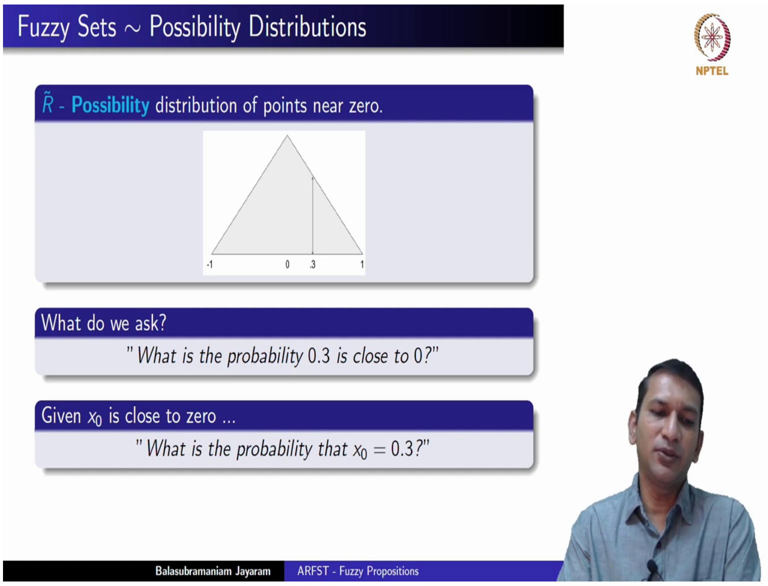

And now we will say, we are looking at a possibility distribution of points near zero.

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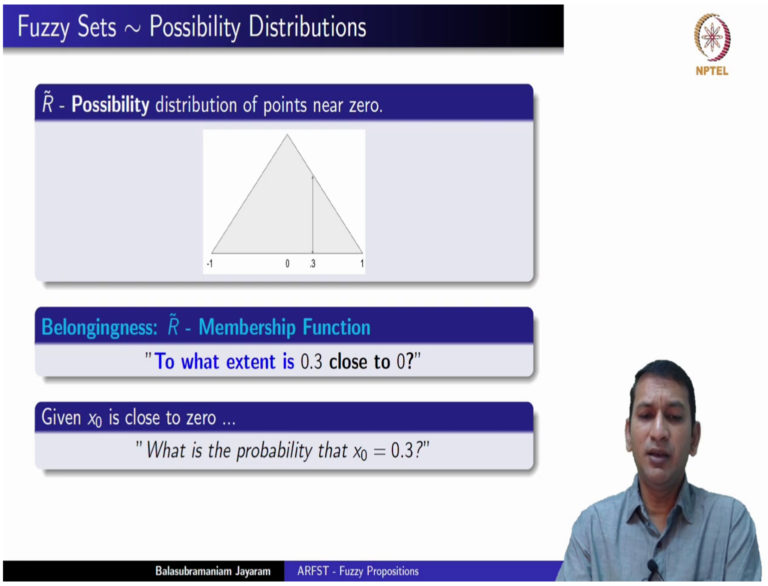

What do we understand by this? We are not asking the question to what is the probability that 0.3 is close to zero, but we are asking the question to what extent is 0.3 close to zero? So, you will see that between this and the earlier, the thick band around 0.3 has actually become has thin line, because you are only fitting the value 0.3 into the function and finding it out.

So, to what extent is 0.3 close to zero? Now, even this is the question that we want answer, then essentially we are looking at R tilde as a membership function. We are discussing the belongingness of 0.3 to this concept. But, we can also ask the same question, given x naught is close to zero what is the instead of probability, we can ask what is the possibility that x naught is 0.3?

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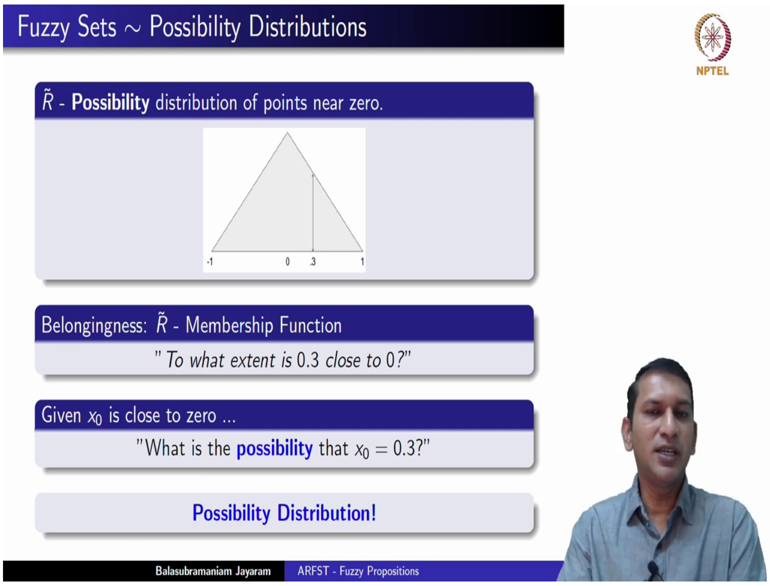

