Approximate Reasoning using Fuzzy Set Theory

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Lecture – 17
Fuzzy Implications – Desirable Properties

Hello and welcome to the second of the lectures in this 4th week under the course titled Approximate Reasoning using Fuzzy Set Theory. A course offered over the NPTEL platform. In the last lecture, we saw what fuzzy implications are we generalized it to a particular axiomatic definition, we have seen some examples in fact, basic fuzzy implications what we call them as those nine important fuzzy implications.

And we have also seen some geometric perspectives of it. In this lecture, we will look at some desirable properties that we would expect the fuzzy implication to have.

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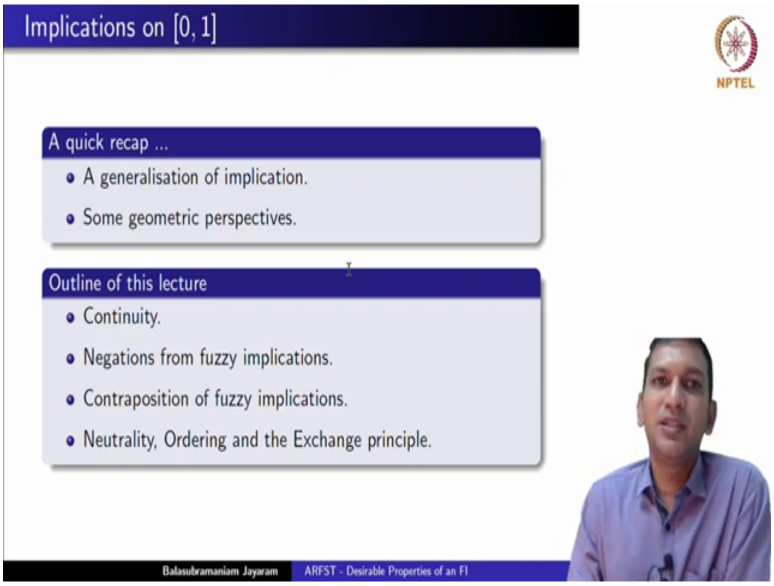

As was told, we saw a generalization of properties from the classical implication two that of fuzzy implication and that is how we have came we have come up with a axiomatic definition of a fuzzy implication. We also saw some geometric perspectives. In this lecture, we would like to elucidate some desirable properties of fuzzy implication of course, continuity is one of them.

We would also see how to get negations from fuzzy implications and once you have negation we could also discuss contraposition of a fuzzy implication. Most importantly we would look at three four particular almost algebraic operation that you could call, that of neutrality, ordering principle, identity and exchange principle.

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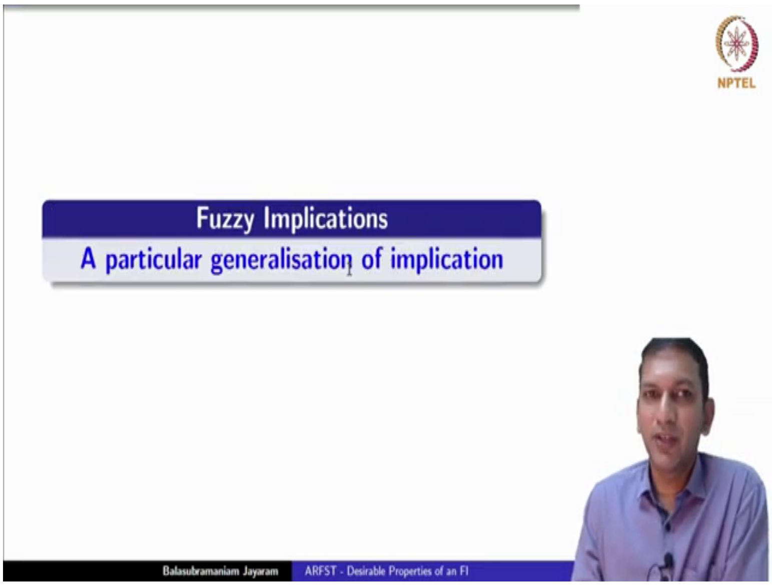

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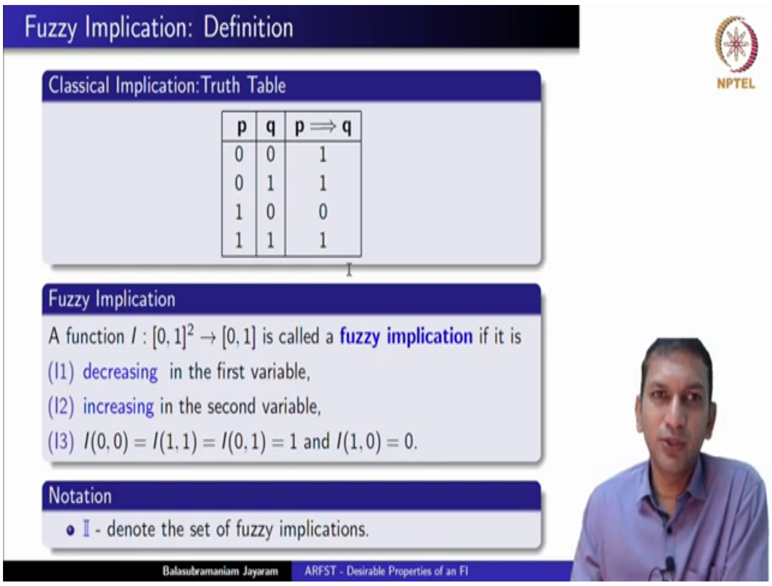

Let us recall what a fuzzy implication is. We have looked at the truth table of a classical implication and from here we say a function I from [0, 1] square to [0, 1]. It is called a fuzzy implication, if it is decreasing in the first variable, increasing in the second variable and satisfies these boundary conditions. And we also said that we will denote the set of all fuzzy implications using this symbol scripter.

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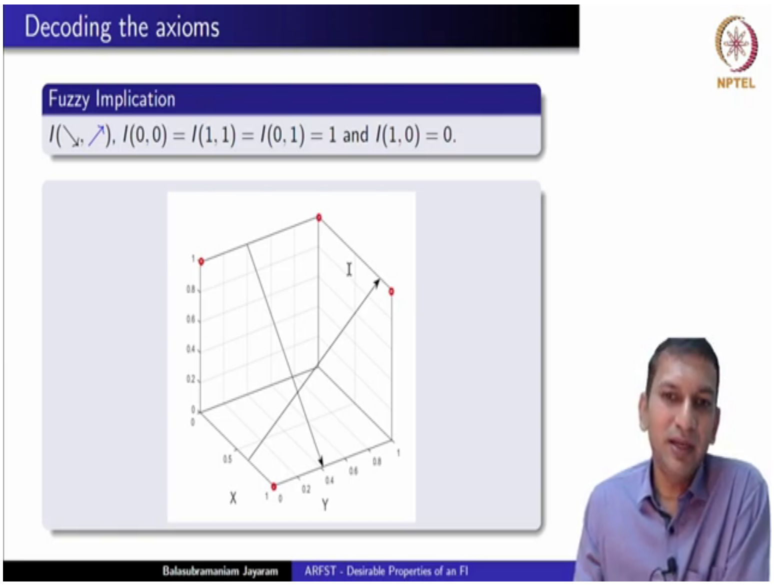

We tried to decode the axioms in a geometric way visually how do they look like. We saw that the boundary conditions translate as these points on the 4 vertices of the unit square and we are looking at decreasing along the first variable and increasing along the second variable.

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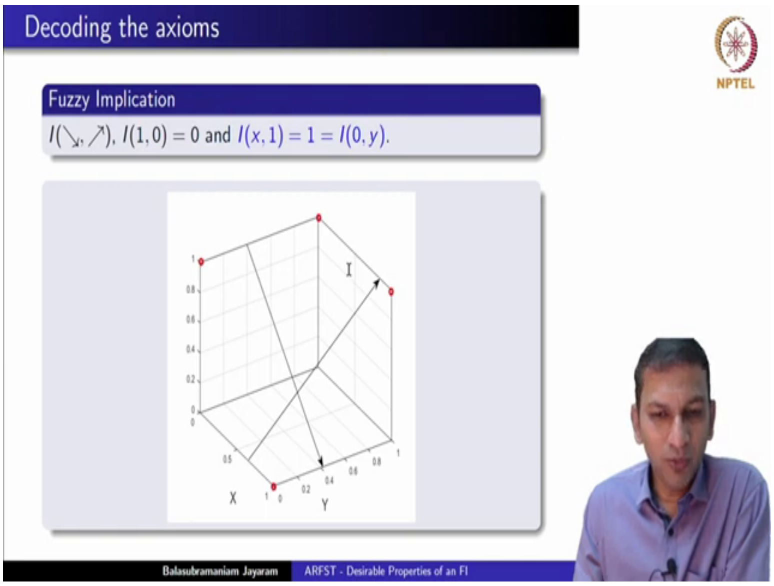

And we have also seen using these boundary conditions, what we get is on the left boundary and the top boundary of this [0,1] unit square we have want that the fuzzy implication should actually take the value 1.

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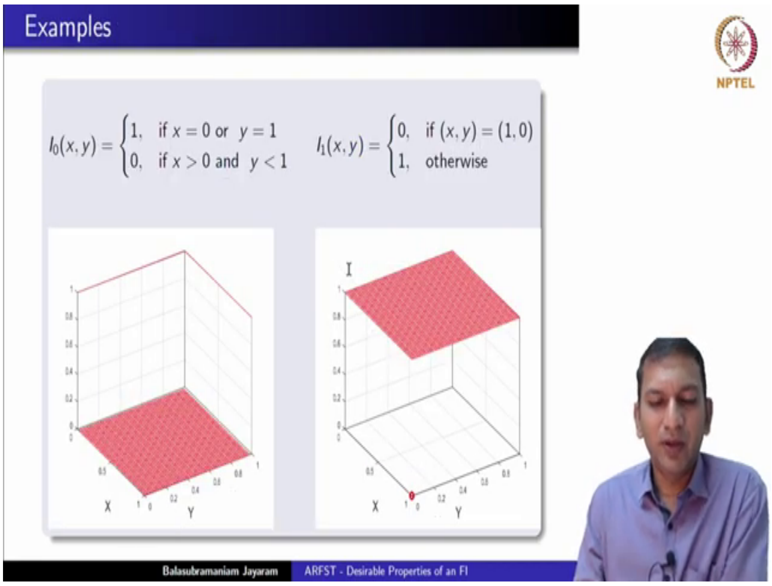

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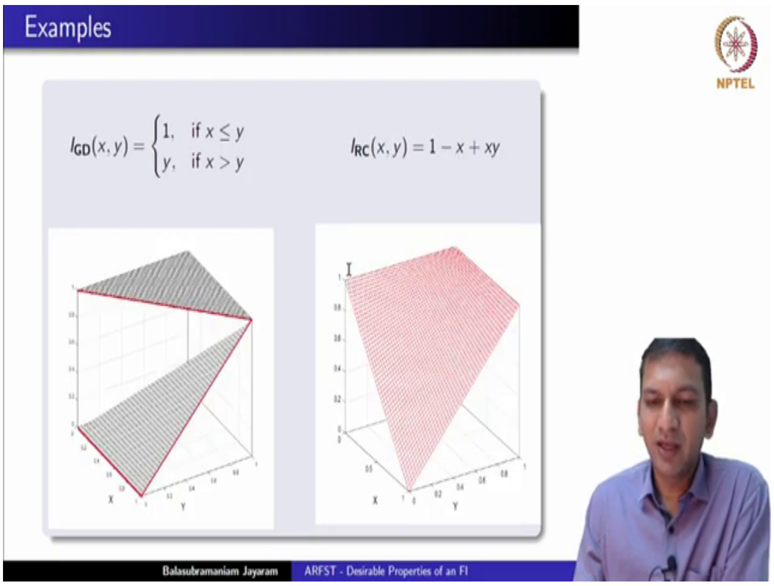

