## Measure and Integration Professor S. Kesavan Department of Mathematics The Institute of Mathematical Sciences Lecture-11 2.6 Errata

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ERRATA Prop. The Borel or -algebra is also the or-algebra guarted by the open sats in R? Proof: 8, = JURI = Boral a-aly Lat I here usual top. on R. To two B, = Stal a, u E 12 (a, u) = [a, b) ( 20 ) E O, Anyopen set is the able min of open intervals =) TCB => 5(1) CB, Conversaly, [a, b) = (a, b) U Sas  $\{a_{3}^{2}=\bigcap_{\infty}(a_{-\frac{1}{2}},a_{+}y_{n})\in \exists(\tau)=\sum_{n}b_{n}(c)\in \exists(\tau),$ Proof: B, = SLRI = Band a-aly Lat To be the usual top. on R. To now B, = 3(=) NPTEL a, u E 12 (a, c) = [a, c)/ (a) E B. Any open set is the attle min of open intervals =) しくめ =) かいくし Conversaly, [a, b) = (a, b) USa3  $\{\alpha_{j}^{2}=\bigcap_{n=1}^{\infty}\left(\alpha_{-\frac{1}{n}},\alpha_{+}\right)\in\mathcal{J}(\mathcal{I})=\mathcal{J}(\mathcal{I})\in\mathcal{J}(\mathcal{I})$ =) やくさに)=) アくさに)=) ざほうほくろに).

Before we go further, I just want to make some corrections on what we did last time. The proof was correct, but I made mistakes when writing it. And so, in case it confuses people, I apologize for that. So, let me write everything correctly.

**Proposition:** The Borel  $\sigma$ -algebra is also the  $\sigma$ -algebra generated by the open sets in  $\mathbb{R}$ .

*proof:* So,  $B_1 = S(\mathbb{R}) = Borel \sigma$ -algebra. So, let  $\tau$  be the usual topology on  $\mathbb{R}$ .

So we show  $B_1 = S(\tau)$ .

So, let  $a, b \in \mathbb{R}$ . Then the open interval  $(a, b) = [a, b) \setminus \{a\} \in B_1$  and any open set is the countable union of open intervals and this implies therefore,

$$\tau \subset B_1 \Rightarrow S(\tau) \subset B_1.$$

Conversely, you have  $[a, b) = (a, b) \cup \{a\}$ . Now

$$\{a\} = \bigcap_{n=1}^{\infty} (a - \frac{1}{n}, a + \frac{1}{n}) \in S(\tau) \Rightarrow [a, b] \in S(\tau).$$
$$\Rightarrow P \subset S(\tau) \Rightarrow R \subset S(\tau) \Rightarrow S(\mathbb{R}) = B_1 \subset S(\tau).$$

And therefore that proves the proposition.