We have picked up an x naught which is close to zero and we can now ask what is the possibility that x naught is 0.3? If you are interpreting fuzzy sets like this, then we call that a possibility distribution. So, we can look at fuzzy sets also as possibility distribution. So, these are two ways to look at fuzzy sets, either as membership functions or as possibility distributions.

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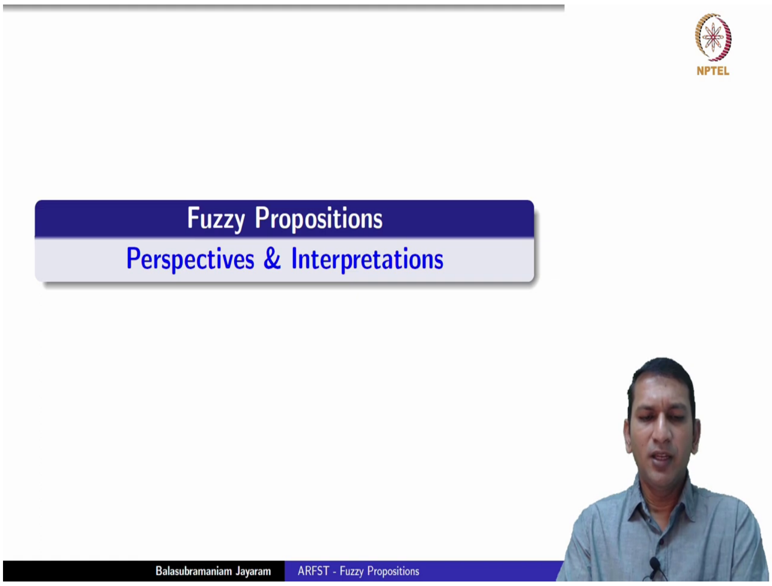

Now, let us look at fuzzy propositions themselves. Once again we will see that there are many different interpretation and perspectives possible here.

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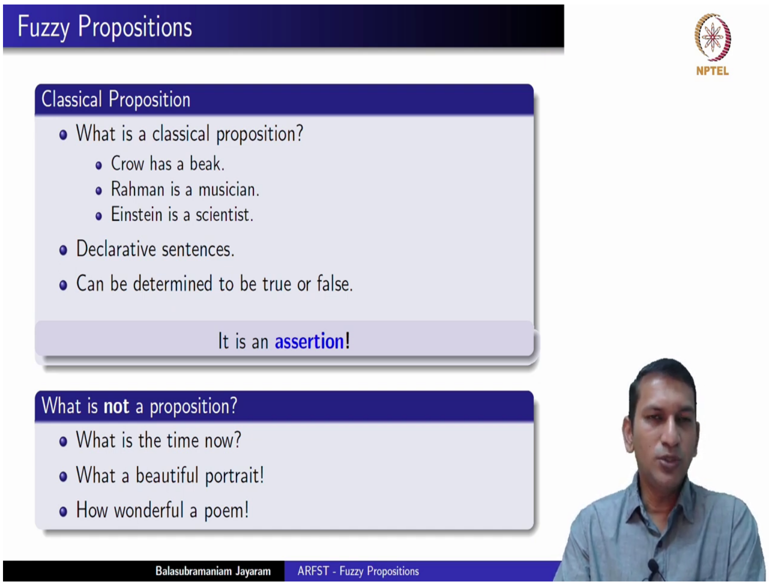

Now, to begin with what is a classical proposition? Crow has a beak. Rahman is a musician. Einstein is a scientist. Now, these are propositions. Why do we call them propositions? They are declarative statements, their truth value can be determined; either they are true or false. In that sense, essentially a classical proposition is nothing, but an assertion.

We should also note, what is not a proposition? A question is not a proposition. What is the time now? That is not a proposition. We cannot ascertain its truth value. Similarly, a rhetorical question is again not a proposition, nor is an exclamation a proposition. So, proposition is an assertion whose truth value can be determined.