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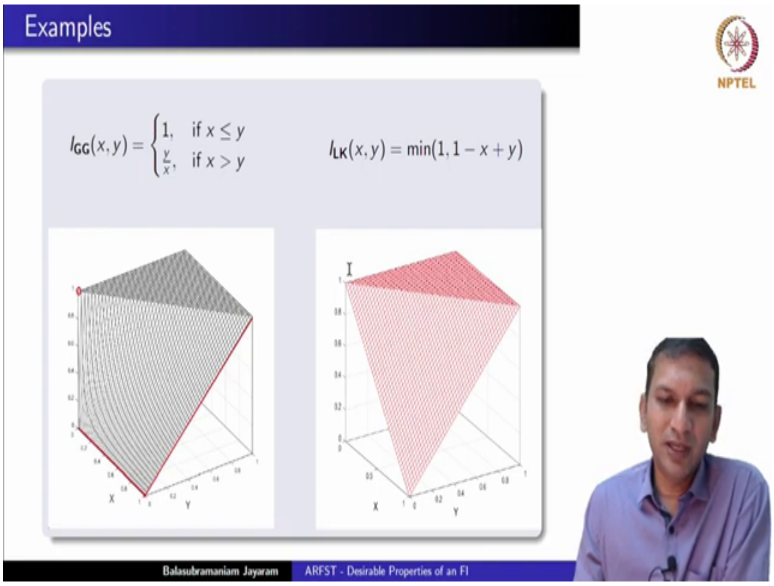

But, now, we have seen a few examples quickly to recall these are the smallest and the largest fuzzy implications and this is the Godel implication on the left and the Reichenbach implication on the right; the Goguen implication on the left and the Lukasiewicz implication on the right.

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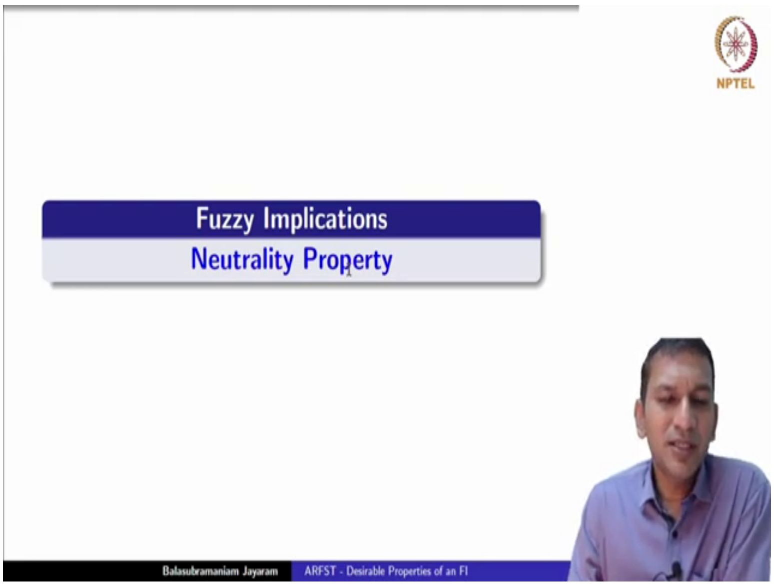

Now, let us look at some desirable properties. In the previous lecture, we have mentioned that in the axiomatic definition of a fuzzy implication we have not asked for any algebraic properties just the boundary condition and some monotonicity properties which you could look at as some order theoretic properties. But, it does not mean that a fuzzy implication is bereft of these algebraic properties. Let us look at a few of them.

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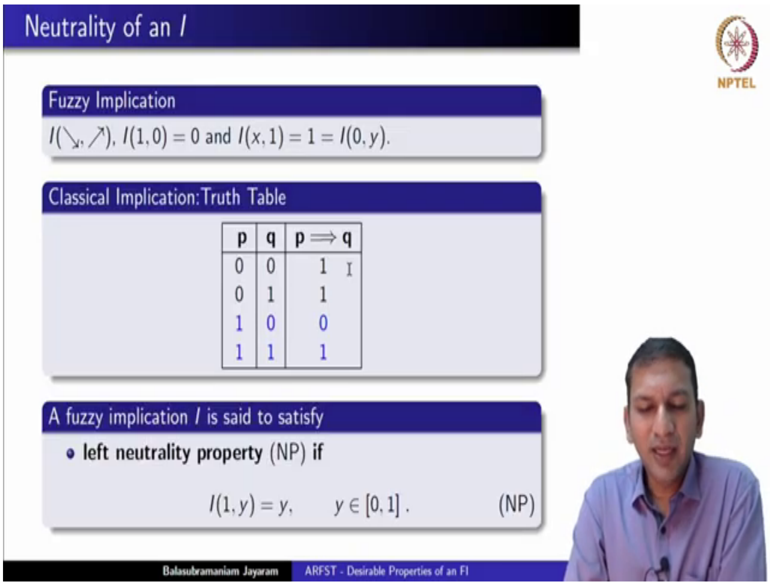

First of them is the neutrality property. Let us look at the classical implication truth table. Now when you look at this truth table it appears that when the first variable takes the value 1 then, the overall truth value of the implication is essentially the value taken by the truth the consequent q, the variable q what happens at the second value. So, it appears that 1 plays the role of a left neutral element.

And this has been captured as the left neutrality property and typically it is also shortened to just the neutrality property. We say fuzzy implication I has this neutrality property if I of 1, y is y for every y in the unit interval 0 1.

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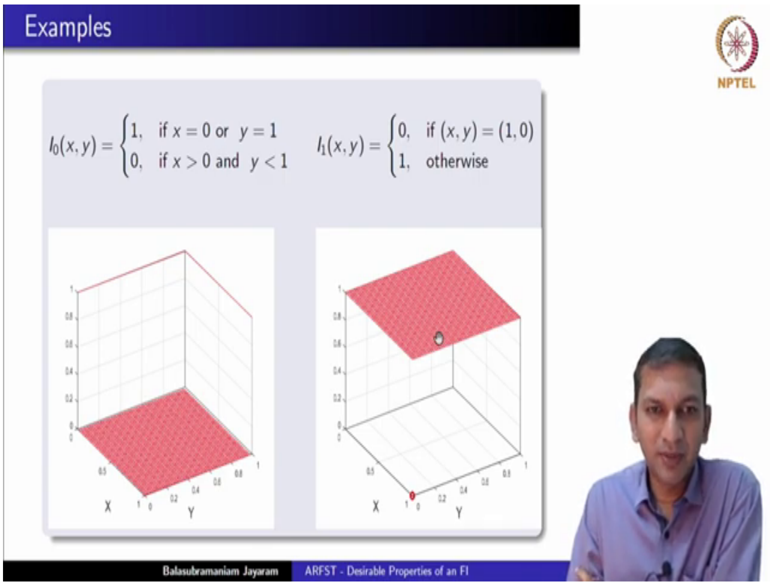

Now, let us look at some of the examples that we have seen including basic fuzzy implications, whether they possess this property. Clearly, when you see here this is the smallest, this is the largest fuzzy implication they do not have this neutrality property. So now, geometrically, visually where do we look for this? So, that is at x is equal to 1 we want that as y increases that I of 1, y should be y.

That means, we are looking at the right boundary. We know that the left boundary is 1 and the top boundary is 1 now we have two more boundaries. So, neutrality property specifies what happens at the right boundary. So, it says that I of 1, y if it is y, then we say that the implication satisfies this neutrality property. We see clearly here that neither of these implications satisfies the neutrality property.

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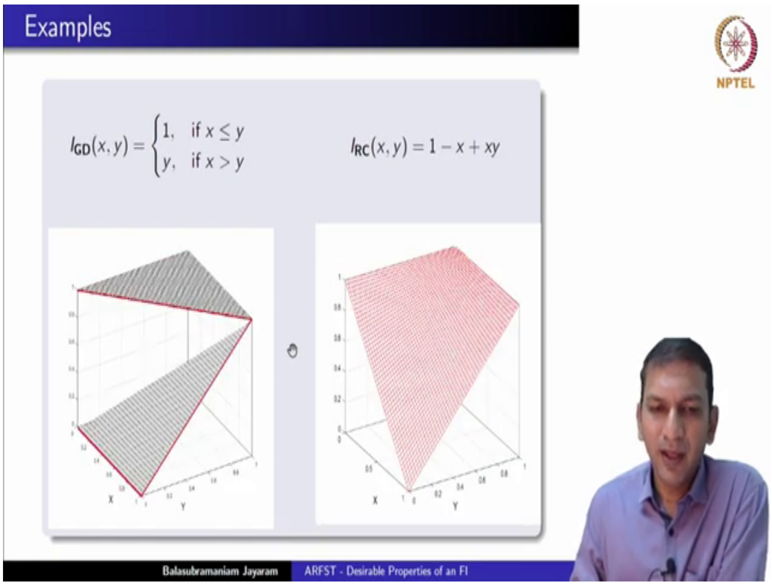

If you take the Godel implication, then we see that at x is equal to 1 it is actually the identity function. That is, I of 1, y is y hence, it satisfies the neutrality property. So, geometrically what we are looking at this boundary the right boundary is that it should take the identity function and we see the same for Reichenbach implication also.

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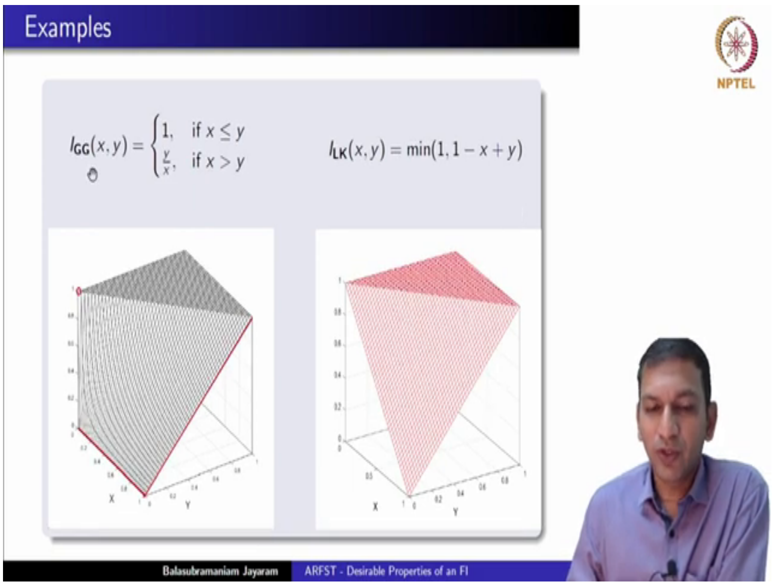

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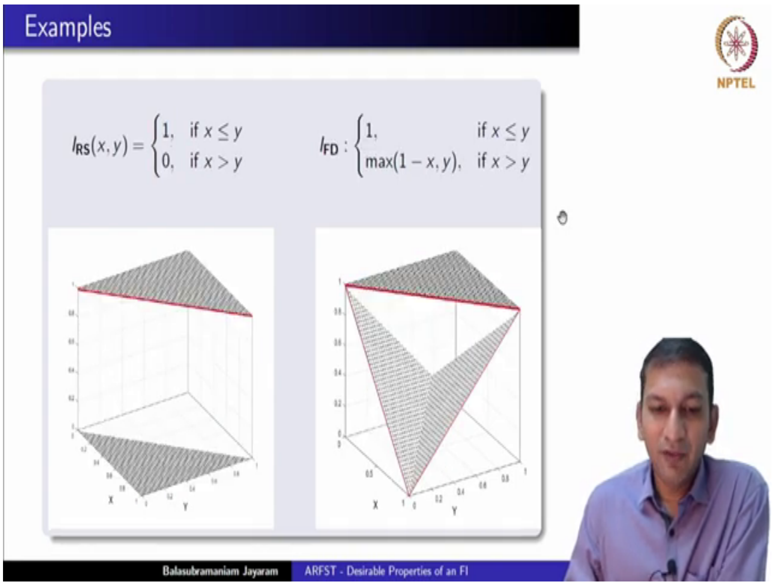

