Introduction to Probability - With Examples Using R
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Conditional Probability

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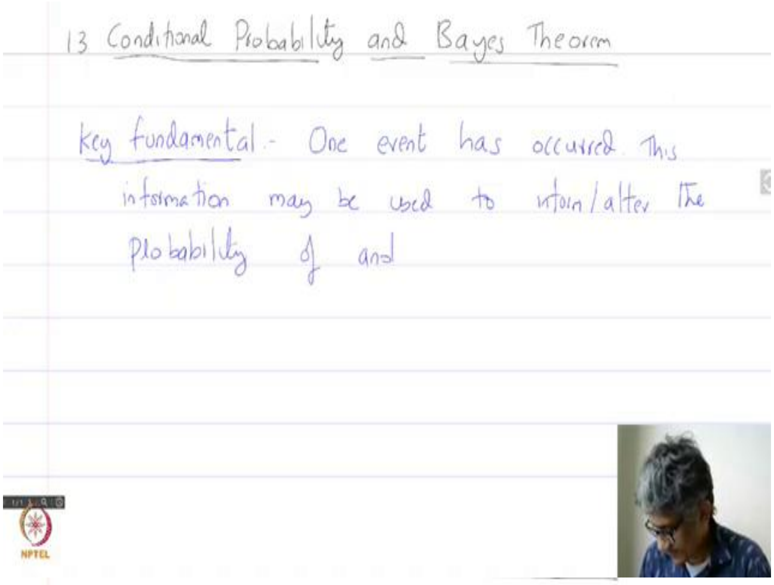

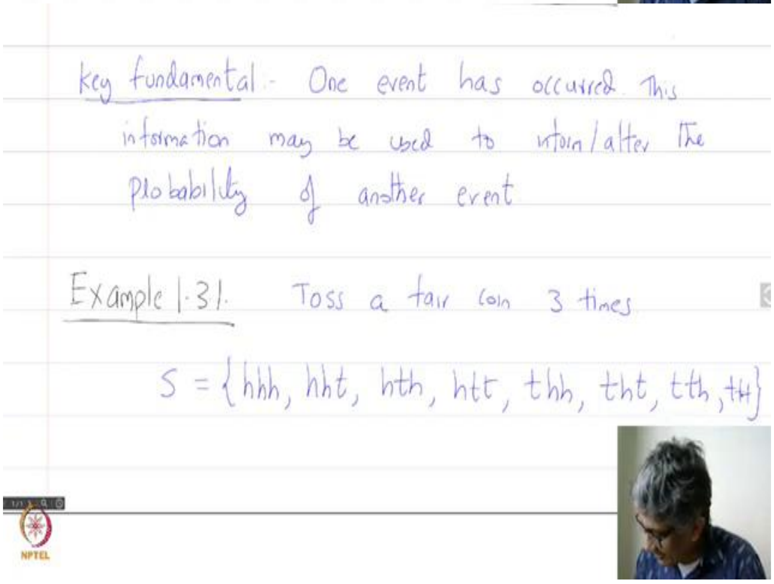

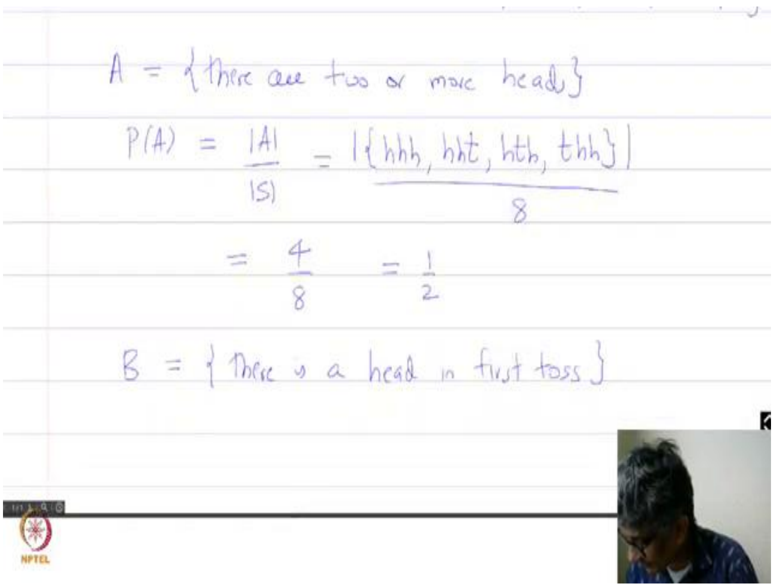

So, today, I want to discuss Conditional Probability. If you all cannot hear me just send on the chat, that cannot hear me, check. Thank you, I have to start now. So, the first thing I want to start this today is, I want to discuss the basic idea in probability. It is called conditional probability and Bayes Theorem.