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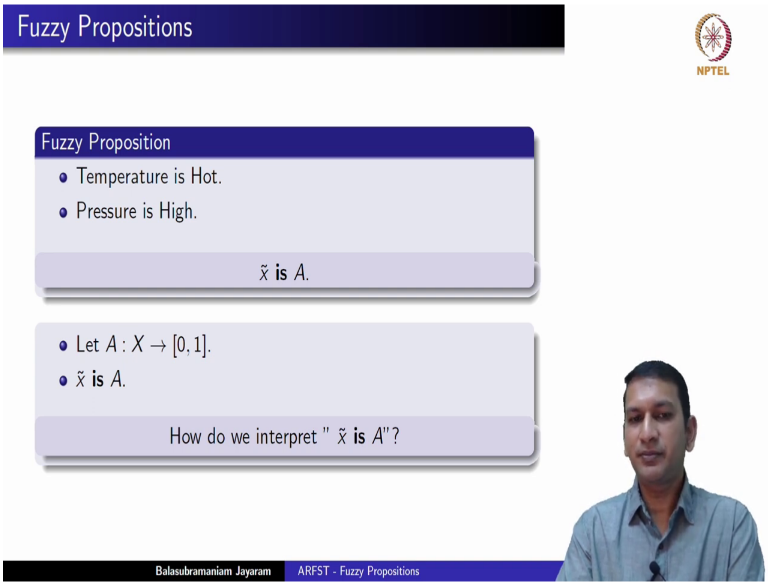

Now, what is a fuzzy proposition? We say temperature is hot. Pressure is high. So, these are fuzzy propositions. Essentially, we have a statement of the form x tilde is A. Now, what is A? It is a fuzzy set. What is x tilde? Is A, this is what we want to interpret. Question is how do we interpret this statement, this assertion wherein we have x tilde is A and A is a fuzzy set?

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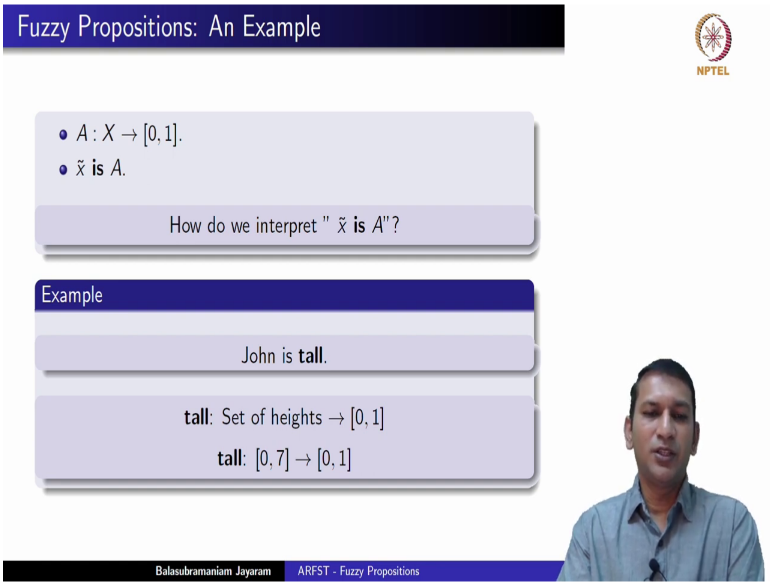

Let us look at that, perhaps beginning with an example. So, once again we have a fuzzy set A, on an appropriate domain x. And the question now is how do we interpret x tilde is A? Take this example: John is tall. Clearly, tall is a fuzzy set. A function from the set of heights to [0,1]. Perhaps, we could also fix the set to be 0 to 7 feet, the interval 0 to 7 representing feet to 0. So, now John is tall clearly falls under this category and we could call it a fuzzy proposition. Question now is how do we interpret it?

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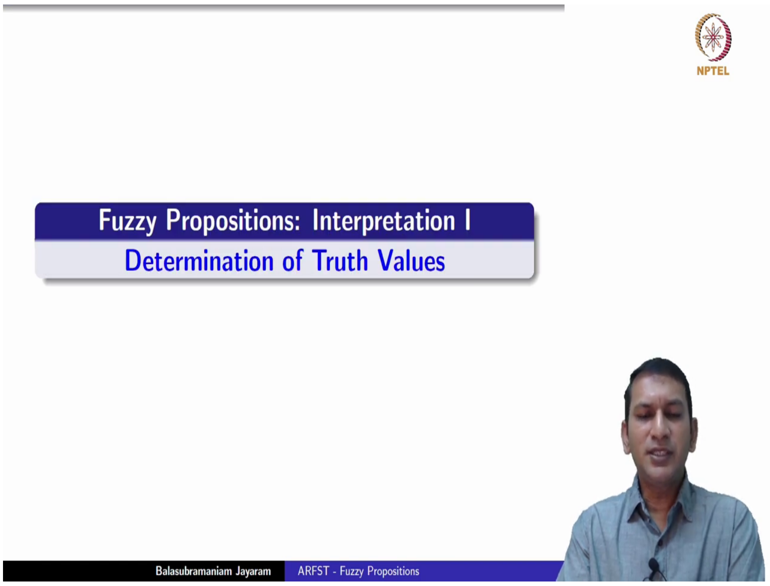

The first interpretation is that of determining the truth value of the statement and how do we do this?