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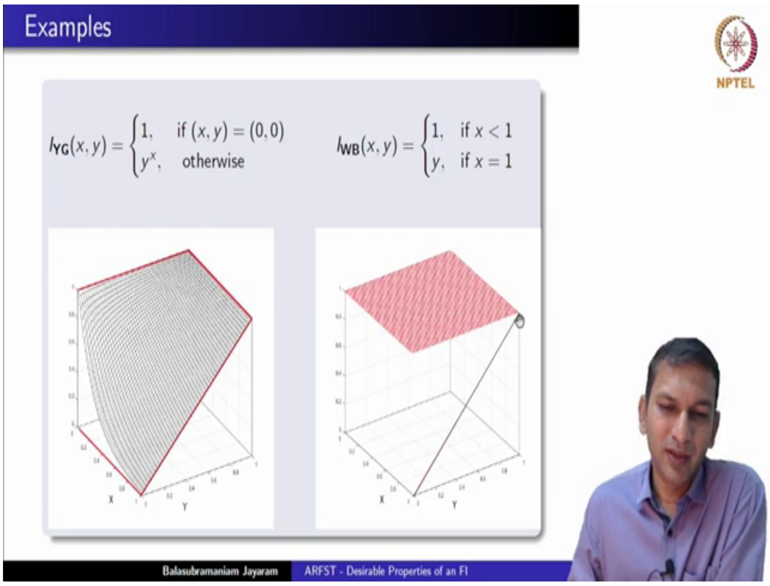

This is true also for the Goguen implication and the Lukasiewicz implication. If you look at the Rescher implication it is no more true, whereas for the Fodor implication again it is true. Now finally, for the Yager and Weber implication for both of them it is actually true. If you recall we got the Weber implication from the largest implication I 1 way to look at it is look at Weber implication is as if it is being obtained from the largest implication I 1 by just redrawing the values at the with the right boundary which is at x is equal to 1.

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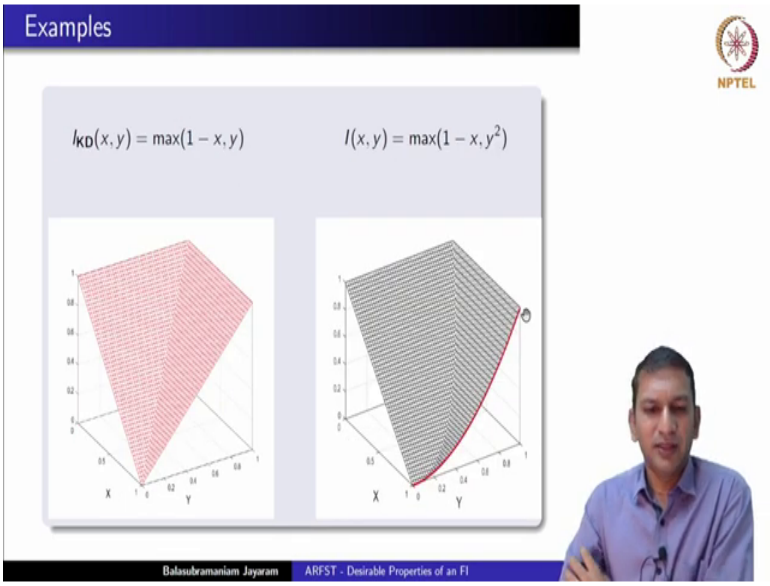

This is the Kleene-Dienes implication clearly it also satisfies neutrality property, but let us not go away with the feeling that we are only going to talk about I of 1, y being equal to y, but there are also lot of implications where I of 1, y could be some function of y, continuous function of y as in this case of I of 1, y being equal to y square.

So, this is again a fuzzy implication you will see just like modification of Kleene-Dienes implications of y we have y square. We will see how this formula this formula has been obtained we will see that this is just a special case of particular way of constructing fuzzy implications in the very next lecture. So, you see here I of 1, y is actually y square. So, this right boundary here it is not the identity function. So, we also have implications of this step not just that here it is 0 or some constant function. So, this is what is a neutrality property.

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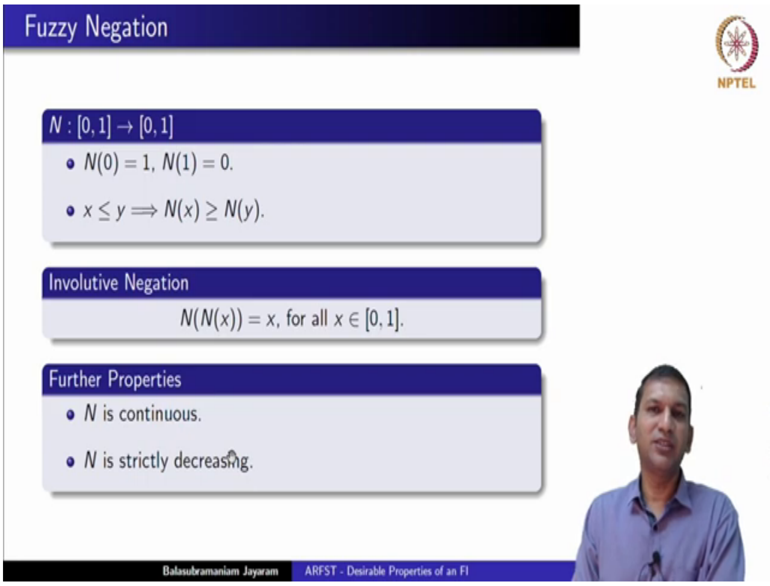

Now, let us begin by recalling what a fuzzy negation is we know it is a unary function from 0 1 to 0 1, such that N of 0 is 1 and N of 1 is 0 and it is decreasing function. We could have different classes of fuzzy negations the involutive ones, which we have also seen earlier N of N of x is x. It could be continuous, it could be strictly decreasing, it could be continuous, but not strictly decreasing, could be strictly decreasing but not continuous, we have seen examples of all these kinds. Now, why talk about fuzzy negation here.

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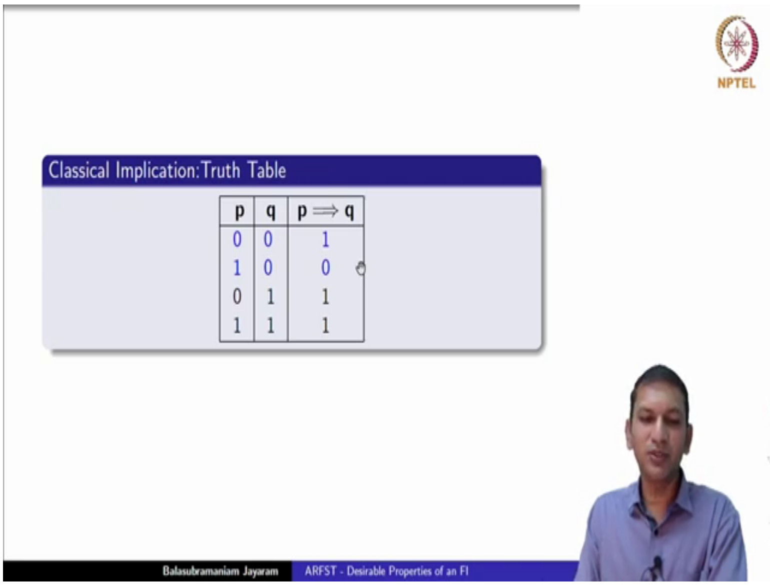

Let us look at the truth table of the classical implication. When you fix the second variable to be 0, it appears the overall truth value of the conditional p implies q is nothing but the inverted truth value of that of p. So, when q is 0 the implication acts as if it is an inverter. So, 0 implies 0 is 1 and 1 implies 0 is 0. Taking q from this, we could look at what happens to implications when you fix the second variable to be 0 that is when y is equal to 0.

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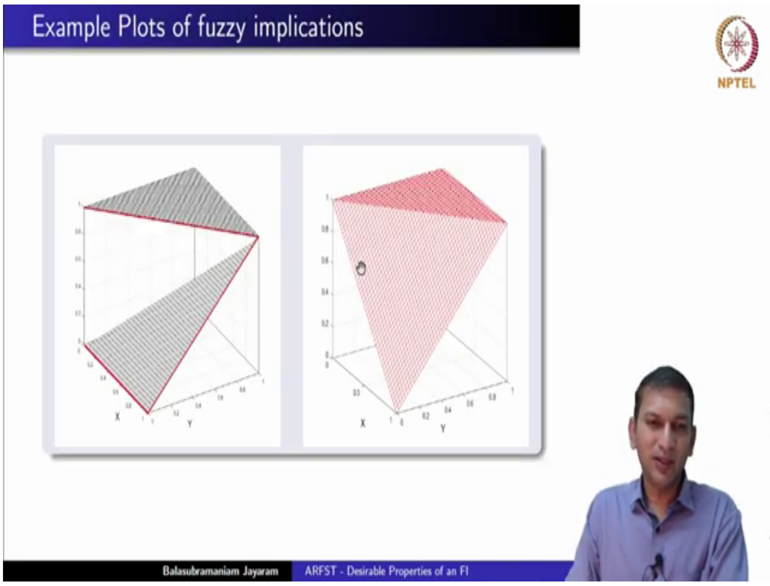

Now, let us look at the plots of some fuzzy implications, this is the Godel implication. Now, when you take when you look at y is equal to 0 that is essentially the bottom boundary you see that at 0 it is 1 and the rest of the places it is 0. And in the case of Lukasiewicz implication which is on the right you see that there is actually a continuous graph here linearly falling from 1 to 0.

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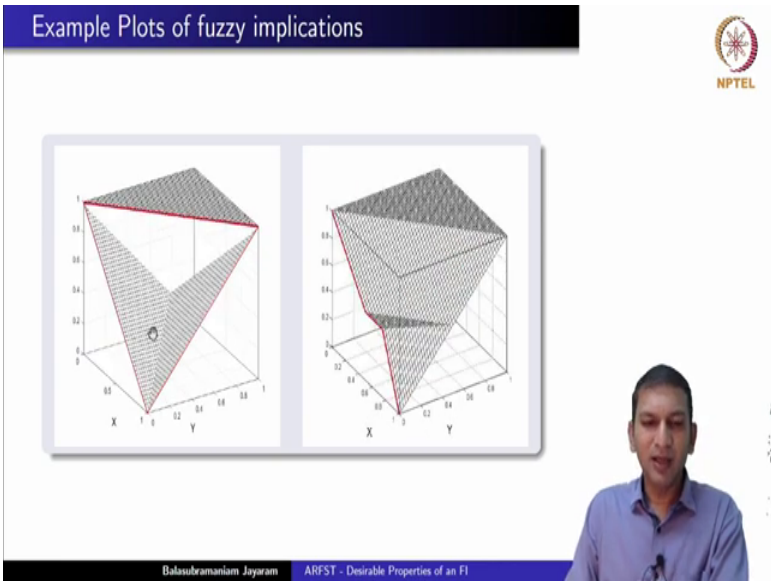

In the case of Fodor implication once again when you look at the bottom boundary it is a continuous function and this is a function which is essentially a modified form of Lukasiewicz implication and here again you see that, in the bottom boundary, the graph of the implication is continuous but of course, clearly it is not strictly decreasing. But, we always see that these are decreasing functions.

