Introduction to Probability with examples using R

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Lecture No. 20

Functions and Independence

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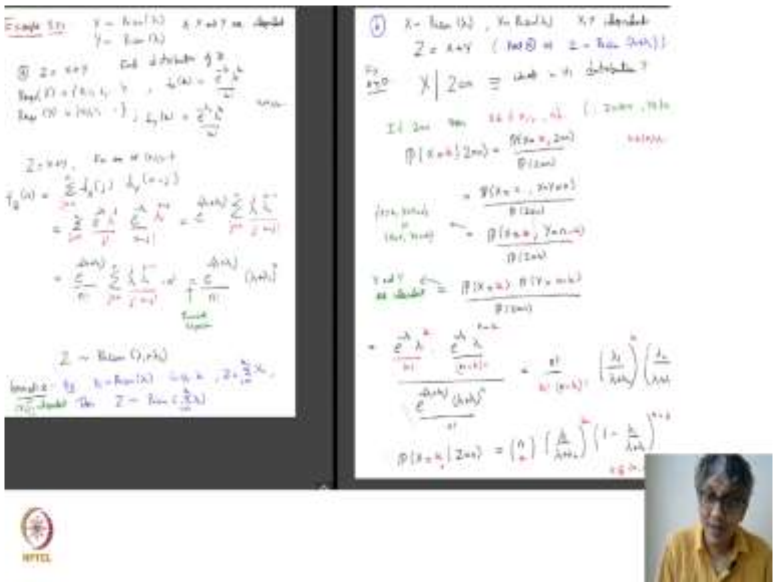

So, now the I just want to quickly recap what we did, we were doing so far. So, then the idea was that in we start with the idea that how to understand sums of random variables. So, we did the example of a to Poisson guys with independents. So, left hand side we immediately see that this let me take independent the sum is given by the combination the distribution of sum is going to convolution and once we have that then we are able to work out that an interesting fact that if Z Poisson adds up, so if X was Poisson lambda 1 one the lambda 2 with a independent then Z was Poisson lambda.

Then you can generalize this, this prove is by induction if you have k random variables there are all independent plus sums and then we also Poisson lambda i in then the Z gives you Poisson of the sum then we come here and then we notice that X Poisson lambda 1 Y is Poisson lambda 2. And I ask you with distribution of X given Z equal to n. We did this calculation using the definitional conditional property and we found out known behold if the chance that x is equal to k given Z equal to n was n choose k lambda 1 by lambda plus 2 equal power k times 1 minus c in fraction then we got n minus t.

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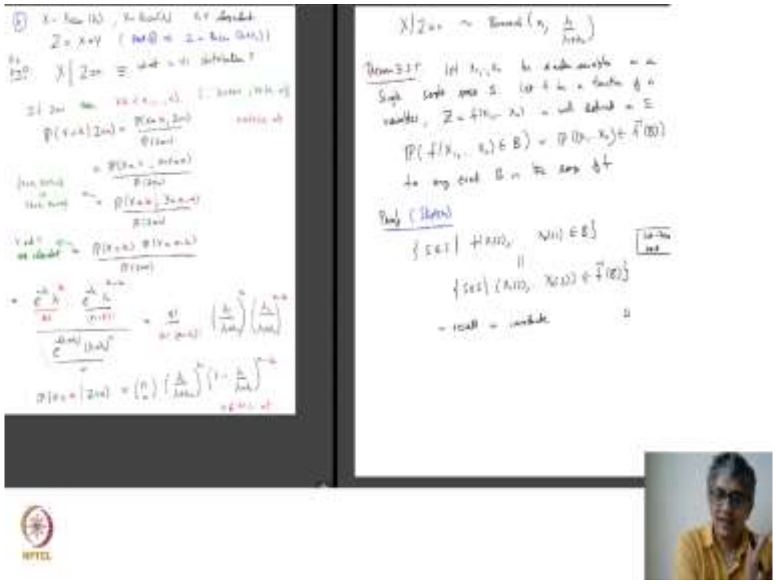

And this we figure out was the same as saying that X given Z equal to n was binomial n of lambda 1. So, an interesting fact if you have two Poisson your conditional sum then the individual Poisson becomes a binomial, so it is an interesting fact. And this whole this understanding distribution of function will be add in the following theorem is which have been using computationally on the example is that if your function and random variables then all is in the factors that then the function of x1 to xn lands in event P is the same as x1, x2, xn landing in the event f inversely. And once you capture this idea and everything as follows by is equal to. Let us move on to the next topic.

Student: Sir, can I just ask you a small question?

