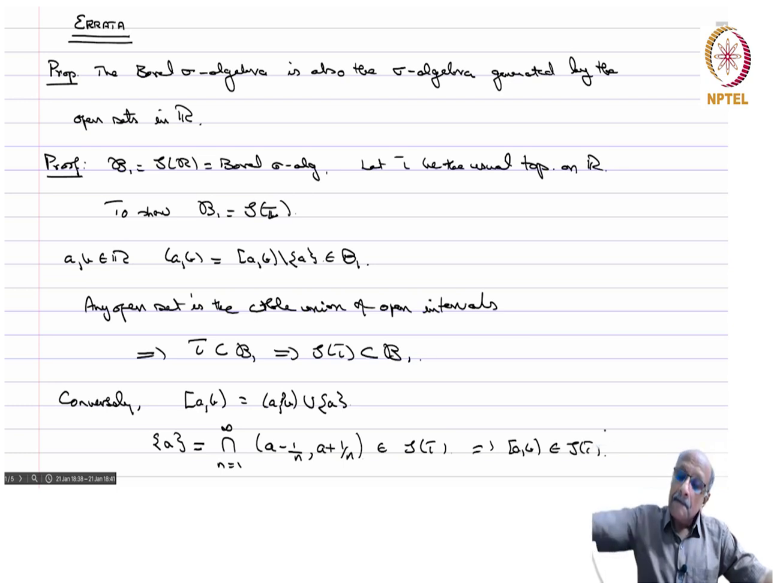
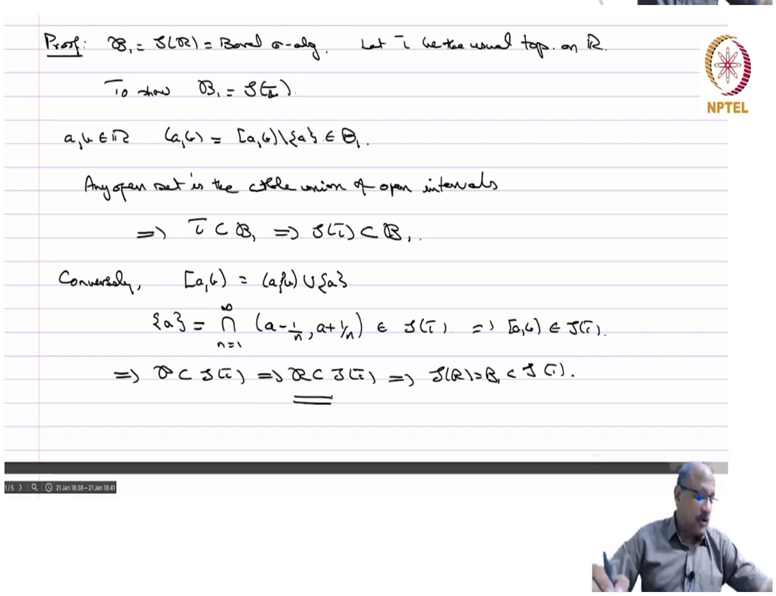
# Measure and Integration
# Professor S. Kesavan
# Department of Mathematics
# The Institute of Mathematical Sciences
# Lecture-11
# 2.6 Errata

(Refer Slide Time: 00:16)




Before we go further, I just want to make some corrections on what we did last time. The proof was correct, but I made mistakes when writing it. And so, in case it confuses people, I apologize for that. So, let me write everything correctly.

**Proposition:** The Borel σ-algebra is also the σ-algebra generated by the open sets in ℝ.

*proof:* So, $B_1 = S(\mathbb{R})$ = Borel σ-algebra. So, let τ be the usual topology on ℝ.

So we show $B_1 = S(\tau)$.

So, let $a, b \in \mathbb{R}$. Then the open interval $(a, b) = [a, b)\backslash\{a\} \in B_1$ and any open set is the countable union of open intervals and this implies therefore,

$$\tau \subset B_1 \Rightarrow S(\tau) \subset B_1.$$

Conversely, you have $[a, b) = (a, b) \cup \{a\}$. Now

$$\{a\} = \cap_{n=1}^{\infty} (a - \frac{1}{n}, a + \frac{1}{n}) \in S(\tau) \Rightarrow [a, b) \in S(\tau).$$

$$\Rightarrow P \subset S(\tau) \Rightarrow R \subset S(\tau) \Rightarrow S(\mathbb{R}) = B_1 \subset S(\tau).$$

And therefore that proves the proposition.