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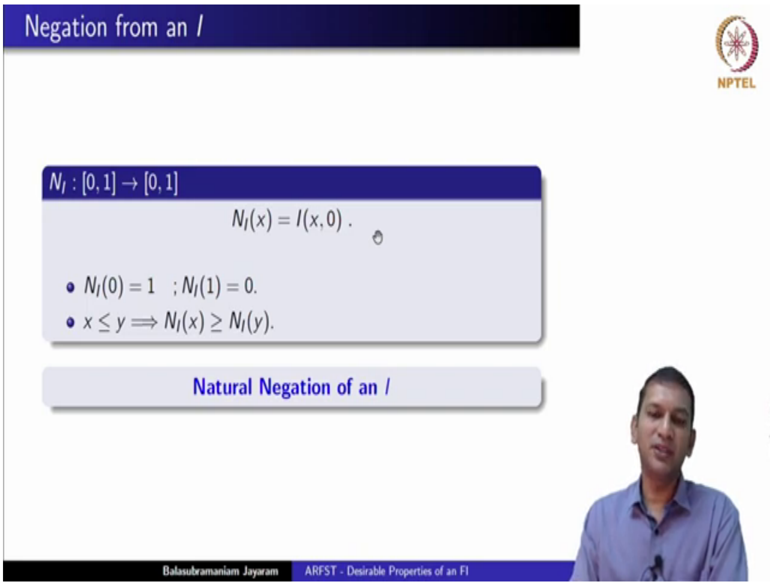

Taking q from here, we could define given an implication an another auxiliary function an like this N_I of x is I of x, 0. So, essentially this is a partial function where we fix y to be equal to 0. Now, clearly from the definition of fuzzy implication it is self we can see that N_I of 0 is 1 and N_I of 1 is 0 and similarly it is decreasing function. This function which is a negation is called the natural negation of an implication I.

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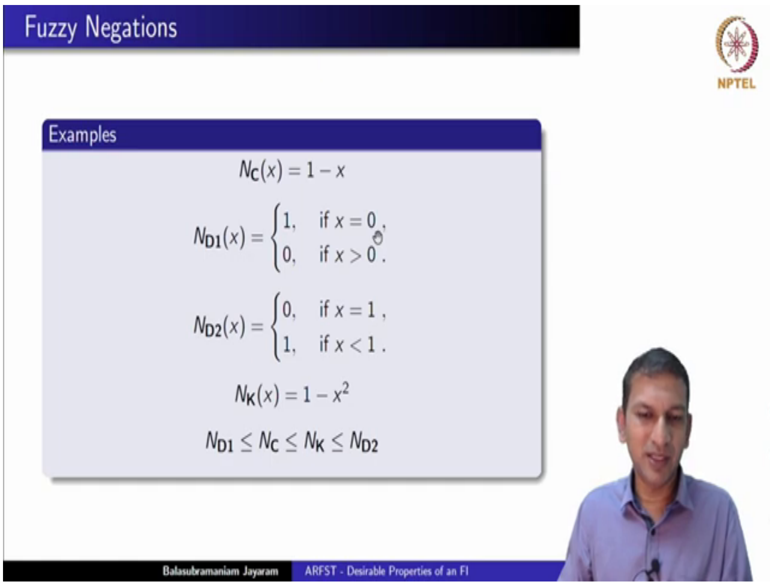

Let us look at some examples of fuzzy negations. So, this is the classical negation 1 minus x which we denote by N_C and if you look at this is essentially the smallest negation that you could have for example, if you look at this formula what it says is when you look a 0, 1.

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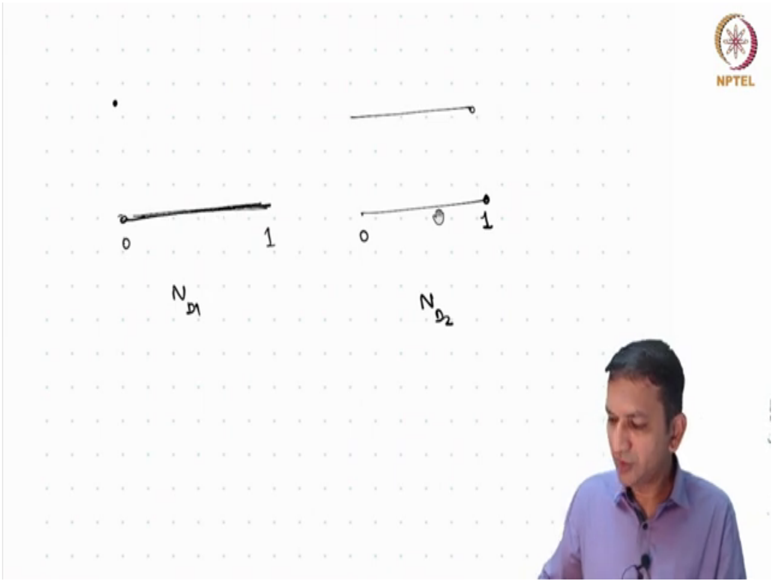

So, then at 0 it is 1 and everywhere else it is 0. So, this is your smallest negation and if you look at the next one N D 2 this is the largest negation on 0 1 at 1 it is 0 everywhere else it is 1. So this is 0 here, it is 1 here it is 0 and everywhere else it is 0. So, this is N D 2 which is the largest fuzzy implication and we could also have all kinds of other fuzzy negation, but let us just look at N K of x which is 1 minus x square. Clearly we have this ordering here 1 minus x is smaller than 1 minus x square.

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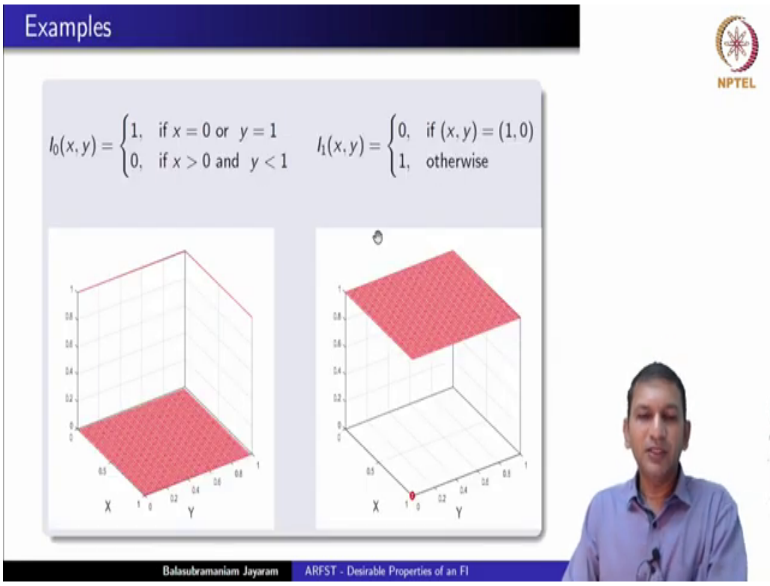

Now, let us look at the plots of this basic fuzzy implication that we have. And see, what is the kind of natural negation that they have. Clearly for the smallest negation at 0 it is 1 everywhere else it is 0 which means, the corresponding natural negation is actually N D 1 and in the case of the largest fuzzy implication everywhere it is 1 for the natural negation except at 1 which is 0.

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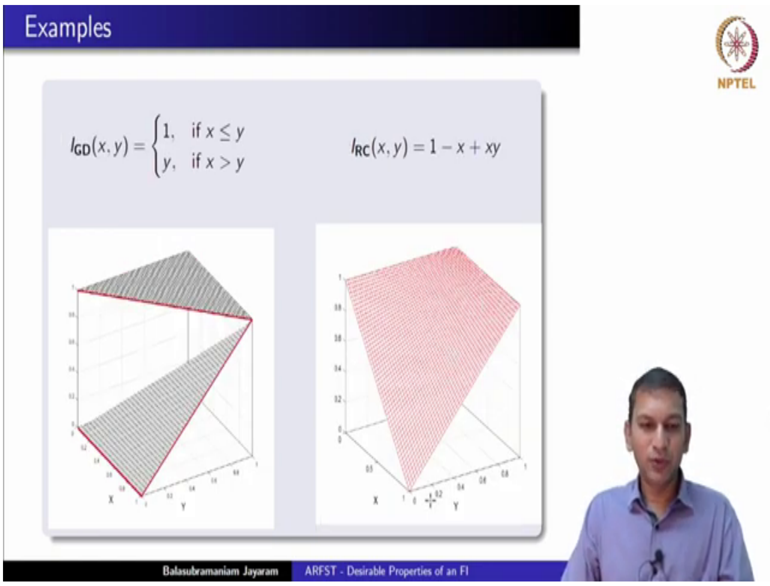

So, this is the largest fuzzy implication. And for the Godel negation as you can see the bottom boundary is almost right here at 0 and only at 0 it is 1. So, it gives you the smallest negation often this is also called the Godel negation because of because it is being obtained as natural negation of Godel implication which as we will see later on is a very important implication fuzzy implication.

In the case of Reichenbach implication, what we see is actually this is the classical negation 1- x. This can also be seen because what we are looking at is fixing y is equal to 0 when you put y is equal to 0 it is essentially 1- x.

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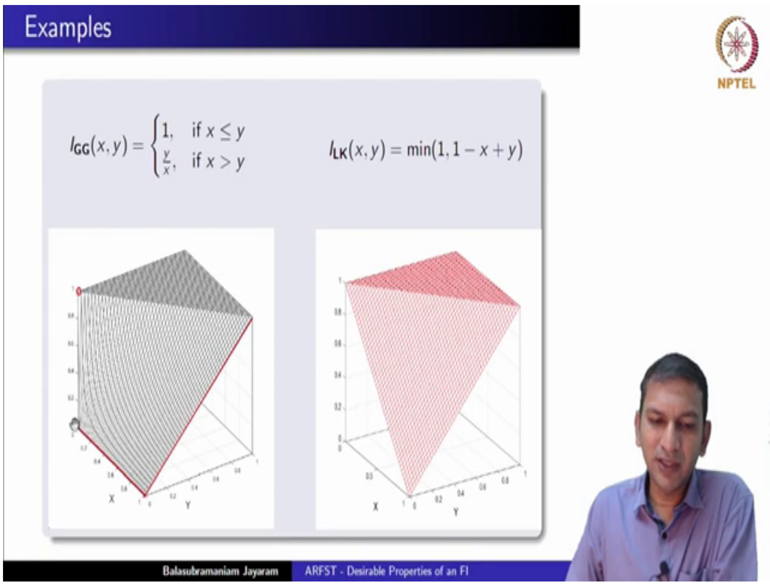

Similarly, for the Goguen implication we get the natural negation to be the smallest one and for Lukasiewicz implication it turns out to be 1 minus x.