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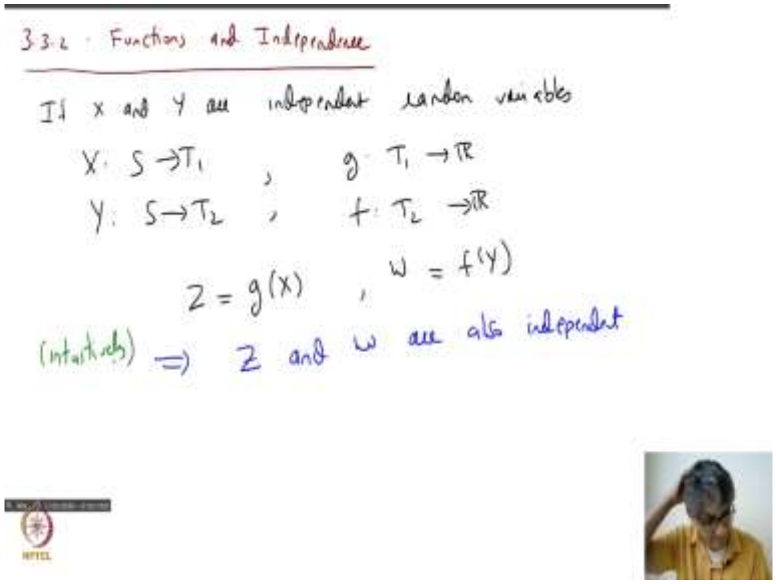

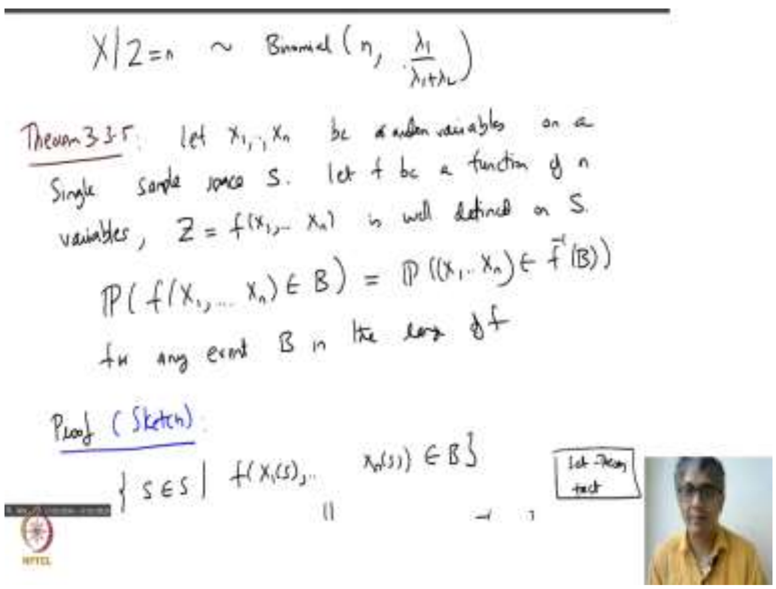

Professor: I want move to the split view split view is good. So, I will not understand this idea of functions and independence. So, it is a 3.3.2 is functions and independence. Sure.

Student: Can you hear me?

Professor: Can you ask it on chat is it possible? Let me carry on I will come back and answer to the question you asked. So now, so there was a question on the previous slide let us go back again let me repeat the answer so the argue was that, so we had shown in this computation that if X Poisson lambda 1 Y was Poisson lambda 2 and Z was X plus Y then distribution of X given Z equal to n was given by binomial comma lambda.

So, what I meant to say in terms of intuition of the following that this is the competition we have done. So, X given Z equal to. So, and the understood Poisson in this class so far as a limiting distribution of binomials. But the idea is that there is a interpretation of Poisson that you can use in the acquiring model where this interpretation on X given Z equal to n being binomial is very useful that is all I meant to say.

So, it is little bit outside the scope of this class but we will come and see there. I will try and see if I can go to any at some point family. Now let me go on to function and independence. So, now here is the idea is the following if X and Y are independent random variables. And then I have function f let say X takes value from S to T1, Y takes values from S to T2 then I have function g from T1 to R I have function f from T2 to R.

I look at Z equal to f of x sorry g of x I look at W equal to f y. So, there is no reason to believe so if X and Y are independent there is no reason to believe that Z and W will not be independent. So, this should actually imply that let me write this limit better so it is clear. Let me go back to black. So, g of x and W is equal to f of y there is no reason to believe that Z and Y are not independent. So, this will actually imply Z and W are also independent.

Because W depends on Y alone Z depends on X alone if X and W are independent X and Y independent then Z and W should be independent. This is something that is all classical into intuiting kind of make sense, so this into intuiting make sense.