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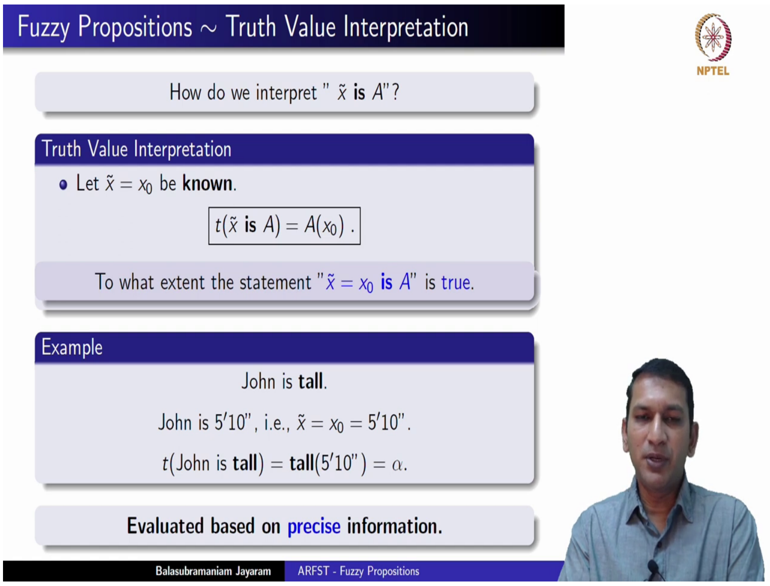

The question is we are given x tilde is A. One way to interpret it as is as the truth value of this statement itself.

Let us be given an x naught and let us know that x tilde is equal to x naught is actually known. Remember, it is known that x tilde is x naught, then we say t of x tilde is A is actually equal to A of x naught. What is t here? The truth value of the statement x tilde is A is in fact, equal to the membership value of x naught in A.

So, essentially we are stating to what extent the statement x naught is equal to x x tilde is equal to x naught is A is true. So, we are finding out the truth value of this statement given x tilde is equal to x naught. In terms of our example, we have this fuzzy proposition John is tall. Now, we know that John is 5 foot 10 inches; that means, x tilde is equal to x naught is 5 feet 10 inches.

Now, this is the truth we value we assign to this statement, this fuzzy proposition. To what extent is John tall? The truth value of John is tall is essentially to what extent 5 foot 10 inches belongs to this fuzzy set tall? If it belongs to the degree alpha, then that is the truth value of this statement; given x tilde is equal to x naught. So, this is one way to interpret a fuzzy proposition. Essentially, we are evaluating the truth value of the statement itself based on precise information.

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There is an alternate interpretation also, a second interpretation; that of determining the compatibility with respect to the given fuzzy proposition.

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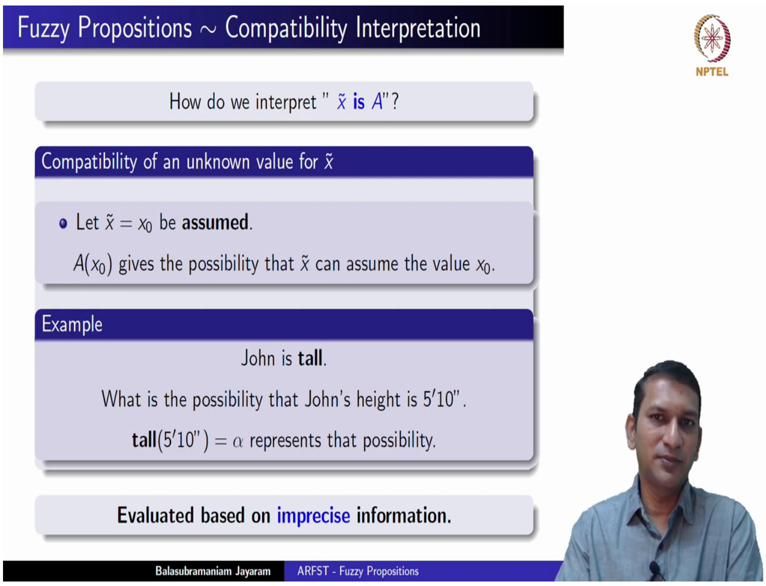

What do we mean by this? Once again, we want to interpret x tilde is A. Now, earlier we were given x tilde is equal to x naught, it was known. Now, let us assume that x tilde is x naught, then what does the statement say? Then, A of x naught, remember A is a fuzzy set, A of x naught is its membership value. It gives the possibility that x tilde can assume the value x naught.

