Approximate Reasoning using Fuzzy Set Theory
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Lecture - 01
Flow of the Course: A not-so-sneak peek

Hello and welcome to the very first lecture in this course titled Approximate Reasoning Using Fuzzy Set Theory. A course offered through the NPTEL platform. You may have seen the brief introduction about the course that is available on the course web page.

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In this first lecture we would like to see the Flow of the Content of this course. Allow me to briefly touch upon approximate reasoning; the way we understand the term in this course and also with an example of a piece of knowledge encoded using fuzzy if then rule. This is necessitated for two reasons, one to set the nomenclature as also to fix the symbolisms.

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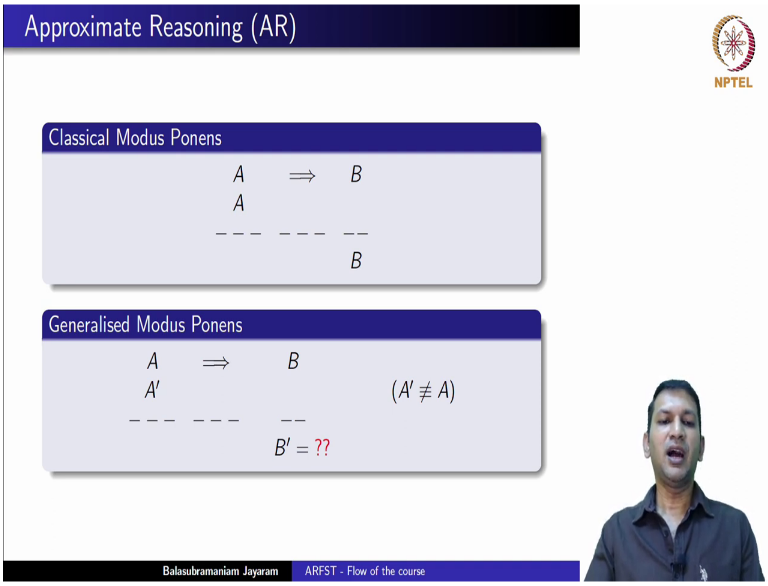

We have seen in the brief introduction to the course, that what we call as exact reasoning is enabled by the classical modus ponens inference. Where a piece of knowledge cast in the form of conditioner A implies B; where A is called the antecedent, and B is called the consequent. Given this piece of knowledge and an input, which matches the antecedent exactly, we can infer B as the output.

In generalised modus ponens which is a generalisation of the upper scheme of inference; we still have the same piece of knowledge. However, the input that we are given A dash may not exactly match the antecedent A. And still the generalised modus ponens allows us to make reasonable conclusions B dash, given this piece of knowledge and the input A dash.

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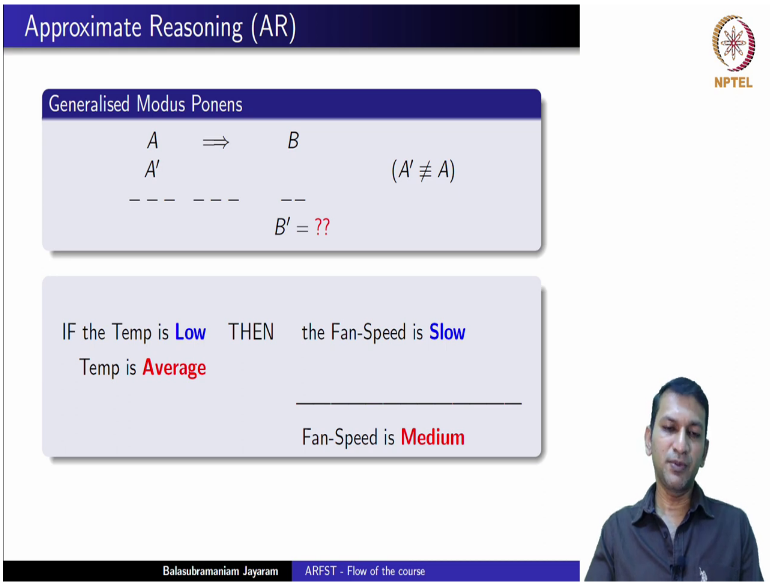

Since, we discuss approximate reasoning using fuzzy set theory. Allow me to offer you a simple example of a fuzzy if then rule which captures perhaps the knowledge that we already have; if the temperature is low, then the fan speed is slow. Now this is cast in the form of a conditioner; given the input temperature is average, we are likely to make a very simple common sense reasoning stating that the fan speed should be medium.

Now, what is low average, slow or medium, how they are interpreted we will see in the due course of the next twelve weeks. But this is a clear example of how we can perform approximate reasoning using fuzzy set theory.

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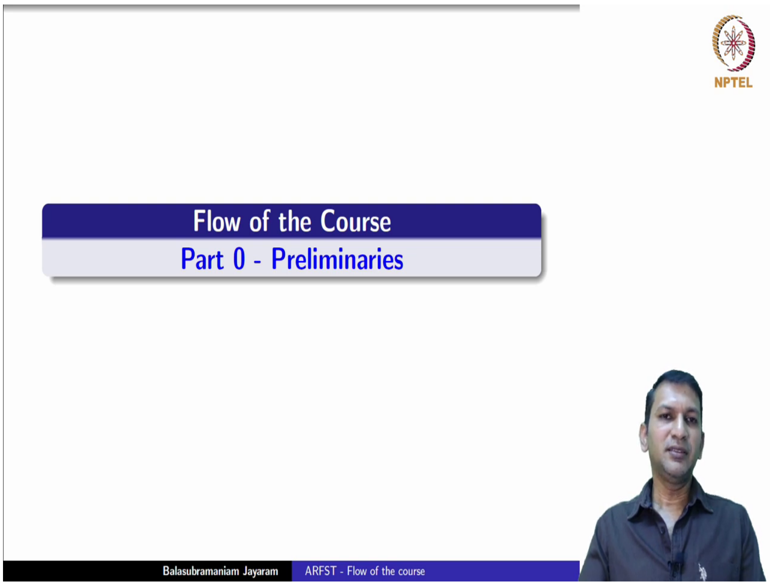

In the next 12 weeks the content that would be covered can be logically broken down into four parts. Part 0 will deal with preliminaries and this is what will be dealt with in the very first week of this course.