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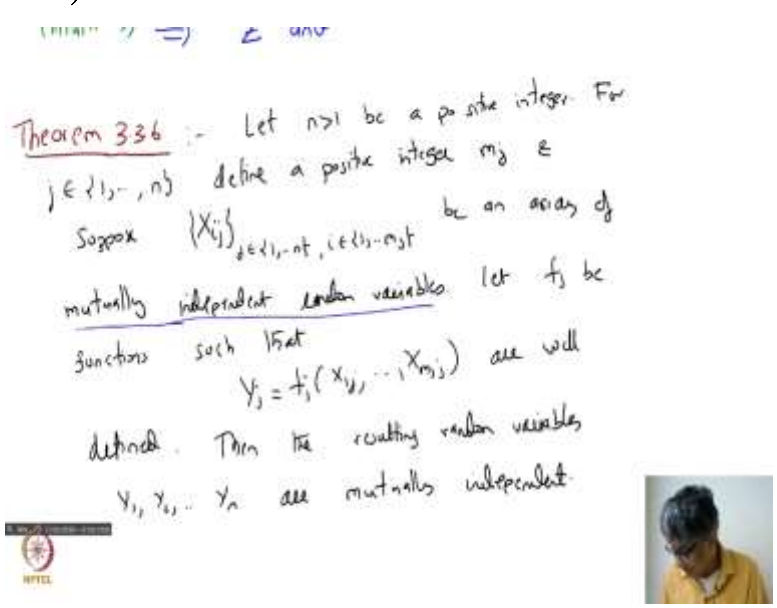

So, let me write a theorem down let us go and classify this properly. Theorem let n bigger than or equal to 1 be a positive integral. So, I take bigger than 1. So, let me add at least two random variables in my collection, so I write the general theorem down let us say I take j in the event 1 up to n I will set one of them.

Let define for j in this set 1 of 10 and define a positive integer m sub j. And suppose I have Xij. So, Xij is what? j is in 1 to n so j goes from 1j is in 1 up and i is from up to n subject. So, be an array of mutually independent and a variables for j in the 1 up to n. So, this already is I already said this no need to save this. Then I let f sub j be functions such that f sub Yj is f sub j of X1j up to Xnjj are well defined.

So, that means f sub j is well defined on this collection of f plan and variables are Xnj to Xmj so each of j acts on different spaces X of mj is perhaps to real line. Then so they are all independence so this crucial they are all independent variables independent collection then if I look at y sub j are also independent. Then the resulting random variable Y1, Y2, Yn are mutually independent. So, general fact about random variables that ones can sub use in sense that.

So, general theorem one can write down so in terms of function and a variables. So, if you have a random variable in array, then and the array is define as such a way that your separate functions and in acting on independent part of the array sorry and disjoint pass the array. Then the resulting sequence is also a mutually independent. So, general theorem now this side up to down tables. We will see recommend of two down tables.

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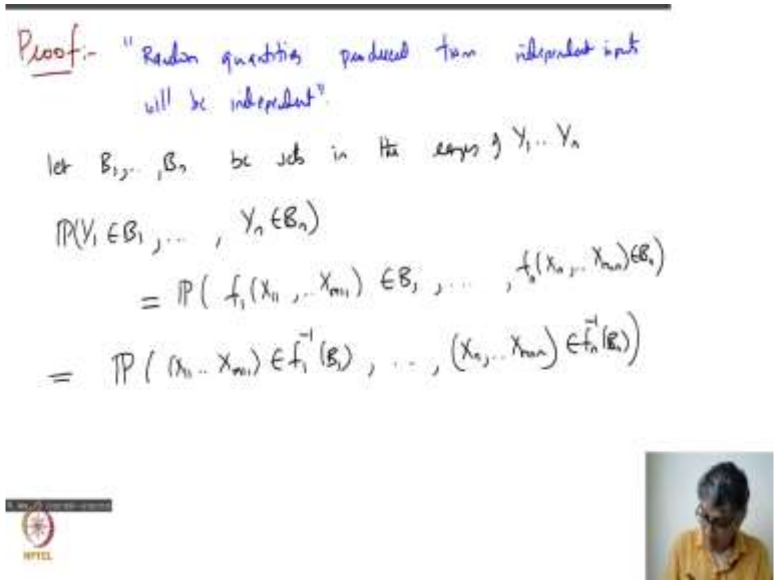

So, proof of this so what is prove in this? So, the prove is a so when add intuition profit theorem what the theorem saying with you the theorem is saying intuiting that random quantities. So, the theorem if I have to state in words random quantities produce from random in from independent inputs will be independent. That is what this theorem is saying. So, then up to for this so let b1 bn the sets in the ranges of Y1 through Yn.