So, now this goes back to the second type of interpretation that fuzzy sets can be given. So, this membership value can be looked at as the possibility that x tilde can assume the value x naught. Once again for the case of our example John is tall. Now, we do not know the height of John, but we are asking the question what is the possibility that John's height is 5 foot 10 inches?

So, now this alpha which is the membership value of 5 foot 10 inches in the fuzzy set tall represents that possibility, possibility that John's height could be 5 foot 10 inches. So, now, once again we are evaluating something about the statement. The possibility that John's height is this based on imprecise information. We actually do not know x naught.

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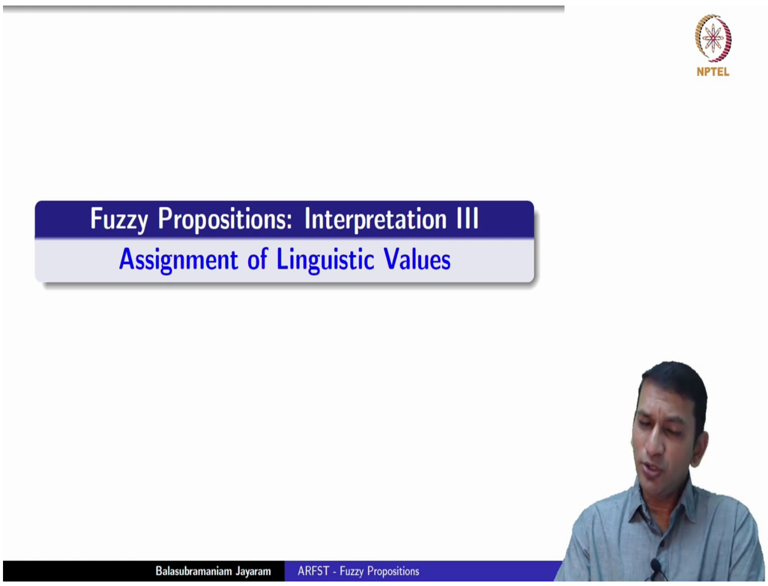

There is also another way to interpret fuzzy propositions as assignment of linguistic values.

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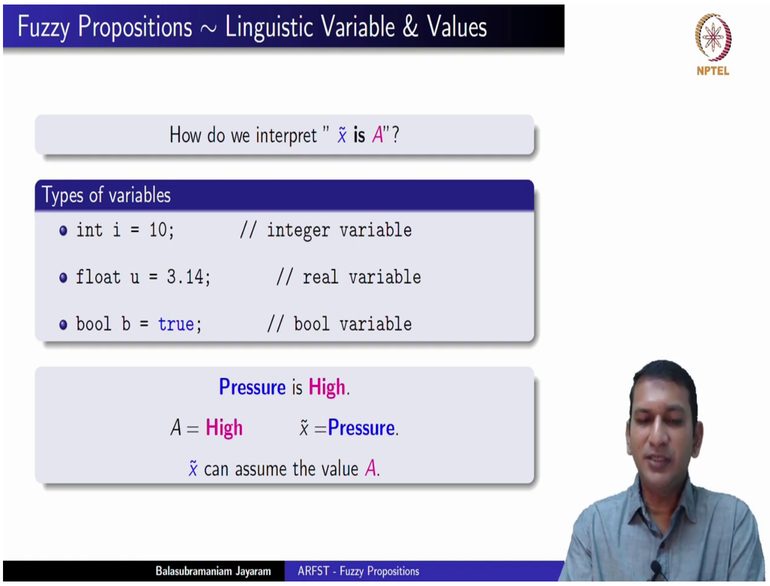

What do we mean by this? So, now, for this particular case we have actually color coded the statement x tilde is A.

Now, look at in typical programming languages, computer programming languages, we declare variables of different types. So, when we write such a statement int i is equal to 10, then the compiler understands that i is of type integer, it is a it is an integer variable and it can assume values only from the set of integers, integer values. If you give it a real of course, it knows how to truncate it. So, i is an integer variable, can assume integer values, float u is equal to 3.14, means u is a real variable. It can assume real values.