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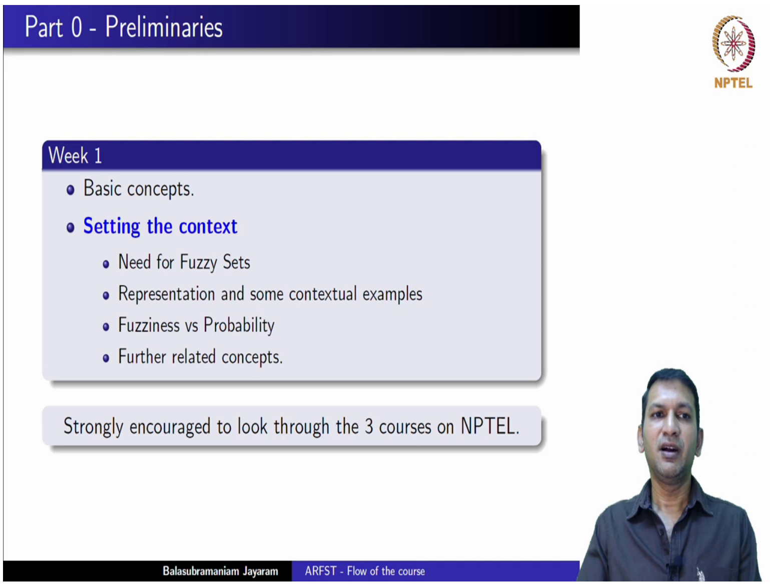

Wherein, we will cover the basic concepts, we will set the context by which we mean, we discuss the need for fuzzy sets, the different representations and some contextual examples, as also see the difference between fuzziness and probability. And some related concepts as generalised from classical set theory to fuzzy set theory. We strongly encourage the participants to look through the 3 courses already that exist on the NPTEL platform dealing with fuzzy set theory.

As has been mentioned in the brief introduction that is available on the course web page, these 3 courses differ in the flavor with which they are offered. However, the preliminaries dealing with fuzzy set theory are likely to be extremely useful for you, even for this course.

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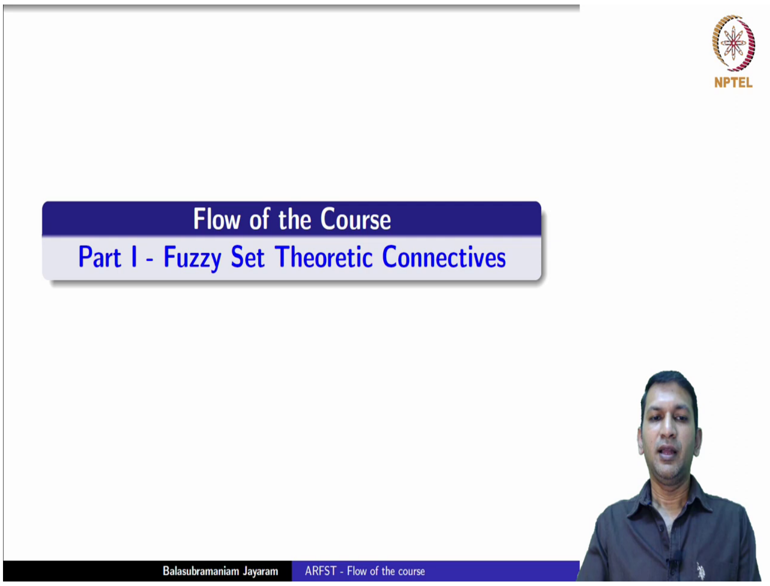

In Part 1 of this course we will deal with Fuzzy Set Theoretic Connectives which play a huge role in fuzzy inference mechanisms.

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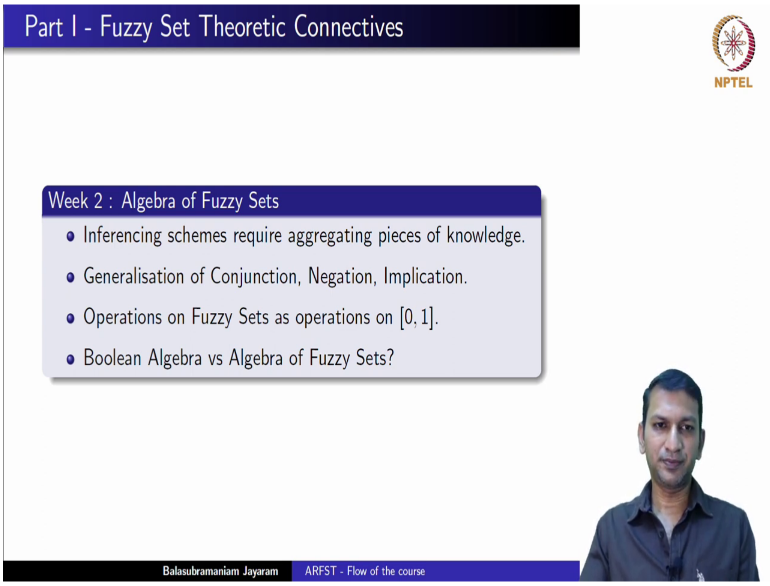

In week 2 of this course, we will take a slightly algebraic look at the set of fuzzy sets, why is it necessitated. Inference schemes require aggregating pieces of knowledge, each piece of knowledge is expressed in the form of a conditioner and we are given an input too. Now, we will have many pieces of knowledge and along with the input we need to aggregate to come up with a reasonable conclusion. This necessitates that we generalise conjunctions, negations, and implications from the corresponding classical set theory.

We will see the operations on fuzzy sets can be seen as operations on the unit interval [0,1]. It is well known that Boolean algebra underlies as the algebraic structure for the classical set theory. Then it begs the question what is that algebra that we can obtain over the set of fuzzy sets? This is something that we would see in week 2 of this course.

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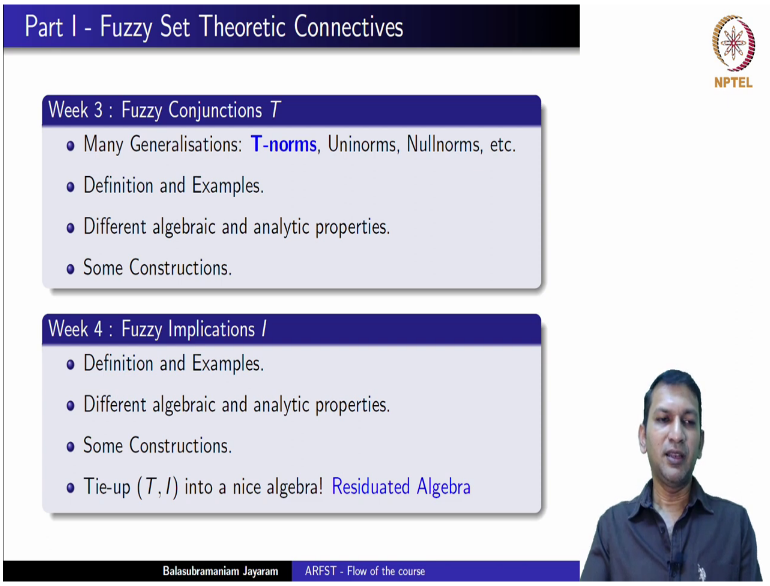

In week 3 of this course, we will concentrate on one particular generalisation of conjunction, which we to the setting of fuzzy sets. There are many generalisations as you will see we will term them as; triangular norms, uninorms, nullnorms, overlap functions, grouping functions and so on. However, in this course we will largely look into triangular norms, shortly T norms, and if there is a need that arises we would also touch upon uninorms, nullnorms, and overlapping functions.