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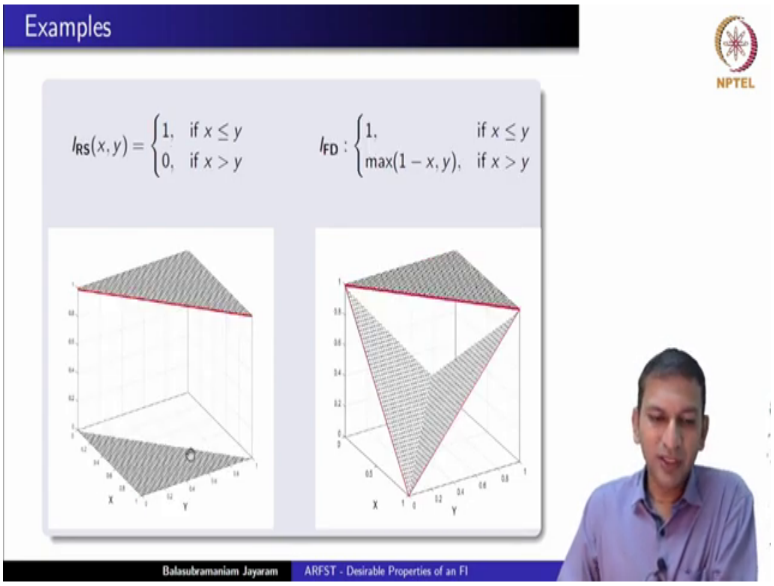

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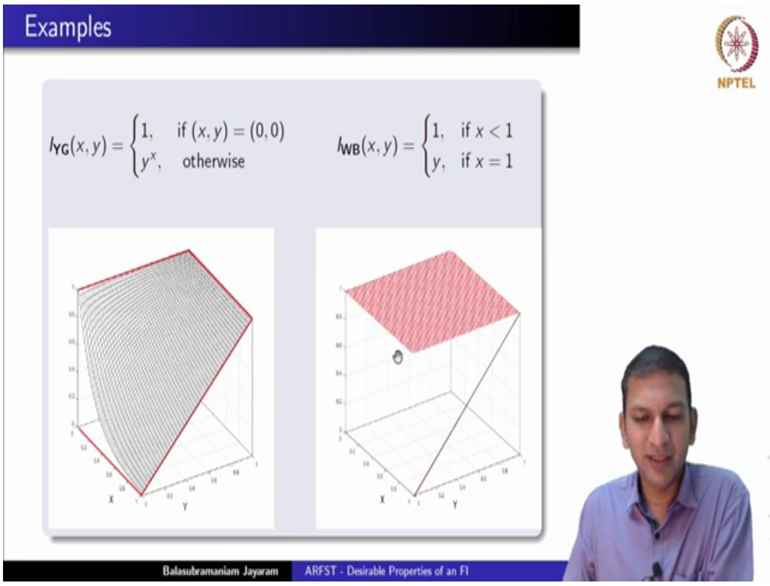

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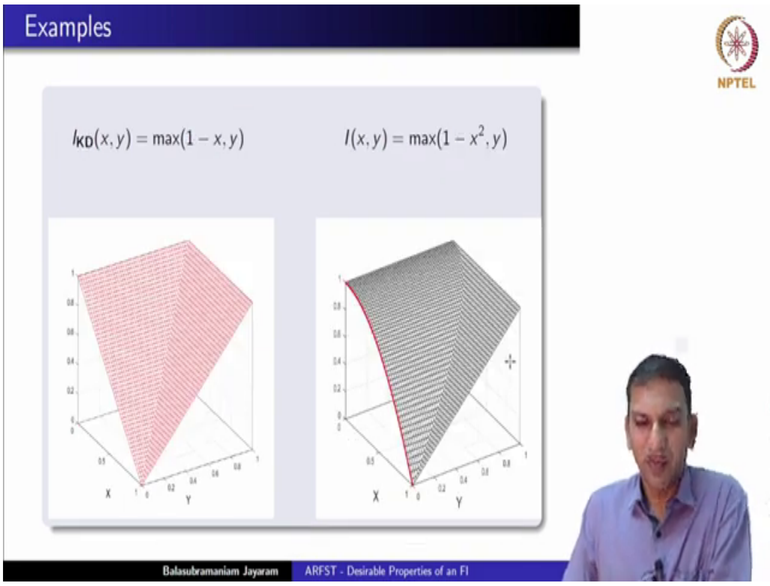

For the Rescher, once again it is the smallest negation fuzzy implication and for the Fodor it is 1- x. Yager it is again the smallest negation and for Weber it is the largest negation. This is obvious because we have seen that Weber can be obtained from the largest fuzzy implications just by redrawing the right boundary. As we have seen, we do not even we do not always need to have one of these constant negation or the smallest or the largest of 1-x we could also have the n t of which is $1 – x^2$ negation, ok.

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So, far we have seen what happens on the left of top boundary which is all the fuzzy implications always 1, on the right boundary we insisted that it should be identity function which means the implication has neutrality property; and at the bottom boundary we looked at it and saw that it was a decreasing function because as x increases it decreases and we saw that it is actually giving us a negation here.

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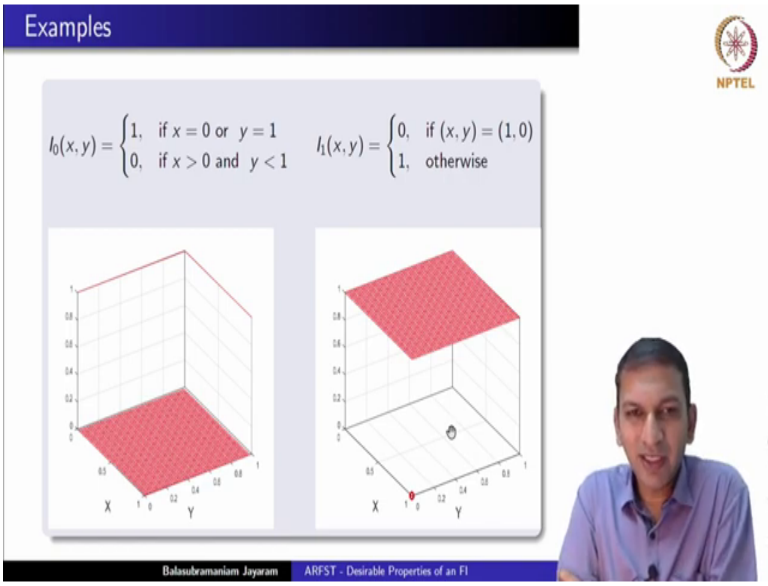

We could also concentrate on the plots and see where it actually attains the maximum value which is that of 1. So, for the smallest fuzzy implication we know that it attains the maximum value 1 only at the left and top boundary. Whereas, for the largest one it is almost everywhere it is 1 except at the point 1, 0.

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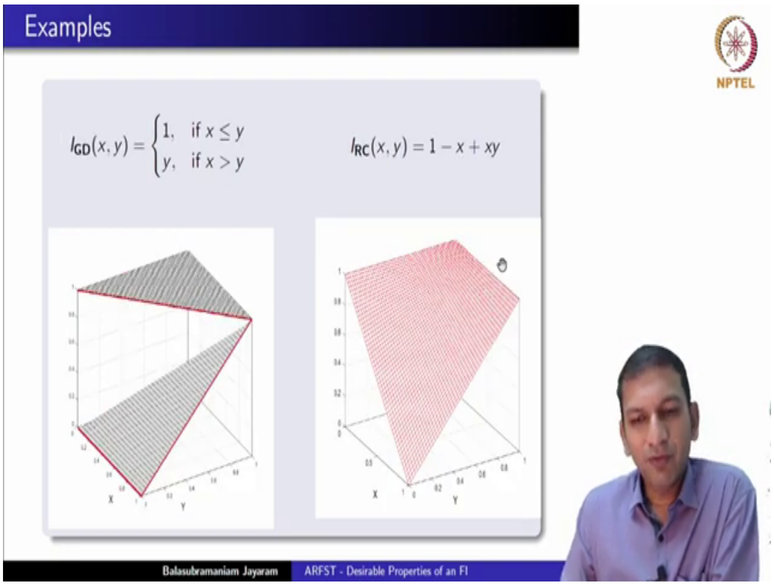

But now, when you start looking at some of the other implications let us restrict ourselves only to the basic fuzzy implication. So, an interesting pattern appears. For instance, it is 1 only above this main diagonal this is x is equal to y and not anywhere else. Whereas, this again it is 1 for the Reichenbach it is 1 only on the left hand top boundaries nowhere else. Wherever it is essentially required to be 1 only there it is 1and nowhere else.

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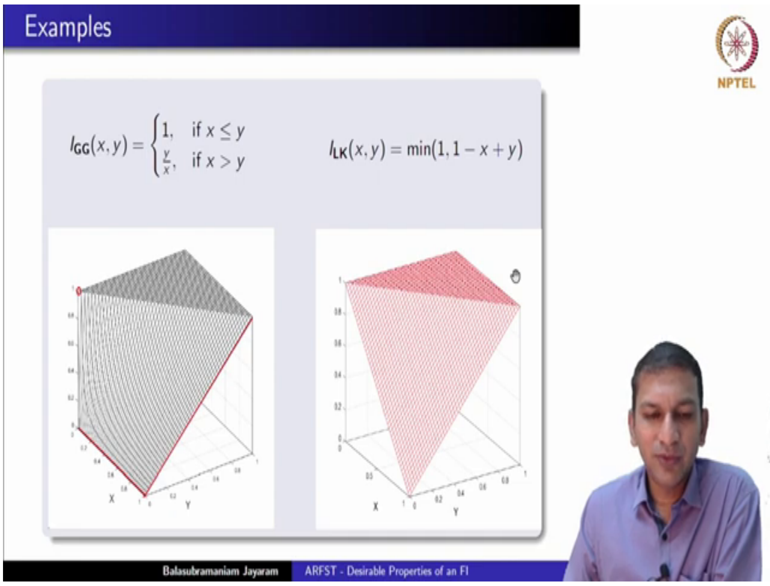

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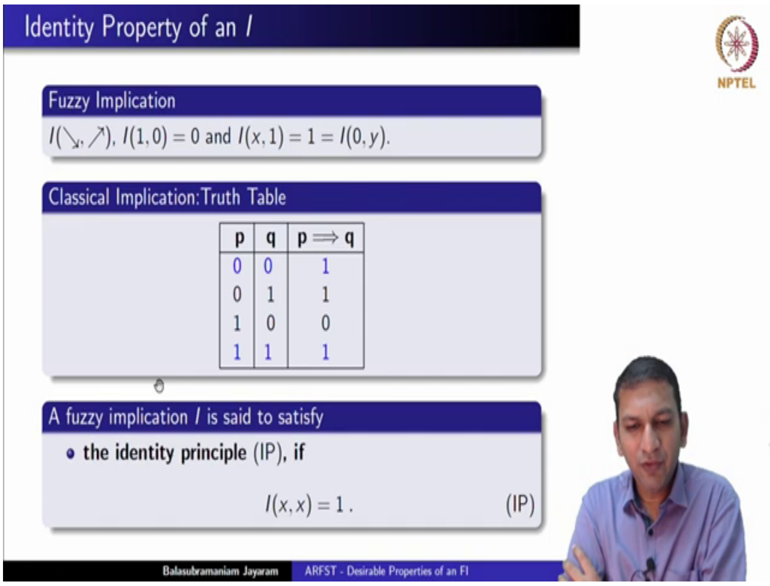

When it comes to Goguen and Lukasiewicz once again above the diagonal we have 1 but, outside of it you know that it is not taking the value 1. If you actually look at this, it gives us a nice interesting idea for this let us go back to the classical implication truth table. You see here, when p is actually equal to q then, the overall truth value is actually 1. We could abstract this as a nice property where we say that I (x,x) is 1.

Remember, in the case of t-norms we talked about idempotence where T(x, x) is x whereas here, going by the examples that we have and also from the classical implication truth table. We see that, a good way perhaps a natural way of abstracting this property is as I(x, x) is 1 and this is what is called the identity principle.

So, essentially we are seeing x implies x should be always true more like a topology that is what we are calling classical logic. But, we also have something more. We see that here, when we have that when p is less than or equal to q once again p implies q is 1 and this is abstracted as the ordering property where, whenever x is less than or equal to y then I of x y is 1 not one way actually both ways it is an if and only if condition.

We say x is less than or equal to y if and only if I of x y is equal to 1. Now, this is this ordering property is extremely important especially in formal logic terms in proving results, perhaps we will touch upon this likely in this setting when we look at conditionals or implications as being related to the sub set and classical set theory sorry set theoretic terms and at that point of time we will make a mention about this property how it could also be seen or interpreted.

For the moment let us abstract it from both the truth table and also from the geometry of fuzzy implication that we have seen. So, an implication is said to have the identity property I(x, x) is 1 and it is said to have the ordering property, if whenever x less than or equal to y then I(x , y) is 1 and on converse I(x, y) is 1; if only if x is less than or equal to y.