So, it is very powerful, Bayes Theorem is very powerful theorem in probability. And, it is very widely applied in statistics. So, the key motion or the key fundamentals is the following. So, one event has occurred. And this information may be used to either inform or alter the probability of another event. That is the promise of conditional probability. So, let me just try and see if I can give you an idea.

So, let us do an example. So, example 1.3.1, So, let us say we toss a coin, a fair coin three times. So, once we do that, then we immediately know that the sample space S, will be these eight people. So, we have, head head head, on the first time, then you will have head head tail, or head tail head, or you have head tail tail, or start the tail head head, tail head tail, tail tail head, tail tail tail. So, I have eight possibility 1,2,3,4,5,6,7,8 eight possibility. Then, let say, I am interested in the event that, let us see, A is the event that there are two or more heads, this we know exactly, we know how to calculate.

Now, we are in this, so, the calculation we are making it, that the fair coin. So, we are in this equally likely outcome setting, that we discussed last time. So, we know the probability of A is just going to be given by the size of A divided by the size of S. And that is going to be one, that is going to be the size of A, size of the set what there are two or more heads, so we go and will pick the two or more responsibility, three heads can happen, two heads can happen in these sections, then THH.

And then the size of S it being cleared. So, now, this gives you that the chance is just 4 divided by 8. And that is a half, chance of two or more heads happening is just a half (())(05:27). Let us look at B is the chance that there is a head, in the first toss, that is the event B.

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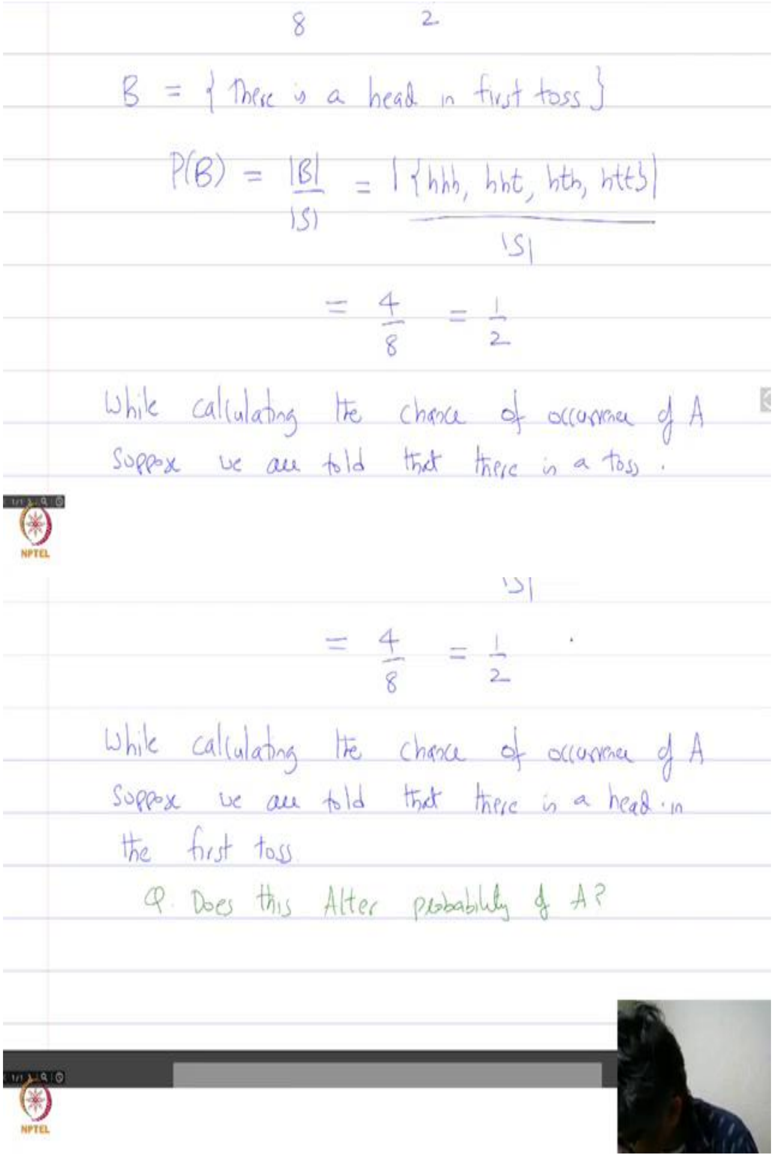

Now, what is this going to be again? Now, the chance of B, we already know is again, the size of B, divided by the size of S. And the size of B is just in the first option, the first one is fixed, the second one be anything, we go and write all the possibilities. Head tail head, and head tail tail, divided by size of S. And that again, you do the whole thing. It is just going to be again, 4 by 8 we get a half, that is clear to you, right? Now, so, the A and B are two different events that we know we understand clearly.