In this we would look at the axiomatic definition of T norms and some very interesting and useful examples. We will also see the different algebraic and analytical properties of these T norms; finally, we will also take a peek into different constructions of T norms. In week 4, we will follow similar line of exploration on yet another important aggregation operation which is this fuzzy implication.

Once again, we will look through the axiomatic definition of a fuzzy implication and some important and useful examples, the algebraic and analytic properties of these operations. Some constructions of these operations and more interestingly we will try to tie up this T norm and an implication into a nice algebraic framework and largely one such algebraic framework that is used is that of a resituated algebra.

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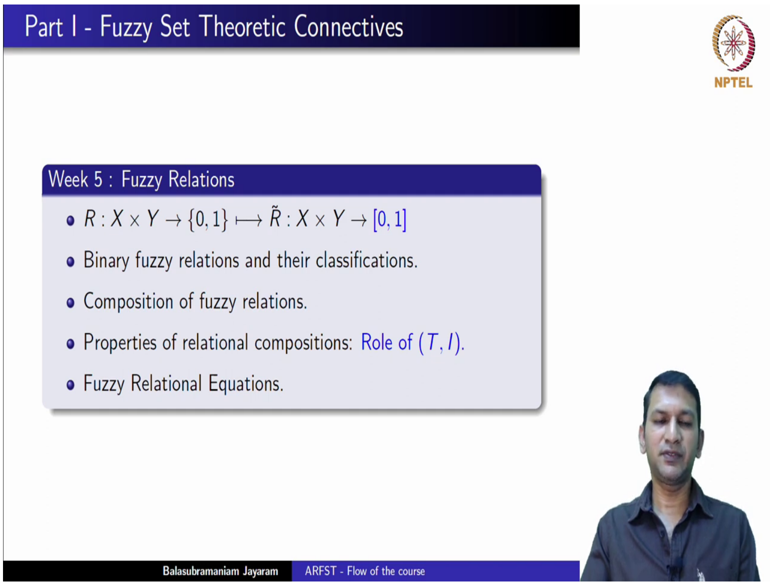

In week 5 of this course, we will discuss fuzzy relations. We know the classical or crisp binary relation is a mapping from Cartesian product of sets to the set {0, 1}. We will see fuzzy relations as extensions of this where the co-domain becomes the unit interval [0, 1]. While we can talk about fuzzy relations on Cartesian products, arbitrary Cartesian products, but largely we will deal with binary fuzzy relations and some interesting and useful classifications of them.

We will also discuss compositions of fuzzy relations and the properties of these relational compositions. Wherein once again you will see the role played by the pair, the couple T and its related implication. Finally, we will also see how to solve fuzzy relational equations and the conditions under which and the same setting under which they can be discussed which will be extremely useful.

Later on in part 3 of this course where we discuss some desirable properties of fuzzy inference mechanisms; so, these 4 weeks will constitute the part 1 of this course, where we discuss fuzzy set theoretic connectives.

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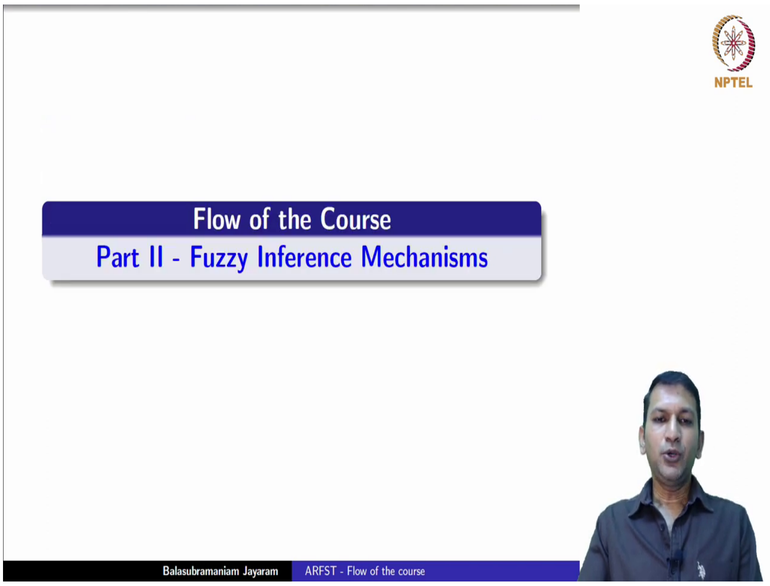

In part II of this course, we will enter into the mainstay of this course which are these Fuzzy Inference Mechanisms.

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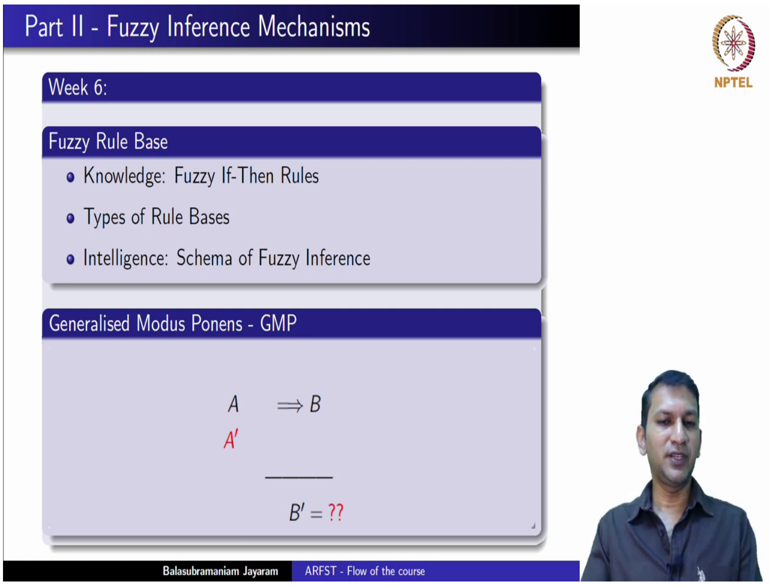

To understand the fuzzy inference mechanism, we need to discuss what we term as fuzzy if then rule bases. We have seen that a knowledge can be encoded in the form of a conditioner; and in this course, they will typically be in the form of Fuzzy If-Then Rules. And as is as was already mentioned not a single piece of knowledge, but many pieces of knowledge; that means, we will have many Fuzzy If-Then Rules which form a fuzzy if-then rule base. We will also take a look at the different types of rule bases available to us, and for reasoning knowledge alone is not enough that is where intelligence comes into picture in the form of different schemes of fuzzy inference.