If you say bool b is equal to true. So, we are initializing, then it understands b is a bool variable and it can only take either true or false. It cannot take 3.14 of course, some compilers may be truncating and understanding what it is, perhaps if it is not 0, it may be giving the value 2. Anyway, the point is you have a variable and the moment you declare that variability of particular type, then it knows that it can only assume values from the corresponding node.

Now, let us consider this statement. Pressure is high, it is much like a specific case of x tilde is A. So, high is A fuzzy set defined on an appropriate domain and pressure is x tilde. Now, what we can interpret this x tilde is A to B is as follows, x tilde can assume the value A, that is all that we are saying; x tilde can assume the value A. So, if we were to give such an interpretation, how do we understand x tilde to A? So, let us stay with this.

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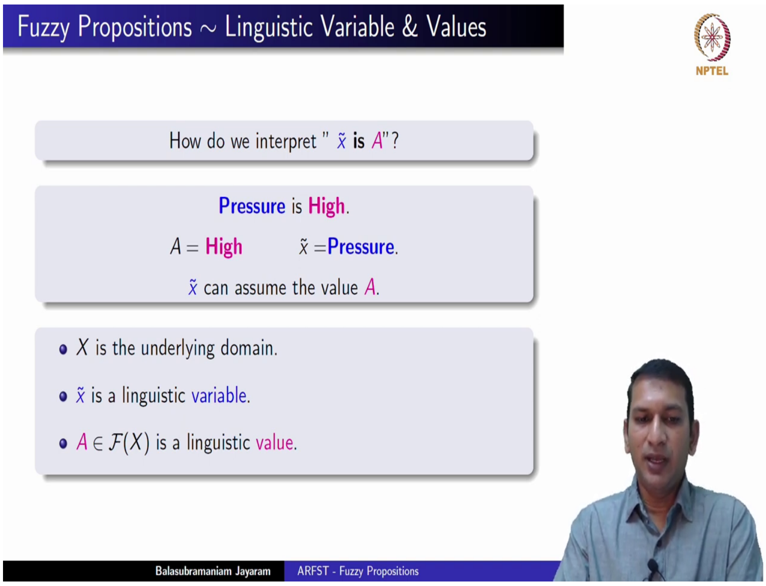

Now, once again x is the underlying domain. We call x tilde to be a linguistic variable and A coming from F(X) which is the fuzzy set, we say it is a linguistic value. So, x tilde is the linguistic variable and A is the linguistic value, that it can assume.

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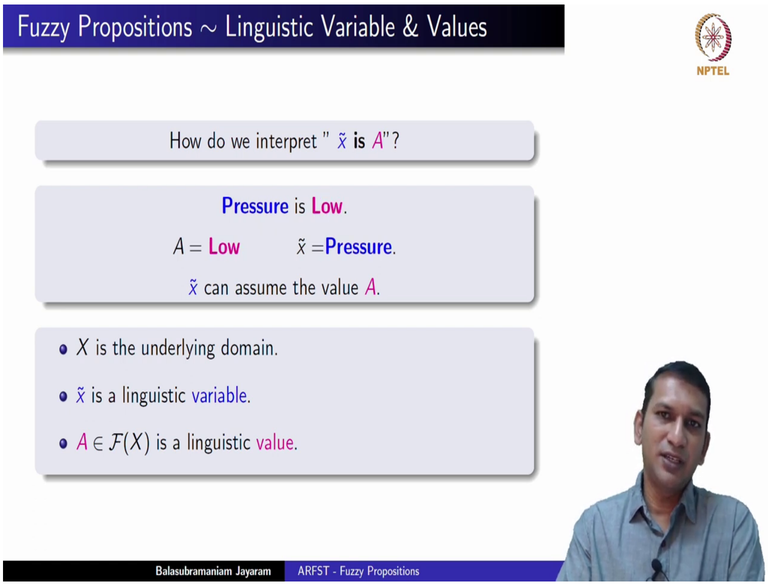