So, while this idea of information, I want to use in the following, so while calculating probability of A, the chance of probability of occurrence of A. Suppose, we are told that B has occurred, that write in this way, in a first toss, there is a head in first toss. And now, how does wants the question is, the question then becomes how does one pursue this probability? So, so does this alter? So, the question is, does this alter the probability of A? So, now, the, this is where conditional probability comes in.

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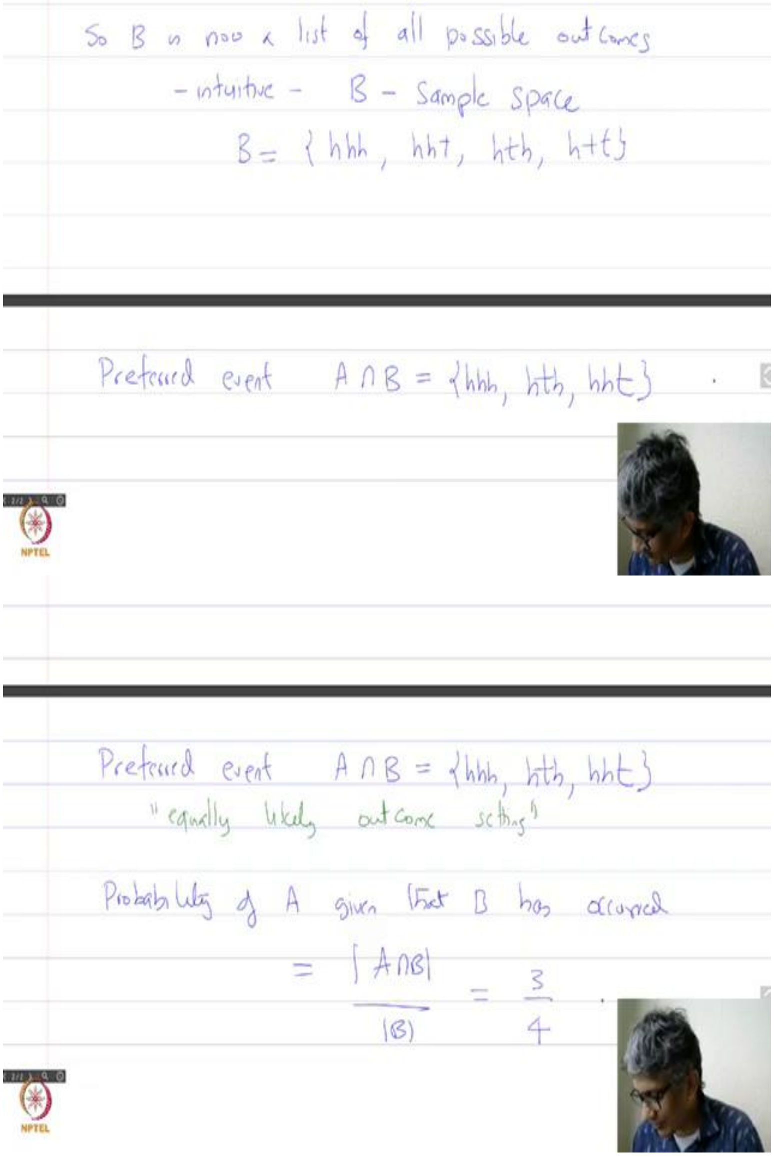

So, what, how does one do this? So, what we do is? So, we again, go back the question again. So, I have I have this, I have this event A and that two or more heads, I B is an event that there is a head in the first toss while doing this. This question I am told that, on Sunday, I am told that, that B has occurred, that is there is a head in the first toss will this alter the probability of A. So, what we do is, so, the key notion is, you restrict your sample space to B. So now, we are understanding A.

So, what we do is we just say B is now a list of all possible outcomes. So, your entire sample space is now B. So, it is intuitive idea, intuitive I mean, it is not quite mathematics yet, but intuitive is to think of B as the sample space now. And what is the preferred event? A preferred event is, A inside B that is A has occurred and B has occurred. So, a preferred event is A on the sample space B, which is going to be equal to.