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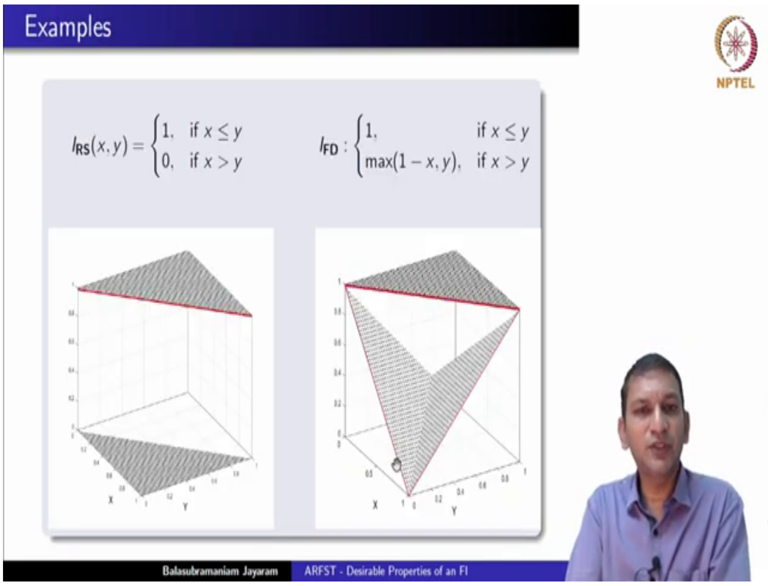

Let us look at some basic fuzzy implications see whether they satisfy this or in principle. As you can see in the geometry Rescher implication satisfies this ordering property and the moment you see that ordering property which is OP I mean abbreviated as OP. If it satisfies OP, then it also satisfies IP which is the identity principle because, at this point when x is equal to y ordering property reduces to that of the identity principle. So, OP implies IP, but clearly IP does not imply OP we will see some implications which actually illustrate this.

For the moment these two fuzzy implications on the screen which are the Rescher and the Fodor implication both of them have the ordering property.

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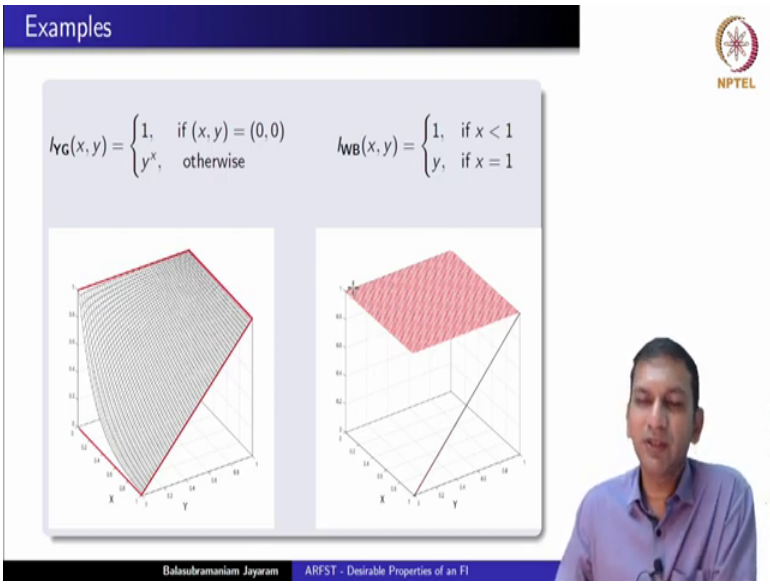

Yager clearly does not have the ordering property, Weber also does not have the ordering property, but Weber has the identity principle; that means, at x is equal to y it is actually equal to 1, it does not have the ordering property because, outside of the diagonal where x is greater than y even there it takes the value 1.

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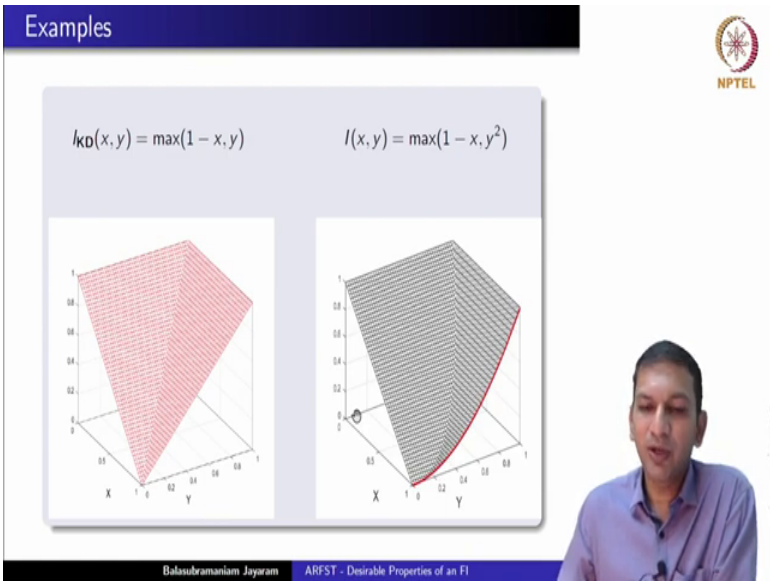

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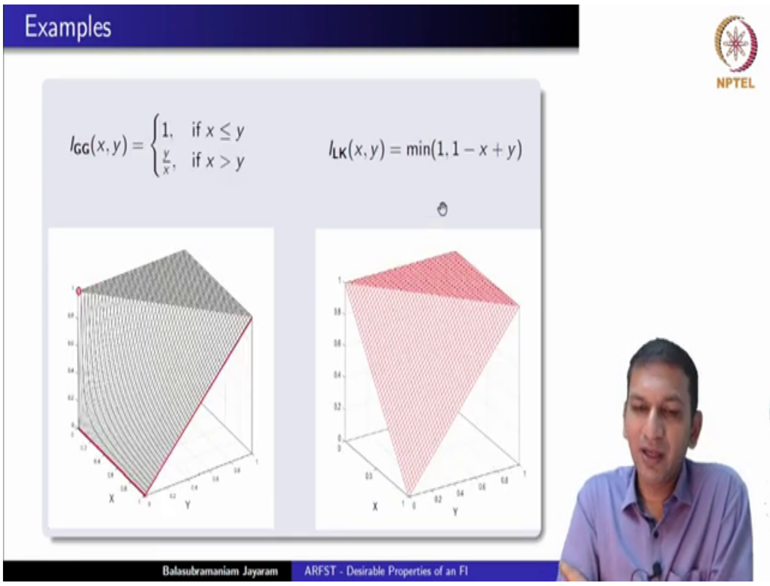

Kleene-Dienes does not have the ordering property. So, does this particular implication. Goguen implication has the ordering property and so does the Lukasiewicz implication. When we introduce Lukasiewicz implication is perhaps the first implication first fuzzy implication in the previous lecture, we have shown two equivalent ways of writing it. Now, that should become clear from the geometry here.

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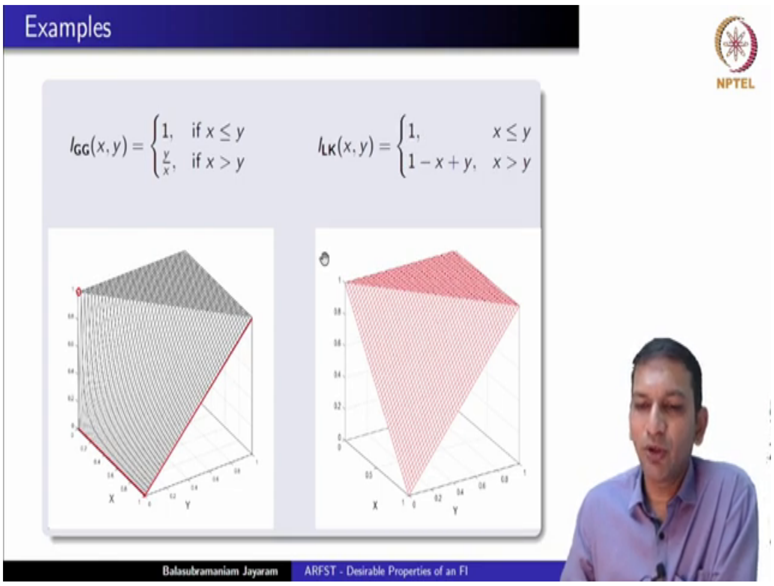

So, you could write this fuzzy implication also this way. When x is less than or equal to y it is 1 this region and outside of it is 1 minus x plus y, ok.

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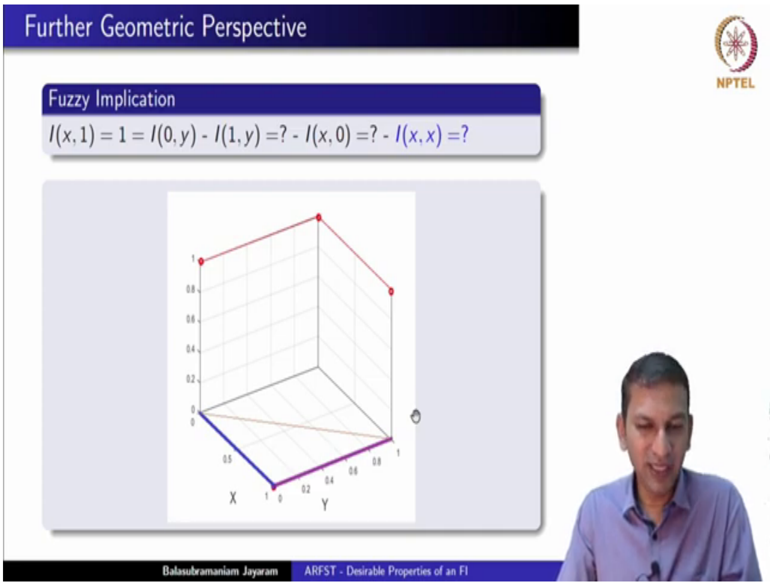

So, what is interesting is, let us do a small recap here. Look at this. We got this these two properties from the boundary conditions and that of monotonocity that is insistent on a fuzzy implication. It told us how does any fuzzy implication will look like on the left and the right boundaries. They have to essentially be 1, this is mandatory. Now, what we have seen now is, what could it look like on the right boundary and what is desirable. So that means, on this line how should it look like.

Of course, it would look like I of 1, y is y the identity function it would look like 1, y is y square, but you wanted to investigate how would it look like on the right one. If it were the identity function then, we see that it has some nice properties where fuzzy implication has nice properties contextually where you want to use and that is what we are abstracted as a neutrality property. And similarly, if you are considering the bottom boundary, then we saw that it actually leads to a negation a fuzzy negation; once again it could be 1 minus x or some such graph or even a discontinuous function.

Finally, what we have seen is, other than these boundaries we have also seen the ordering property the identity principle it is related to what happens above this main diagonal which is x is equal to y. From purely a geometric perspective, this is what we have seen and abstracted and these can also be looked at as some algebraic property because, left neutrality and the right it is not an annihilator but you can look at it as an inverter and what happens when both the arguments are same.

In the case of t norms as we said, we insist a idempotence x star x is x, whereas here we are seeing that x implies x should be equal to 1.