We have seen generalised modus ponens as follows, given a piece of knowledge in the form of conditioner A implies B. And the input A dash we would still like to obtain B dash even when A dash is not an exact not exactly the same as A the antecedent of the conditioner. There are two ways two major ways of doing this inference using fuzzy set theory, these are called the fuzzy inference schemes.

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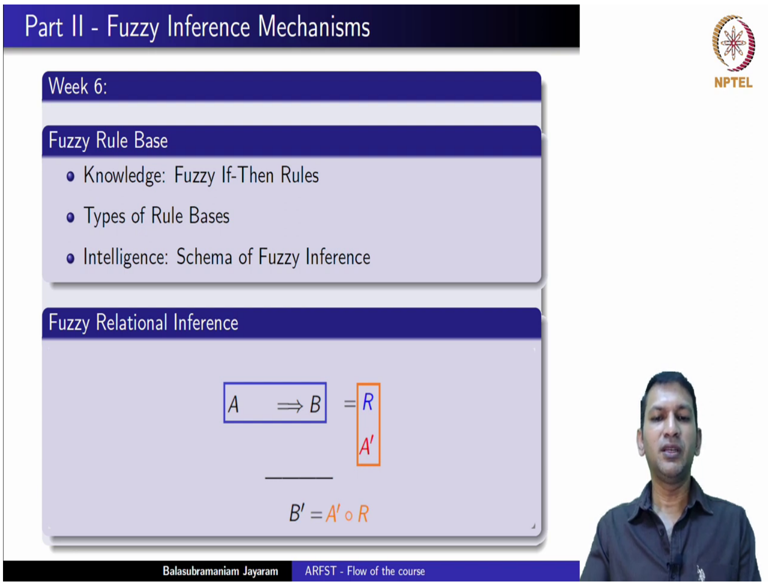

The first of them is what we call a fuzzy relational inference. In this scheme we convert this knowledge in the form of conditional A implies B into a fuzzy relation. Then using the given input, A dash; we obtain the B dash, the output B dash as composition of A dash with the obtained relation R.

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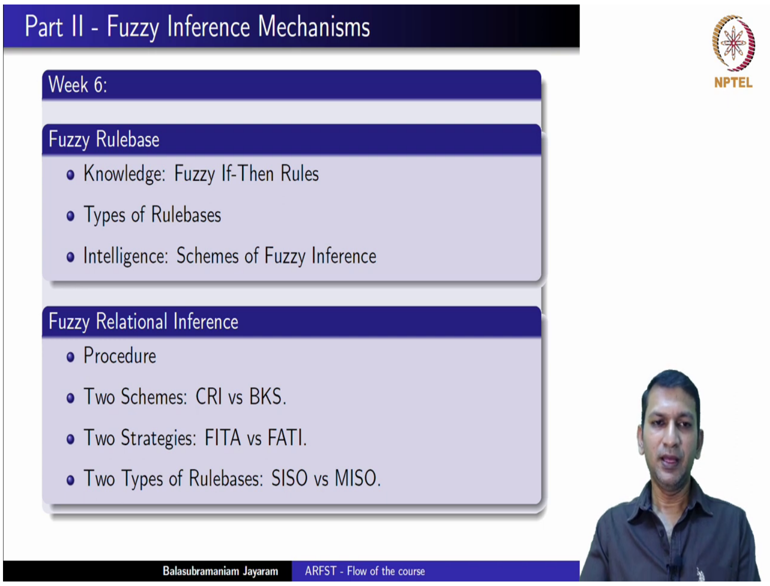

Of course, this is in a nutshell what we do, but as they saying the devil is in the details. So, we will look into the procedures that enable us to do this fuzzy relational inferencing. In fact, there are two such schemes, the compositional role of inference which is shortened into the term CRI has also the Bandler Kohut Subproduct which is abbreviated as BKS. There are also two strategies first infer then aggregate and first aggregate then infer.

There are also two types of rule bases to consider, whether it is a single input single output, or multiple input single output rule base. We will take a look at all these things related to fuzzy relational inference in week 6 of this course. This is the first major scheme of fuzzy inference.

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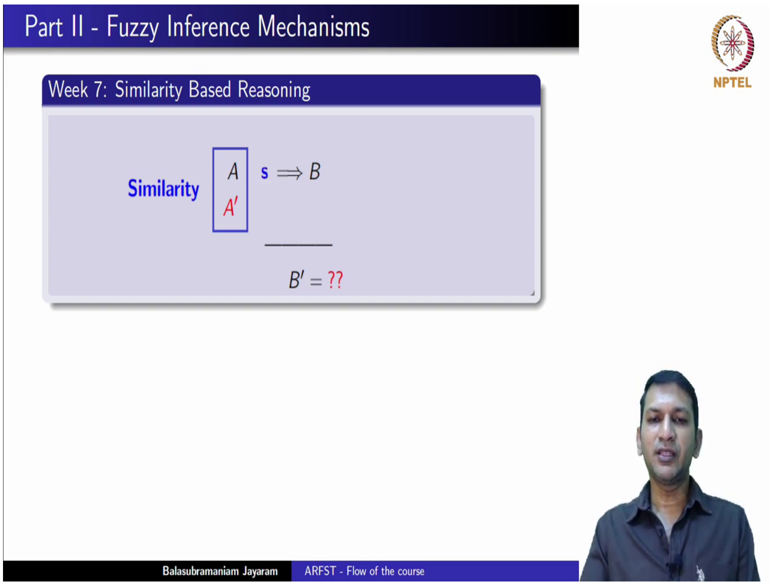

There is also another type of fuzzy inference mechanism called similarity-based reasoning; once again look at this, this is the general schema of generalised modus ponens. In similarity-based reasoning, as against coming up with a relation from the piece of knowledge, what we do is take the input A dash and try to see the similarity between A dash and the antecedent of the rule A.

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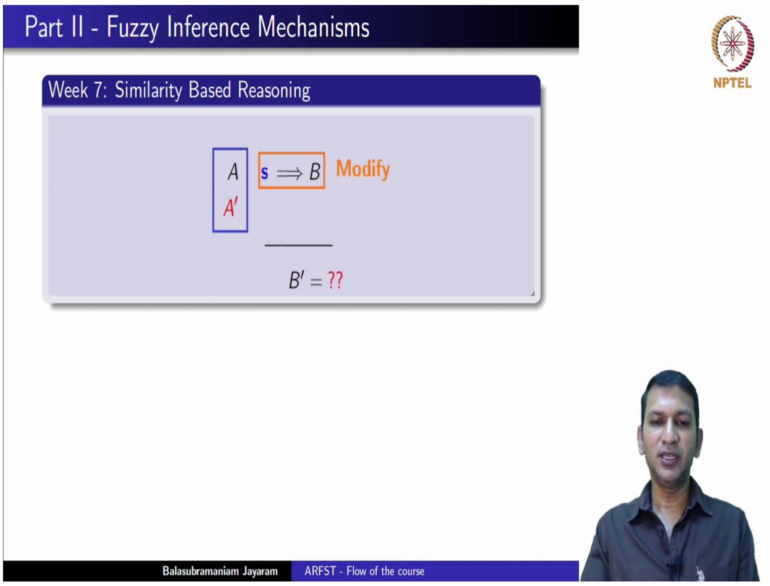