So, B is what? The B is just going to be the same as before, this head in the first toss, h h h, h h t, h t h, and h t t. That is B. So, A and B. And what is A? A will just head in three tosses. Which is going to be head head head, two or more head, h t h, and h h t that is the end. Once you do this, again your restricted sample space B, your preferred event is this event.

And again, we have to go back to our idea that we are still in this equally likely setup. Outcome setting. So, our probability should be equal to, so the probability of A given that B has happened, B has occurred should be the size of A and B divided by the size of B. And that turns out to be what? Size of A and B is 3, size of B is 4.

So, this is one way of understanding this problem. So, suppose you want to know, like I said, you have this simple experiment of tossing a coin three times. And then you sort of, you know that there is a head in the first toss, and you interested in the chance of occurrence of A, if there is two or more heads, what you do is, you restrict sample space should B, that B has already occurred. So, you are inside B now, then what you are interested in is A and B, you will be equally likely outcome setting. So, the probability of A given B is just A and B divided by B as 3 by 4. That is one way of solving this problem.

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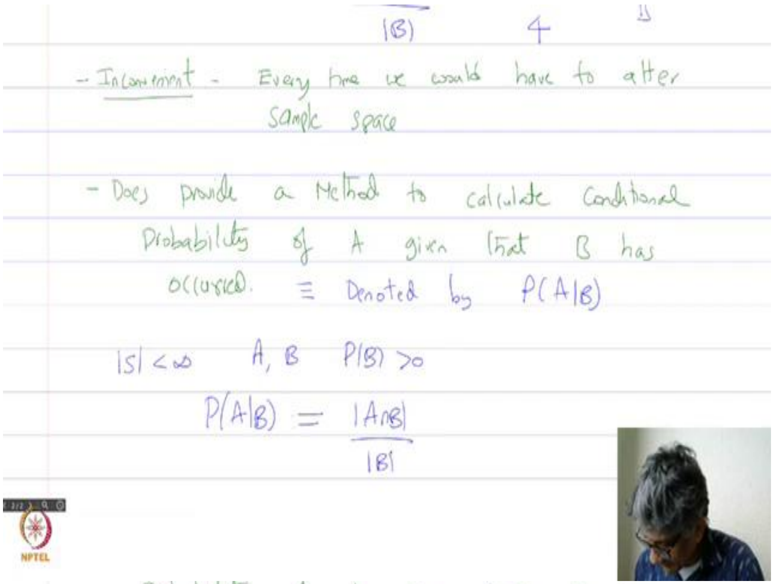

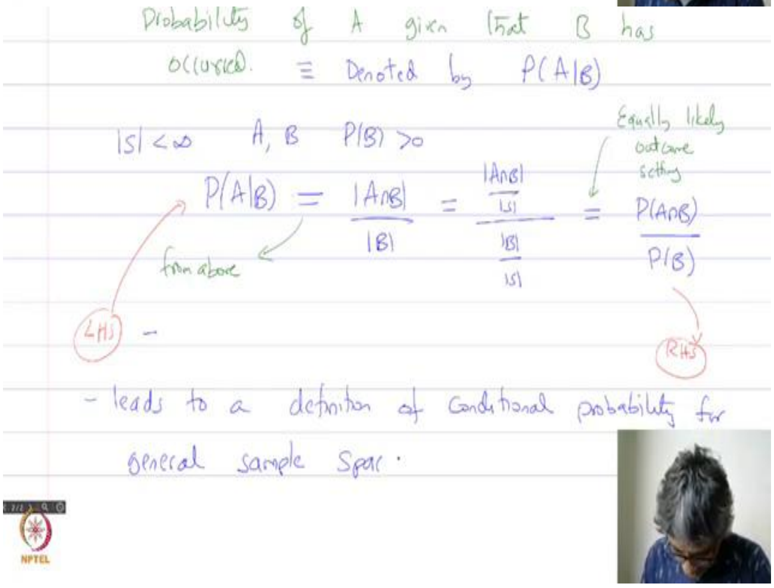

So, what are the pitfalls of this idea? So, one is that, it is a bit inconvenient. So, it is a bit inconvenient, because to solve a problem this way, every time we would have to change sample space. So, moment (())(12:46) is given, and you notice, if I rewrote the sample space, we will call it events, change your probabilities. But the idea is that the key is it does provide, a method to calculate the conditional probability, probability of A given that B has occurred. So, how does one generalize this?

Let us just see how to sort of it makes it a little bit more makes it provides a framework, let us try and see if we can grab the framework properly. So, the idea is that, this is denoted by probability of A given B. So now, how does provide the framework? So, the framework is the following. Suppose, so, the framework is, suppose mod S is finite, then, so this is, so, I want to understand I have two events A and B. And let say chance of B is positive, that is B has occurred. And probability of A given B.