So, then what I have to show I have to that the chance that Y1 in B1 all the way up to Yn and Bn this splits up as a problem. So, what is first line let see so here is interesting catch I will leave it as an open question for you guys to understand. So, the first thing is we write down this that is easy, the first thing is that watch Y1, Y1 is f of X11 up to Xm11 this is in B1 and all the way up to fn of X1n all the way up to Xmnn is in Bn, that is what this is of this is the same as the chance that if you look at each of these events look at X11 up to Xm11 is in f1 inwards of B1 all the way up to Xn Xmn and in fn inwards of Bn that this theorem.

Now you know that the Xi. So, the Xi is an independent Xij are independent are mutually independent. So, this becomes a product of j equal to 1 to n the probability of each of these of guys. So, you take Xj1, X sub mj1 so mjj. Let me say it is a Y let me just do the reverse in this. Because notation that will be say i equal to 1 to n. Let me just go at this matter m is equal to j, so j equal to 1 to m Xj at nj mjj is in the event fj inverse of B sub j this product.

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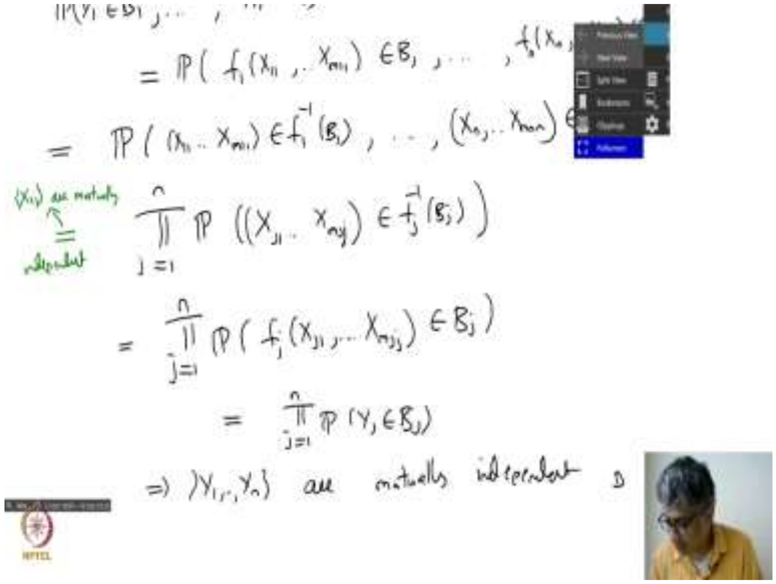

Now you just go back lap where you get the same as product of j equal to 1 to n probability that f sub j of Xj1 this is clearly little lagging behind X sub j of X sub mjj is in Bj. And that is the same as the product of j equal to 1 to n probability is in that Y sub j is in Bj. And this I have shown for any possible events Bi's which proves that is we imply that Y1, Y2 to Yn are mutually independent. Let me quickly recap the prove will in goes double split view and show it both sides.

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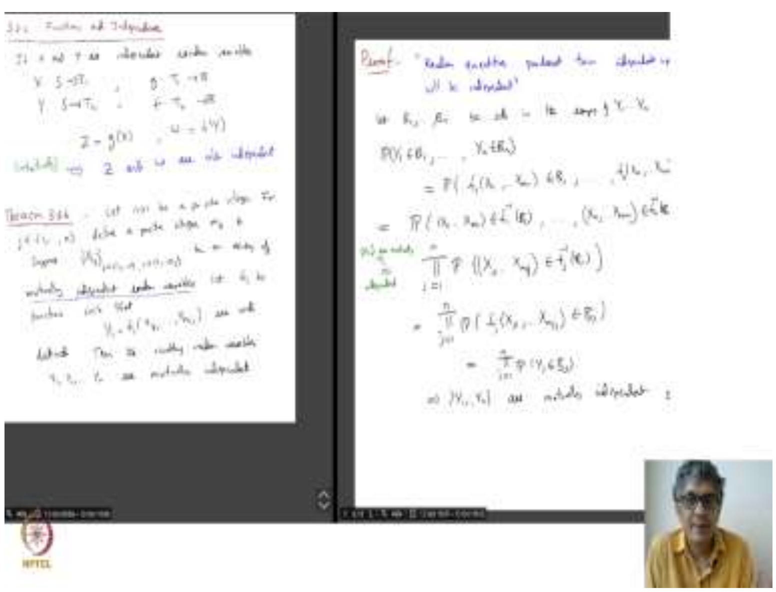

Clear? We will be doing this down little bit you can see the prove may be down you get let me go to previous page. So, this is what I trying to show. So, I said that functions are independent if this happens. If you have an array Xij and you have functions acting on disjoint pass the array these random variables acting under the independent. So, this what is the prove is very simple but there are the small exercise to do in the prove that the green part has to justify little bit.

But I will leave this exercise for you think about and the prove follows that the Y1 Yn are also mutually independent. So, the idea is that random qualities produce some independent inputs will always be independent.