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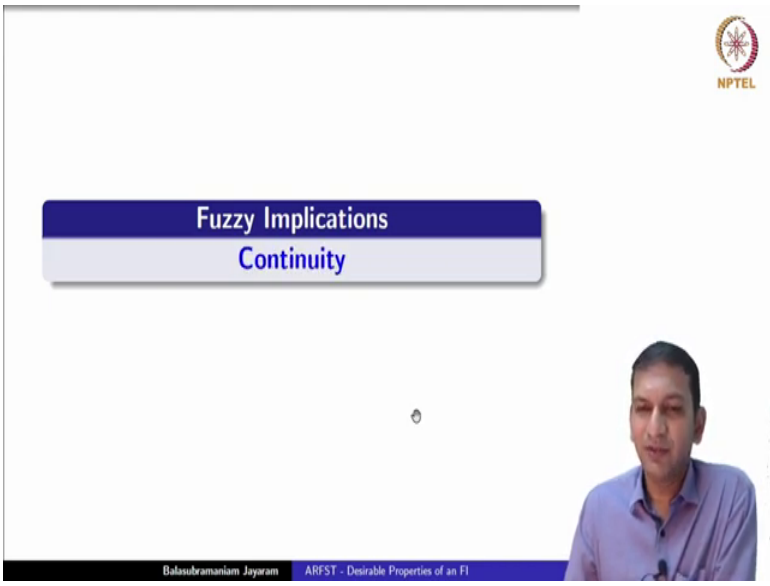

A few further desirable properties, that we expect from fuzzy implications are these- first of them is continuity. Clearly when you look at this Lukasiewicz implication is continuous, Goguen is almost continuous, except at the point 0 0 where it is 1, rest of the places it is continuous, Rescher is not continuous, Fodor is not continuous, Yager definitely is not continuous, so is Weber.

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Once again we had said that in the axiomatic definition we did not insist on many algebraic properties, but we have already seen a few. It is clear that we cannot have symmetry, because of mixed monotonicity it is decreasing in the first variable and increasing in the second variable, but what about associativity.

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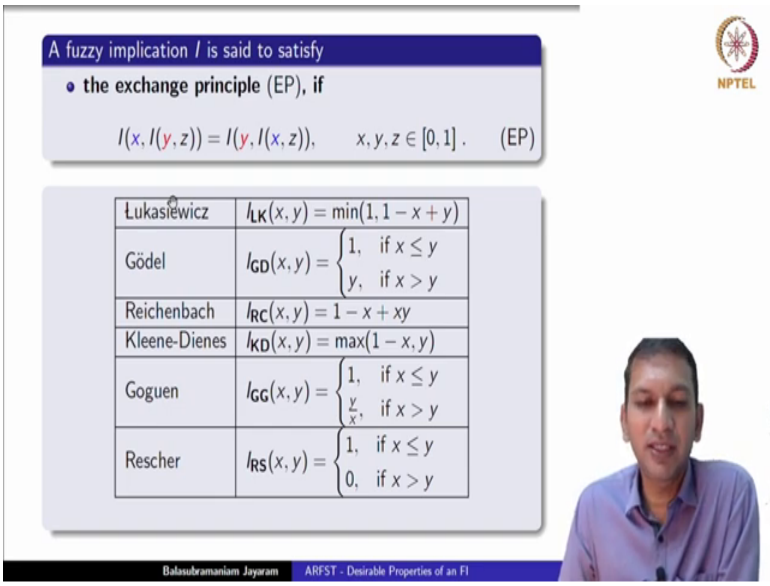

Well, this exchange principle is some kind of an associativity. In fact, it almost plays the role of associativity in this in the in when you consider it in terms of implications. And why do we make this comment that would become clear as we go forward and look at some algebraic properties that fuzzy implications can have and also in an application of context.

So, what is the exchange principle? We say a fuzzy implication satisfies the exchange principle, if it satisfies this equation for any x y z coming from 0 1. So, I of x, I y z should be equal to I of y, I of x z. So, why is it called the exchange principle? Obvious, we are able to exchange these two arguments. So, keeping z fixed if you can swap x and y exchange them we call it the exchange principle.

Now, just like Lukasiewicz associativity it is little difficult to give a geometric idea of it a perspective of what an exchange principle is. So, we take recourse to actually taking the formula and checking it for instance.

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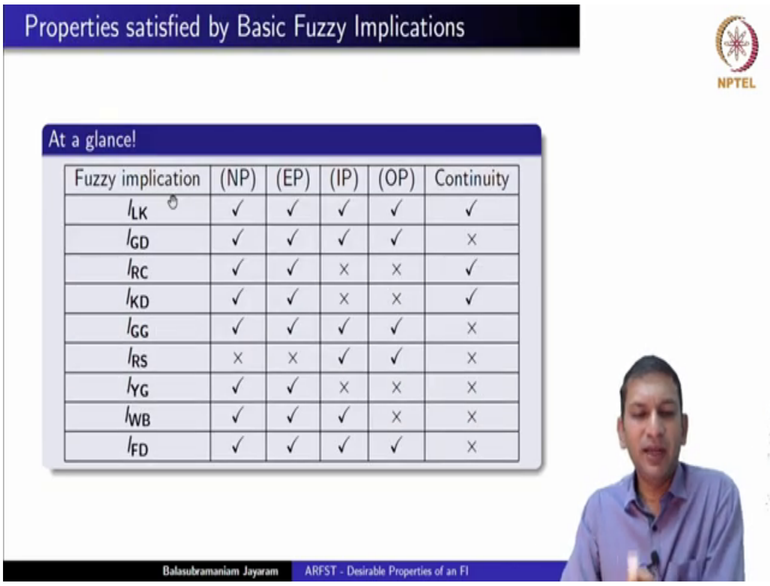

If you look at this is these six basic fuzzy implications. Except for Rescher implication it can be shown that all the others do satisfy the exchange principle. Now, if you look at entire nine basic fuzzy implications and the properties that we have discussed so far. The neutrality property, exchange principle, identity principle, ordering property and also continuity this is how it looks like. So, if you look at exchange principle. Almost all the basic fuzzy implications do satisfy them, but there is not one property that all of them satisfy.

So, neutrality is not satisfied by Rescher, exchange principle not satisfied by Rescher for identity principle there are many fuzzy implications that do not satisfy. So, is the case if it does not satisfy IP where you cannot satisfy OP also and there are also very few basic fuzzy implications which are in fact continuous. So, you see here non continuous fuzzy implications, we have somehow present them as basic fuzzy implication which tells you that in spite of not being continuous they play a vital role when it comes to looking at using fuzzy implications in a particular context.

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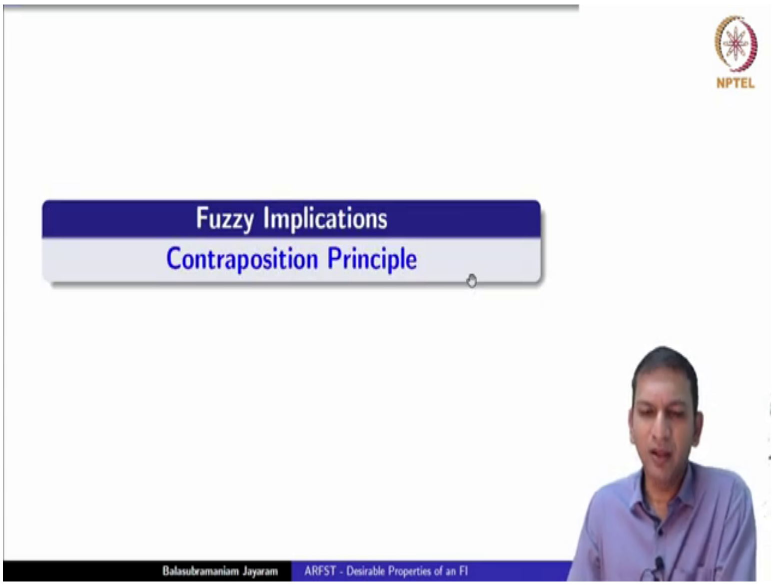

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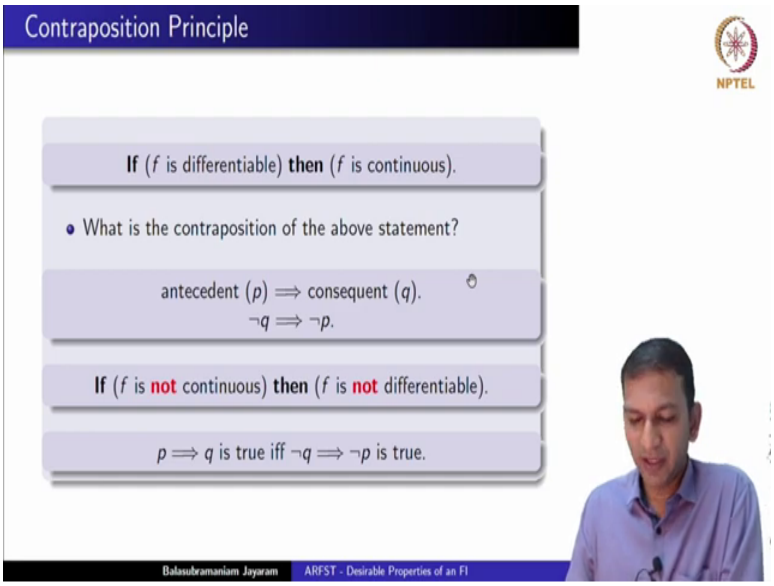

And let us talk about the contraposition principle. Once again let us go back to this conditional statement. If f is differentiable then f is continuous. If I can ask the question what is the contraposition of the above statement. So, we know that you can look at it as p implies q. So, the contraposition is nothing but negation q implies negation p.

So in this particular context, the contrapositive statement of the above conditional statement is nothing but if f is not continuous then f is not differentiable. So, essentially we say that p implies q is true if and only if negation q implies negation p is true. So, this is how you could equivalently express it in the case of classical implication. And now this is what we want to carry to that of fuzzy implication.

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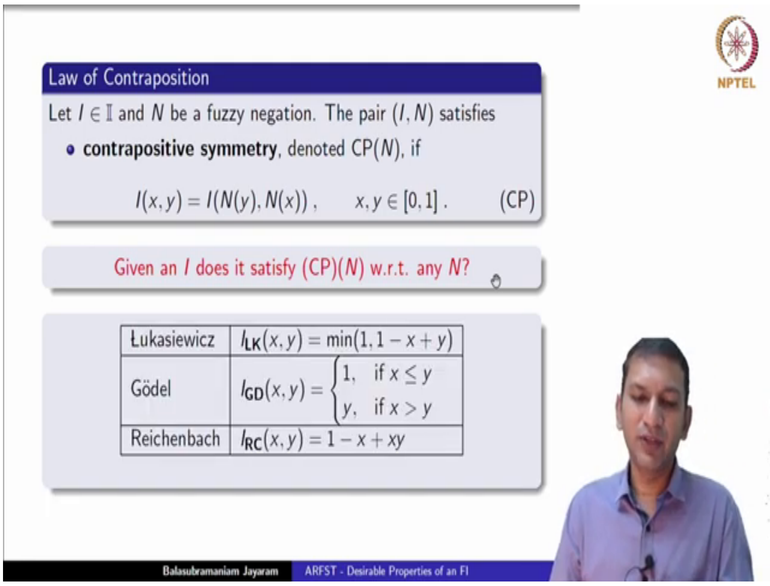

How does that how does this translate in the case of fuzzy implication. So, we start with an implication N and then fuzzy negation N and we say that it satisfies the contra positive symmetry with respect to a negation N we will denote it as CP of N. If this equality is satisfied clearly I of x y is equal to I of N y, N x.

Now the question is, given an implication how do we know whether it satisfies contrapositive symmetry with respect to any negation some negation for instance. Let us consider these three fuzzy implications.