The above says, is going to be A and B divided by A, is what the above says, this is from above. But what we do is, we should think about it a little bit more carefully. You divide top and bottom by S. We get A and B divided by S, the size of S, the whole thing divided by B and the size of S. Now, you know, if you are equally likely to the outcome setting, suppose equally likely to the outcome setting, this is going to be the probability of A and B at the top. And the bottom is going to be probability of B.

So, this leads to this idea. So, you can look at the left-hand side, that is what we are interested in understanding from here, and look at the right-hand side of the end up with. This provides a certain framework of understanding Conditional Probability. So, this leads to a definition of Conditional Probability, for general sample space. So, let me define it now.

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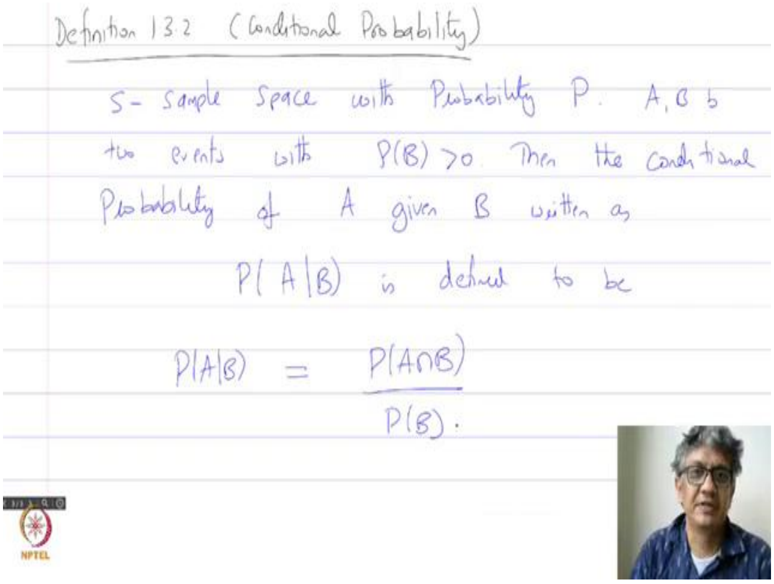

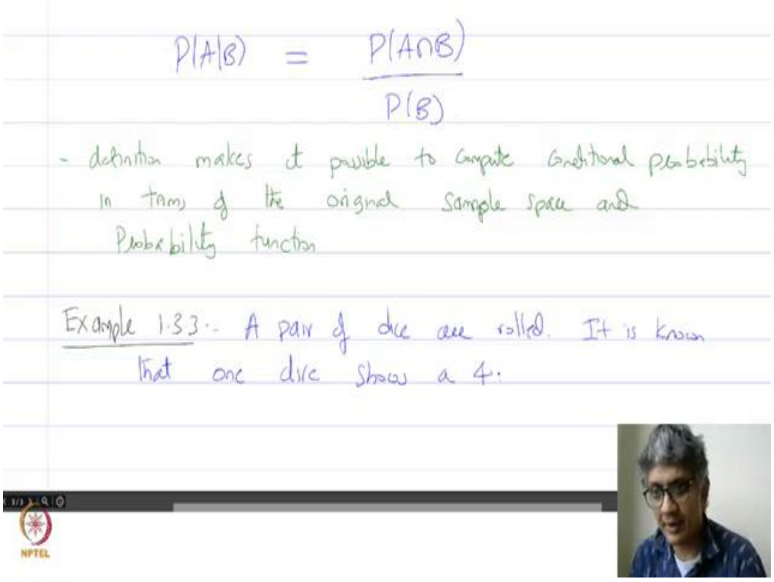

So, here is the idea. So, here is the key definition. The definition is 1.3.2. This is called Conditional Probability. So here I go and say that, let S be a sample space. So, any countable set with probability B. So, I am sorry, I just say a countable set. But a priory we do not have to assume that for this definition. S can be any sample space with probability P, A and B be two events, with probability of P is positive, then the Conditional Probability of A given B, which is written as the probability of A given B, so it is defined to be probability of A given B is given as probability of A and B divided by probability of B.

That is the definition of Conditional Probability we will use. So, the key idea is that, this definition allows us to sort of, definition makes it possible to compute Conditional Probability. The key is that you do not have to keep on changing sample space, in terms of the original probability, original sample space and probability function. Let us look at a quick example and just come back few steps. So, now, let me try the following, it is a really good example 1.3.3, here the example is following. A pair of dice are thrown or rolled and then let us say we have the event be known as it is known that one of the dice shows up shows a 4.