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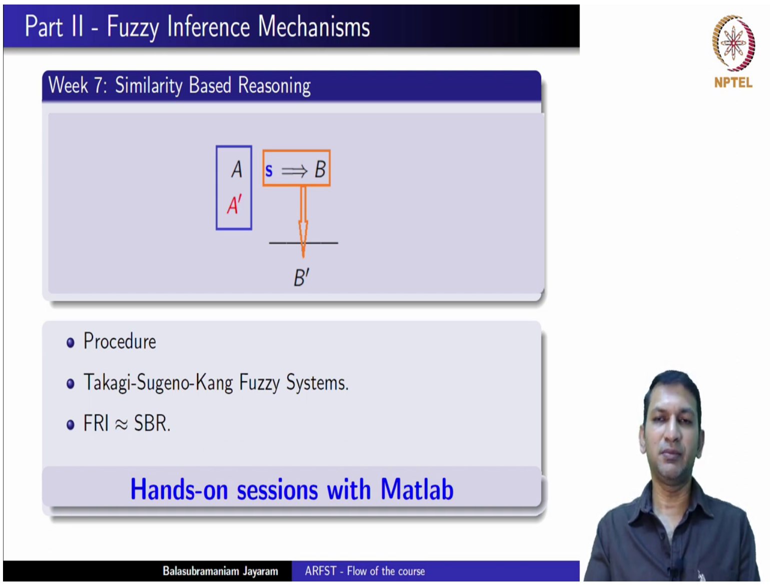

So, we find the similarity between A dash and A, take that similarity value s and modify the consequent B using this to obtain the B dash. So, in some sense in similarity-based reasoning, we first go vertically up and then horizontally across; as against fuzzy relation inference where we go horizontally across first and then come down vertical to obtain the inference.

Once again, we will look into the procedure in detail; one of the most important useful and often applied fuzzy inference system that falls under the similarity-based reasoning is the one proposed by Takagi Sugano Kang often abbreviated as TSK fuzzy systems. We will also see some situations where in a fuzzy relational inference is actually equivalent to a similarity-based reasoning scheme.

And this usually this equivalence has very interesting implications, especially in computational complexity. But what is most important about this week 7, which happens to fall back in the middle of the course.

Is that we will also have Hands on sessions with MATLAB wherein we use the fuzzy logic toolbox in MATLAB to build fuzzy inference system especially TSK and Mamdani Fuzzy Inference Systems which will show you how to approximate a given function. Thanks to the NPTEL organizers who will ensure that the participants of this course will have access to MATLAB at the appropriate juncture. These 2 weeks constitute part 2 of this course dealing with fuzzy inference mechanisms.

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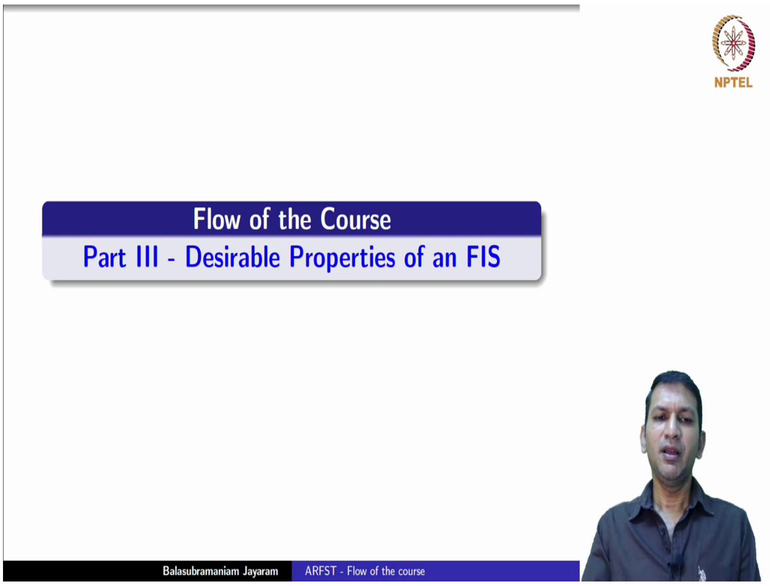

Part 3 of this course, we will discuss the Desirable Properties of an FIS of a Fuzzy Inference Scheme. This part of the course is important for two reasons; one it talks about what a fuzzy inference scheme is expected to possess not only that this also nicely ties up the previous two parts.

The theoretical underpinnings of fuzzy set theory that we have probably covered by them in part 1 of the course, two the fuzzy inference schemes that we have discussed that we would be discussing in part 2 of the course.

How the theoretical constructs discussed in part II, play a role in ensuring guarantee these desirable properties are possessed by the corresponding fuzzy inference schemes. What are the desirable properties of fuzzy inference systems that we will be discussing in this course.

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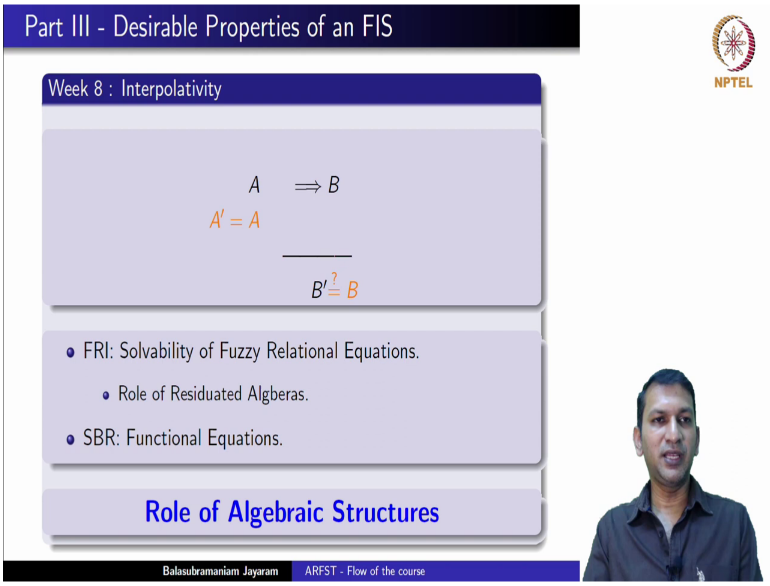

The first of them is what we call interpolativity, once again this is the general schema of generalised modus ponens A dash is the given input. We know that in generalised modus ponens, A dash need not be equal to the antecedent and we still intend to obtain a reasonable conclusion in the form of B dash. But the question now is, what if A dash is actually A, would we get B dash to B? If we do then, we say this fuzzy inference system possesses interpolativity.

