Algebra-II
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Lecture 45
What is category theory (And why is it important)?

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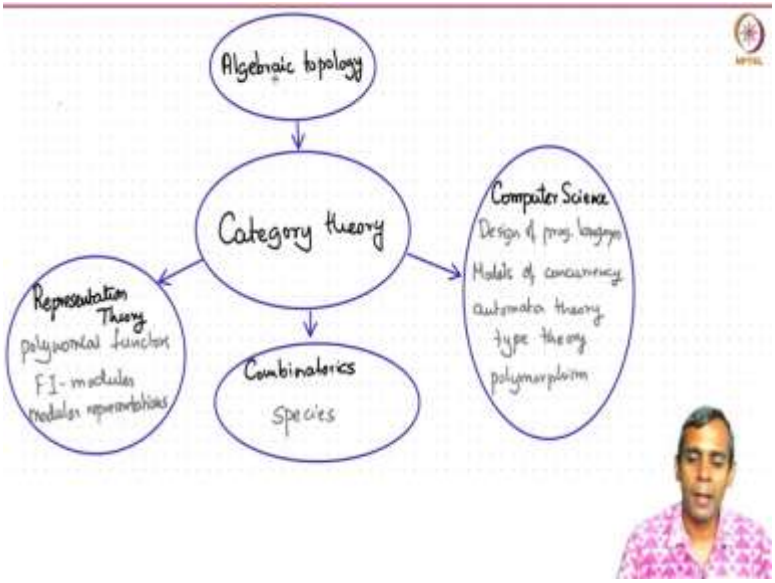

In this part of the course we are going to study category study category theory. Before we begin let me tell you what category theory is about and why we study it. Category theory has its origins in algebraic topology, which is an area where you try to attach algebraic invariants to topological spaces. So, you have a topological space and it is this floppy thing.

I mean it is a geometric object and to get a grip on it you try to attach to it an algebraic object like a group or a ring or a module, and it was very important in algebraic topology to keep track of how under continuous functions of topological spaces you have got homeomorphisms of the algebraic objects. And that was called funtoriality.

It was also very important to keep track of how different algebraic objects related are to each other, and this was done by the notion of naturality. So, functoriality and naturality are two important concepts in category theory. It is relatively young subject and this aspect of category theory developing from algebraic topology happened in the 1960s.

And it led to unification of a lot of ideas. Now, most of you would have seen the theory of groups, the theory of rings, the theory of modules and in all of these theories you must have seen the notions of products and quotients. Category theory allows you to unify all these definitions, but more recently category theory has had many interesting applications which are not so abstract.

So, the representation theory, category theory has been used to define notions like polynomial functors, FI-Modules and also helps us study Modular Representations. In combinatorics category theory gives a very nice framework for studying what are called species, which help us understand how different counting problems have different generating functions and their properties. In computer Science there has been a flurry of activity involving category theory.

Category theory has been used for the design of programming languages, models of concurrency, automata theory, type theory, polymorphism and so on. I have only touched at a few areas where category theory is used, mostly areas that interest me but I think the list would be much larger if one look for an exhaustive list. And I think not category theory no longer seems to be this abstract meta theory but it is a tool that mathematicians use just like group theory or ring theory.

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In fact, for this part of the course I am going to use as the main reference a book which is written for computer scientists, it is called Basic Category Theory for Computer Scientists and it is by

Benjamin C. Pierce. It is published by MIT press in The Foundations of Computing Series. The classical textbook for category theory, which I used when I was learning the subject is by Saunders Mac Lane, it is called Categories for the Working Mathematician. It might be worth taking a look at that as well if you are able to do so.