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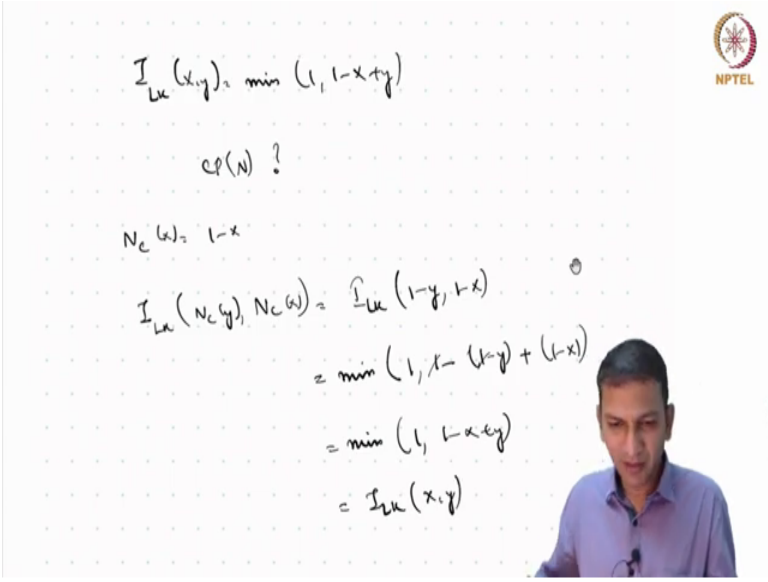

We can ask the question, does the Lukasiewicz implication this formula looks like this. Does it satisfy contrapositive symmetry with respect to sum N.? Let us consider the N to be 1 minus x. So, what we want is let us look at what is I L K of N c of y, N C of x is nothing but I L K of 1 minus y, 1 minus x.

From the other formula we see that this can written as minimum of 1, 1 minus 1 minus y plus 1 minus x this will turn out to be minimum of 1, 1 minus x plus y which is nothing but I L K of x, y. So now we see that, the Lukasiewicz implication does in fact satisfy contrapositive symmetry with respect to the classical negation 1 minus x. But the question is, how do we know given a fuzzy implication whether it satisfies the CP with respect to any element of for instance. Let us do the same with the Godel implication.

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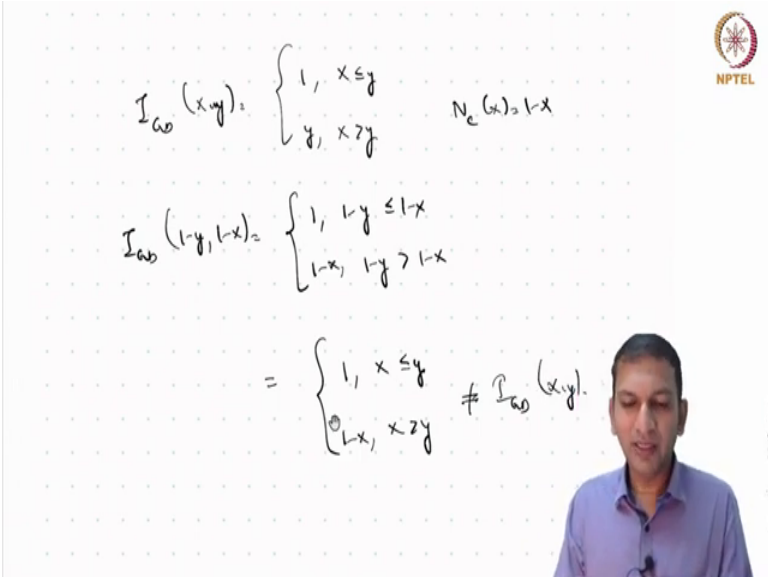

So, the Godel implication is 1 if x is less than or equal to y and y if x is greater than y. So, let us take the same 1 minus x consider it is 1 minus y 1 minus x this is 1 if 1 minus y is less than or equal to 1 minus x. And the second argument, if 1 minus y is greater than 1 minus x, clearly it could be written as 1 if x is less than or equal to y and 1 minus x if x greater than y. So, we see that, this is not equal to I GD of x, y.

So, what we have shown now is that Godel implication does not satisfy contrapositive symmetry with respect to the classical negation 1 minus x, but perhaps there is some other negation with respect to which it does satisfy the contrapositive symmetry. Can we say no there does not exist any negation with respect to which it satisfies contrapositive symmetry or if it says there does exist how do we say that and if there does exist a negation with respect to which it satisfies CP, how do we find that out.

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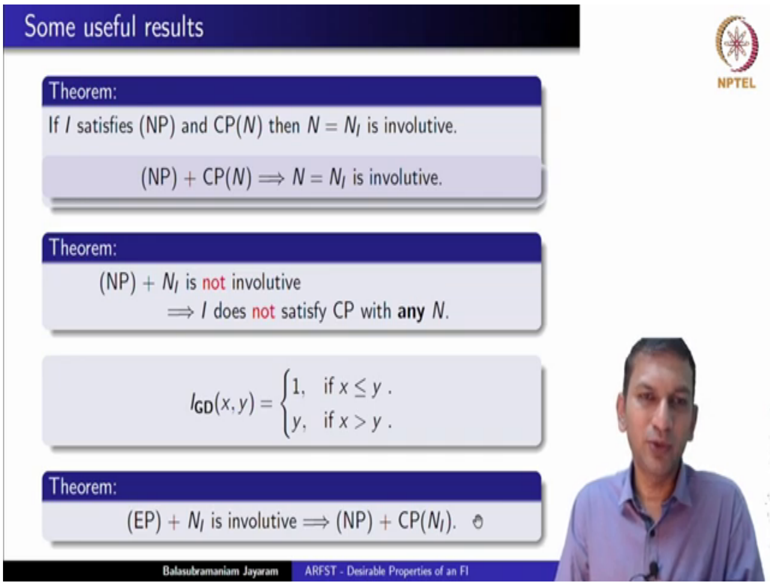

Let us look at some useful results. There is a result, which says that if an implication satisfies neutrality property and contrapositive symmetry with respect to some N, then that N has to be actually the natural negation obtained from it and it has to be involutive. So, this is the result we will not try to prove it, but I will give you a reference where you can find the corresponding proof. For the moment just to save space and to highlight the importance the important aspects of the result.

Let us decode this result or rewrite this result in this form if an I has NP plus CP of N that is neutrality property and contrapositive symmetry with respect to sum N. We see that, it means it implies that N has to be equal to it is natural negation and it has to be involutive, ok. Another result says that, if an implication has neutrality property and the and it is corresponding natural negation is not involutive, then we have a much stronger result which says that I does not satisfy contrapositive symmetry with respect to any negation N.

Now, if you take the Godel implication, we know that it satisfies neutrality property when x is equal to 1 that is y. However, how what happens to it is natural negation. We know it is natural negation is in fact the least negation that is it takes the value 1 and 0 and everywhere else it is 0. Which is essentially not an involutive negation N of N of x is not equal to x.

It immediately tells you that the Godel implication will not satisfy contrapositive symmetry with respect to any fuzzy negation that is the power of this result, ok. So, this a nice result, but in some sense it has a negative connotation. It tells you when an I cannot or does not have contrapositive symmetry with respect to any negation. Is there more like a positive reason, yes there is there are many, but let me give you just one.

If a fuzzy implication satisfies this EP which is the exchange principle and if the corresponding natural negation, the natural negation that you obtained from if it is involutive then what we do know is it will also satisfy NP, but most importantly in this context we can say that it satisfies contrapositive symmetry with respect to it is own natural negation. So, that is quite interesting and that is how we found that the Lukasiewicz implication which satisfies exchange principle.

If you look at it is natural negation is in fact, the usual negation 1 minus x and which is involutive and we see that immediately it that it will satisfy contra positive symmetry with respect to it is natural negation which is 1 minus x. And that is how we actually picked up that negation 1 minus x to illustrate this fact with the Lukasiewicz implication. You could do the same also with Reichenbach implication; you could you would see that it satisfies contra positive symmetry with respect to 1 minus x, ok.

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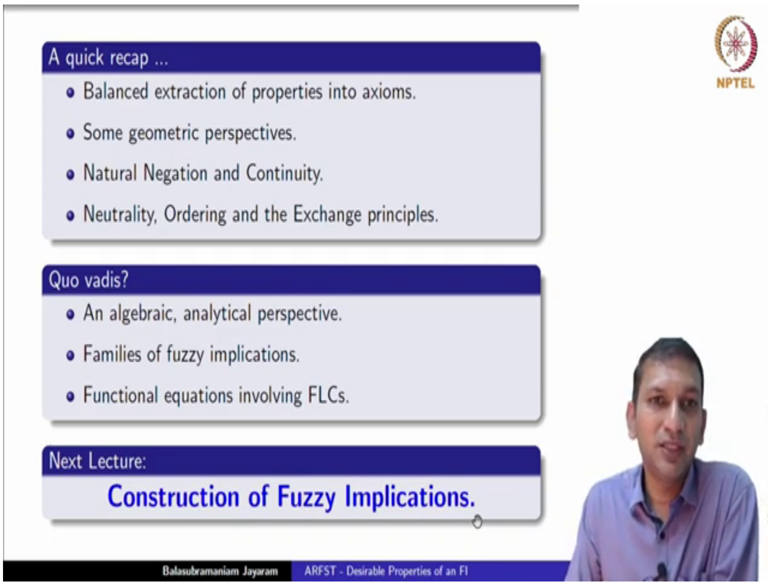

A quick recap of what we have dealt with in this lecture or so far we have actually axiomatized, the definition of fuzzy implications, in the last lecture we saw some geometric perspectives and not just one in the last lecture we carried that forward to this lecture also looking at how a fuzzy implication can behave at different boundaries or also on the diagonal.

And what we would prefer to have along those boundaries. In that sense, we talked about the natural negation from it and of course, continuity is always a desirable property, but more importantly, we have seen about this neutrality ordering principle and exchange principle. As you would have seen that they could also be look viewed as algebraic or logical properties that an implication should have and also in terms of with their geometry.

Finally, where do we go from here? In the next few lectures on fuzzy implications in this week we will look at some algebraic perspectives in terms of relationships that fuzzy implications enjoy with negation and triangular norms which are other important fuzzy logic connectives; we will see some families of fuzzy implication the way we construct them.

And finally, we will also seen some functional implications involving fuzzy logic connectives mainly implications. We only look at them from a theoretical perspective a few of them there are many many of them, but a few of them in a very introductory way, but all of these will have a bearing later on when we discuss properties of the approximate reasoning schemes that we will discuss later.

What next, we will in the next lecture; we will look at how to construct fuzzy implications. So, far we have only arbitrarily given some fuzzy implications we have stayed true to those nine basic fuzzy implications. We hope that these formulae are quite readily decodable by you. In the next lecture, we will look at a very systematic way of constructing fuzzy implication. There are many ways we will at least see couple of them.

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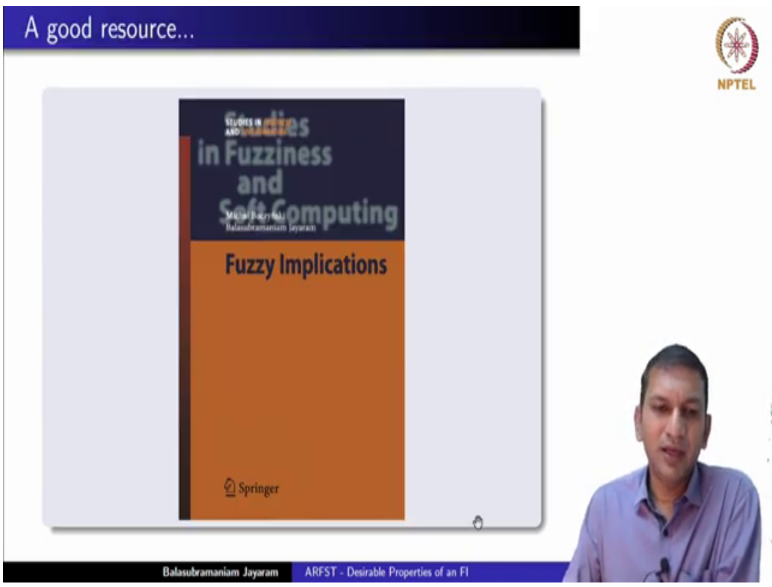

Finally a good resource for what has been covered in this lecture also is this book of book on fuzzy implications. Thank you once again that you could join for this lecture and hope to see you soon in the next lecture.

Thank you again.