Now, there are two major schemes of fuzzy inference systems; the fuzzy relation inference, and the similarity-based reasoning. We will discuss interpolativity of both these major schemes of inferences. You will see that to discuss interpolativity in the setting of fuzzy relation inference deals with solving fuzzy relational equations.

Once again this highlights the role played by Residuated Algebras, in the case of discussing interpolativity for similarity-based reasoning we will see that some functional equations play a role. On the whole discussing interpolativity of a fuzzy inference system shows the role played by the Algebraic Structures in ensuring the same.

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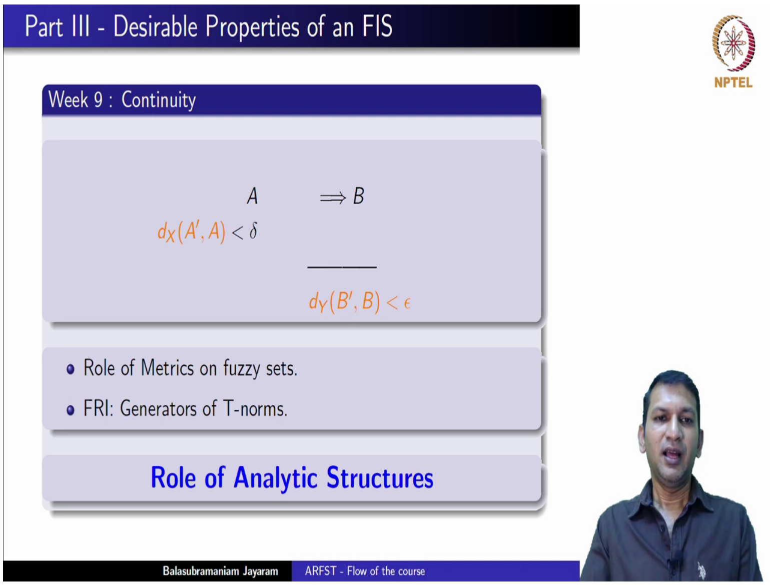

The next property that we will discuss of a fuzzy inference scheme is that of continuity. Once again consider this general schema, what we mean by continuity is? The given input A’ what if it is close to A would we get an output that is also close to B, that is if A’ is close to A with respect to some distance or metric that we have in mind will B^' also be close to B with respect to some metric or a measure.

Clearly this shows the role of matrix over fuzzy sets. Interestingly, if you want to discuss continuity of fuzzy relation inference schemes, you will see that one particular construction of triangular norms especially the generators of them they come into play to discuss this continuity and help us in discussing this continuity. Clearly this highlights the role of analytic structures in the study.

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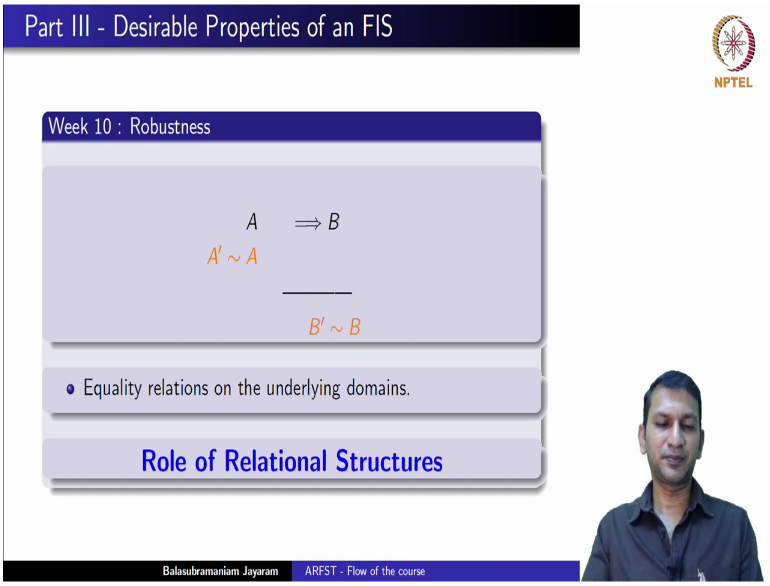

The next property that we will deal with is that of robustness. While continuity discussed whether the closeness between A' and A is being carried over to B between B' and B in the case of robustness, we discuss a slightly different concept. Here we assume there is some kind of a relation that is available both on the domain, in input domain and the output domain.

And we ask this question if A dash enjoys a relationship with A, will that relationship be carried over to B' also; If A' is related to A will B' also be related to B. In what sense these are related that will be made precise during the course of this these lectures. However, it suffices to say that we expect there are equality relations on the underlying domains and we expect this fuzzy inference scheme to preserve it. Once again, in this case when we discuss robustness it will highlight the role of relational structures.

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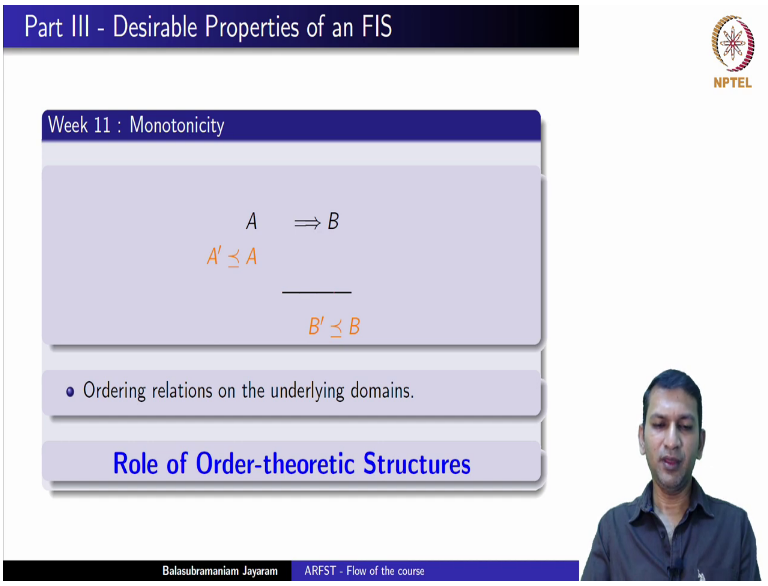

The final property that we will discuss is that of monotonicity; once again, this is the general scheme of generalised modus ponens. In the case of continuity, we discussed how close A' is to A and if that closeness proximity is preserved by the mapping. In the case of robustness, we discussed the relation between A' and A and whether that would be preserved and carried over as relationship between B' and B.

In this case, we expect there is an ordering on both the input and output domains. And ask the question if A' is less than or equal to A, where B' also be less than or equal to B. And this operation or relation ordering relation itself will be made precise during the course of these lectures. So, the assumption is that there are ordering relations present on the underlining domains, and we ask the question is this ordering preserved by the mapping. Once again this highlights the role of order creating structures.