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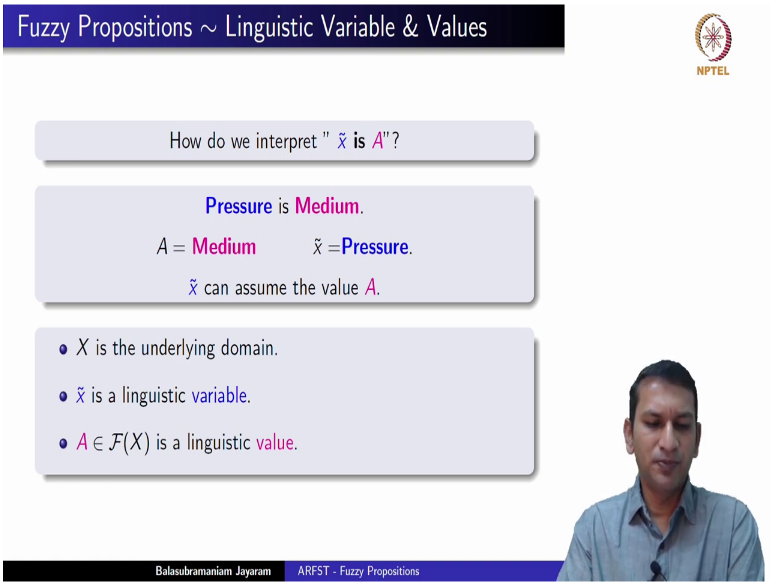

Now, in this case A was high, but a could as well be low which is again a fuzzy set defined on x, it could be medium. Another fuzzy set defined on x. Now, note that when we say x tilde can assume the value A. So, we are somehow interpreting and essentially keeping the semantics of x tilde with respect to the context as a fuzzy set defined on x. You could also say can A be cheap? Pressure is cheap or can A be fast? Pressure is fast.

Well, as long as we have a decent way of interpreting these fuzzy sets it is ok, but otherwise we need to keep the context in mind. And, it goes back to one of the earlier lectures where we said the context has to be kept in mind while defining the fuzzy set. But, now the context also has to be kept in mind, while choosing the set of linguistic values, a linguistic variable can actually assume.

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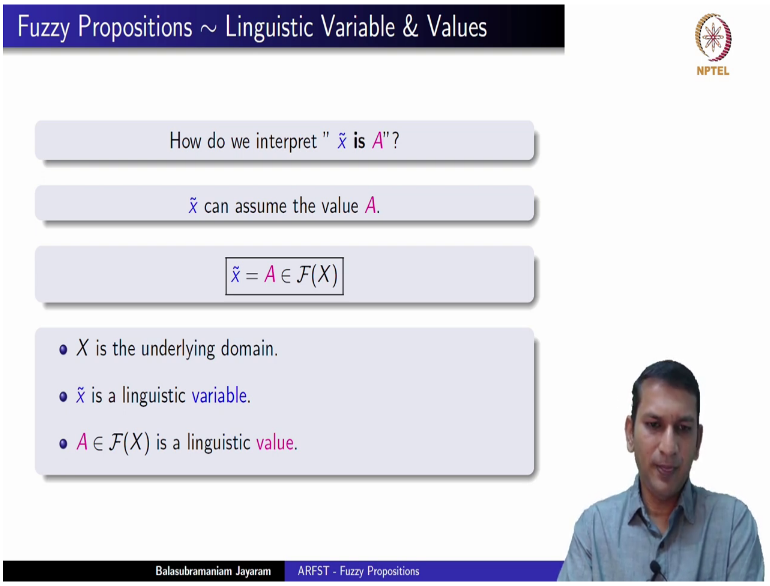

So, what we have is x tilde is A can also be interpreted as assigning a linguistic value to a linguistic variable, x tilde can assume the value A. The most important thing is x tilde is equal to A and it is coming from F(X). In the context with respect to the context, we have fixed the domain X and we will interpret by giving semantics accordingly. So, that x tilde is equal to A or x tilde is A, makes sense in that context.

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Well, a quick recap. We recalled generalised modus ponens. We saw a couple of interpretations of fuzzy sets. One of them which we always already knew which was that of a membership function. But, we saw that it could also be seen as a possibility distribution. And, clearly the distinction between the possibility distribution and the probability distribution was explained.

And, we saw fuzzy propositions and we again had at least three different interpretations. First of them enabling us to discuss the truth value of the statement itself, if x tilde is precisely known. If x tilde is not precisely is known, then we could look at it as possibility of x tilde assuming that value; essentially going back to the second interpretation of fuzzy sets.

And finally, we also looked at it as a linguistic variable assuming a linguistic value. So, with this the basic building block of fuzzy propositions which go into building a fuzzy if then rule has been explored, to perhaps a sufficient detail of what is required for us. In the next lecture, we will look at fuzzy if then rules.