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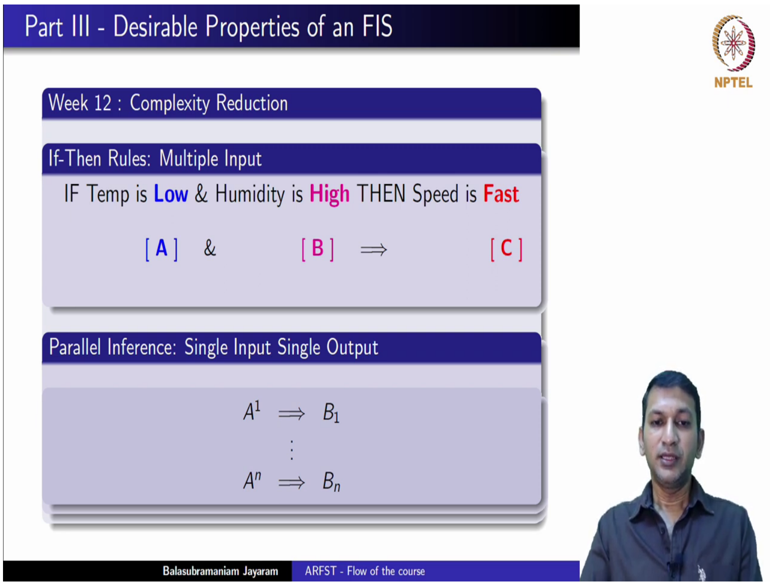

In week 12 of this course in the last week of this course, we will discuss interesting offshoot of what we would have seen till now, till then. When we talked about the properties of continuity or robustness, we are essentially discussing a theoretical property dealing with its efficacy or accuracy. And a property that we expect the fuzzy inference mechanism to have, these are easily explorable theoretically.

But there is also the issue of efficiency when you are actually and practically implementing them. And this computational complexity can arise in different ways; we will see at least two such issues and try to address them. As has already been discussed, a single fuzzy if then rule will not be able to capture the working of an entire system which means we need multiple of multiple fuzzy if-then rules. Often also multiple input single output rules for instance consider this fuzzy if-then rule.

If the temperature is low and humidity is high, then speed is fast; perhaps, there is a piece of knowledge that you would have about the working of an air conditioner. Here, when we when we write speed, we are referring to the fan speed inside the air conditioner. So, here you see that you can abstract it in the form of a following conditioner A and B implies C; so, this is an example of a multiple input single output rule.

Now, fuzzy inference schemes actually do parallel inferencing; by that we mean, they take a set of rules and consider all the rules that are given to the system in obtaining an inference or output B' from the input A'. Now, the rule bases themselves could be single input, single output rule bases; so, A^1 implies B_1; so on to A^n implies B_n; where, n is the number of rules. They could also be multiple input single output rule bases of this form.

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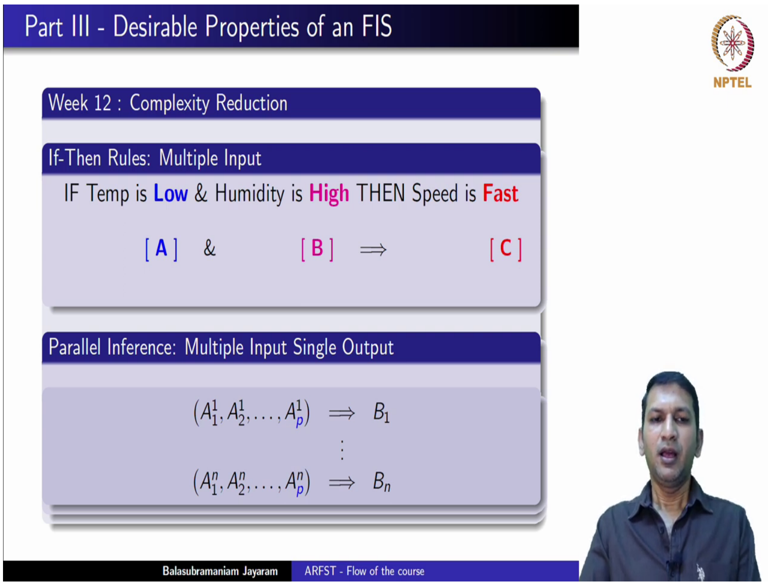

Where each antecedent is actually coming from a Cartesian product of p different domains; so p here refers to the dimensionality of the antecedent term.

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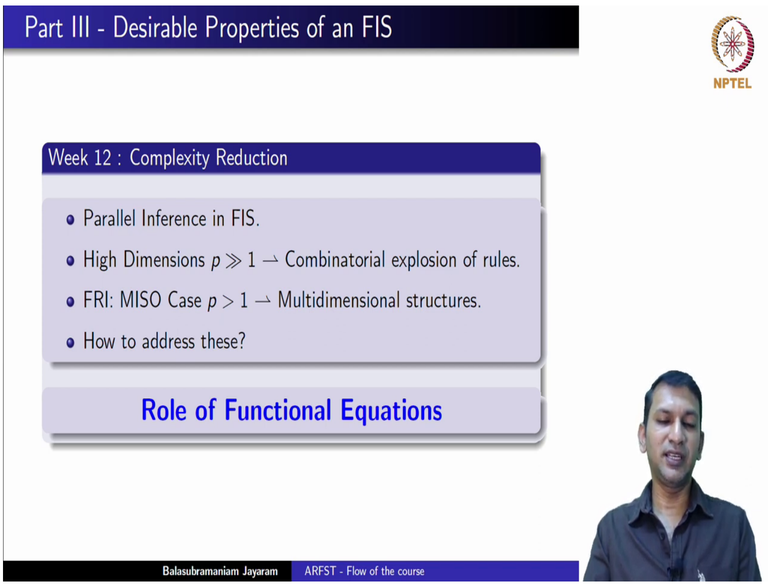

Now, as was mentioned fuzzy inference schemes perform parallel inference. Now, in high dimensionality; where, p is far greater than 1 this leads to a combinatorial explosion of rules, we have far more rules than can be easily managed. So, this; obviously, immediately has a bearing on the processing time of the inference.

However, in the case of fuzzy relational inference, when we consider a multi input single output case the p does not have to be very high; even when the dimensionality is low, it leads to dealing with multi dimensional structures which once again have a bearing on the amount of memory that we use while implementing it and also the processing time. Now, how do we address these? Yes, we have contexts or situations where these can be handled and some computational efficiency can be gained.

Here, predominantly we see the role played by functional equations; what functional equations, what are functional equations, and what are these functional equations involving fuzzy set theoretic connectives this again will be made precise during the course of these lectures. So, this in a nutshell is what we will be covering in these 12 weeks under the course titled approximate reasoning using fuzzy set theory.

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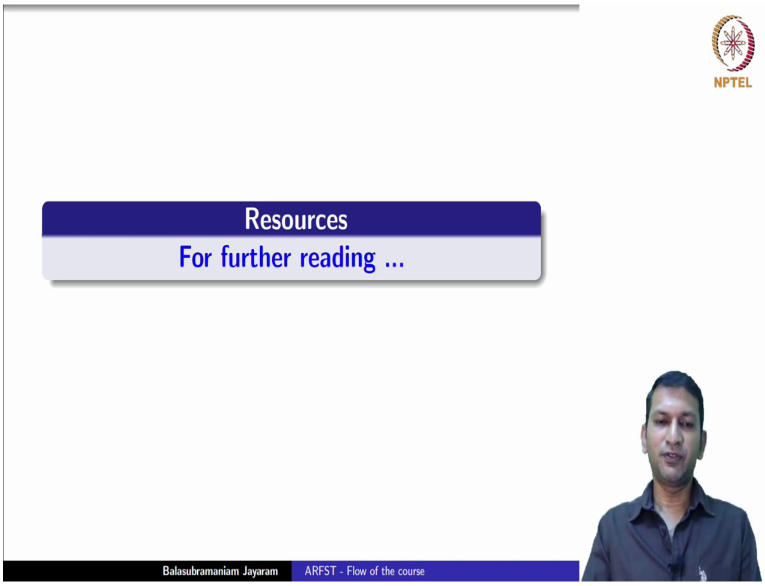

Allow me to present you some resources for further reading which through which you can both supplement and complement the content covered in these lectures.

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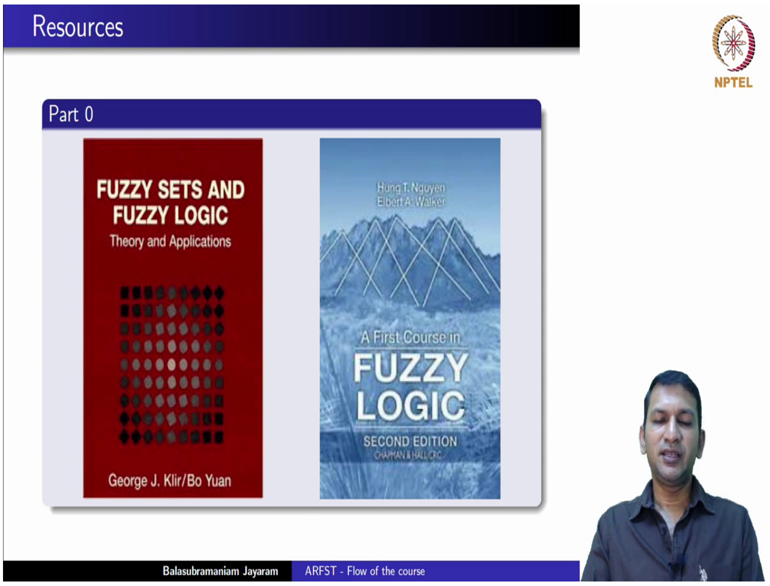

These are two excellent resources for you, if you want to read more on basic concepts of fuzzy set theory, which we will be dealing with in the very first week under the part 0 of this course.

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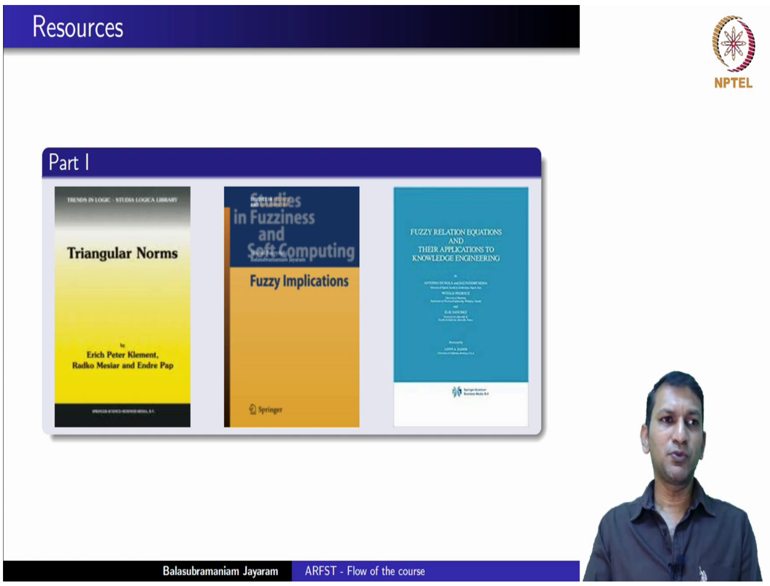

In part 1 of this course, where we discuss fuzzy set theory connectives especially those of conjunctions, triangular norms, and fuzzy implications and fuzzy relations. These are three excellent resources again for you to depend to.

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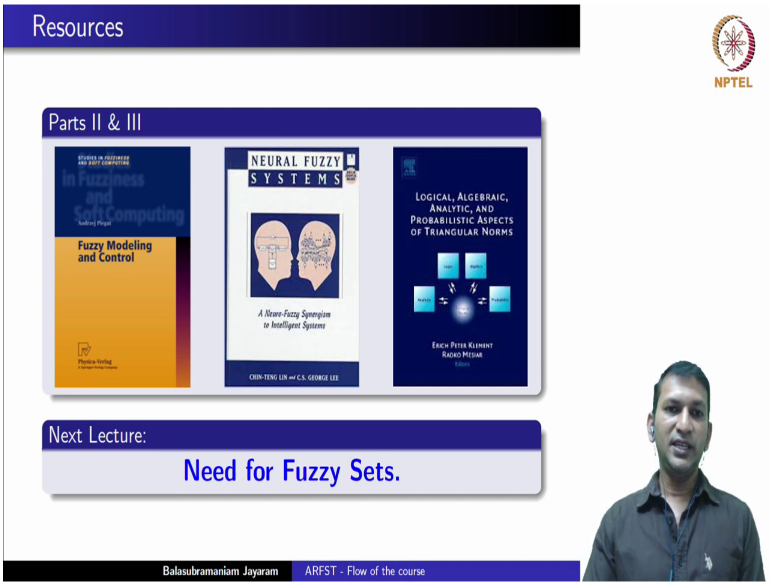

And when it comes to parts 2 and 3 of this course where we discuss fuzzy inference mechanisms and the corresponding desirable properties of them and the computational complexity aspects of it these are three again good sources for you to read through. Of course, these are not non overlapping some of these resources also address concepts which we may have covered in part 1 and 0 of this course.

In the next lecture of the series we will look into the Need for Fuzzy Sets; wherein, we will see fuzzy sets as a generalisation of classical sets, also in a way which will showcase fuzzy sets as a natural component when we want to perform approximate reason.

Thank you.