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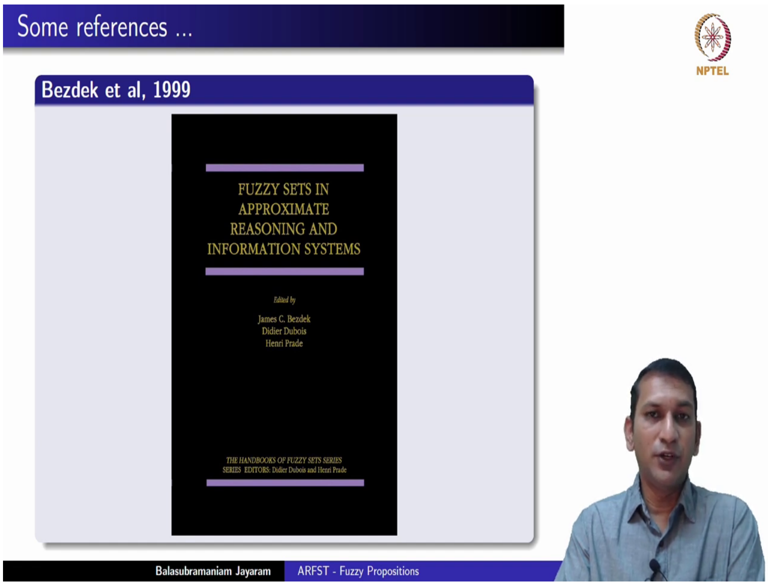

Now, this is an excellent edited volume by Bezdek, Dubois and Prade. For more on the topics dealt with in this lecture, you could refer to this book.

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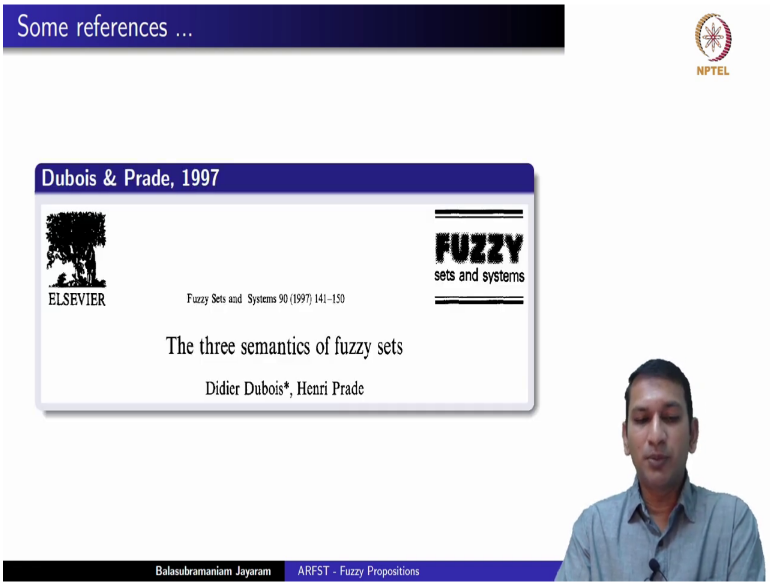

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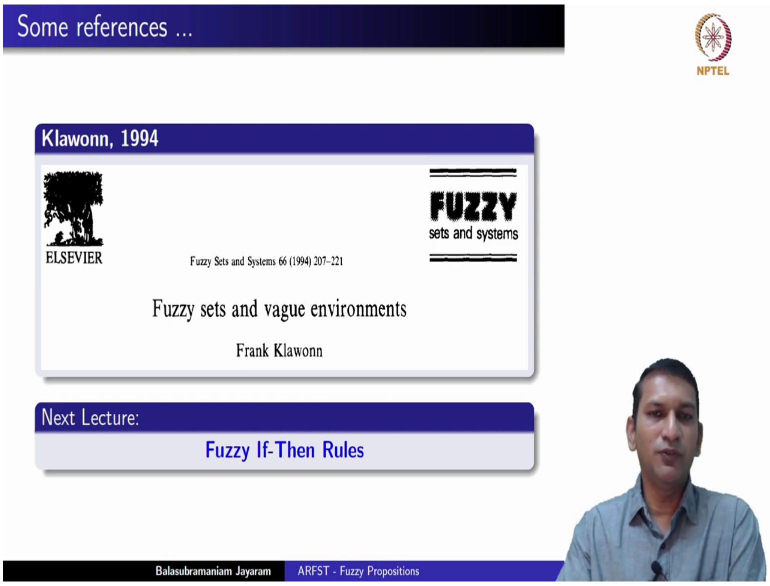

As also to the papers of Dubois and Prade, one of them being three semantics of fuzzy sets. And, also an earlier paper of Frank Klawonn which deals with some of the things that we have discussed, Fuzzy sets and vague environments.

Next lecture, we will discuss fuzzy if then rules, with that the basic building blocks of fuzzy inference system must be in place; so, that we can looked into fuzzy inference systems. Glad that you could join us for this lecture and looking forward to seeing you soon in the next lecture.

Thank you once again.