Mathematical Methods 1
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Ordinary Differential Equations
Lecture - 70
Ordinary Differential Equations

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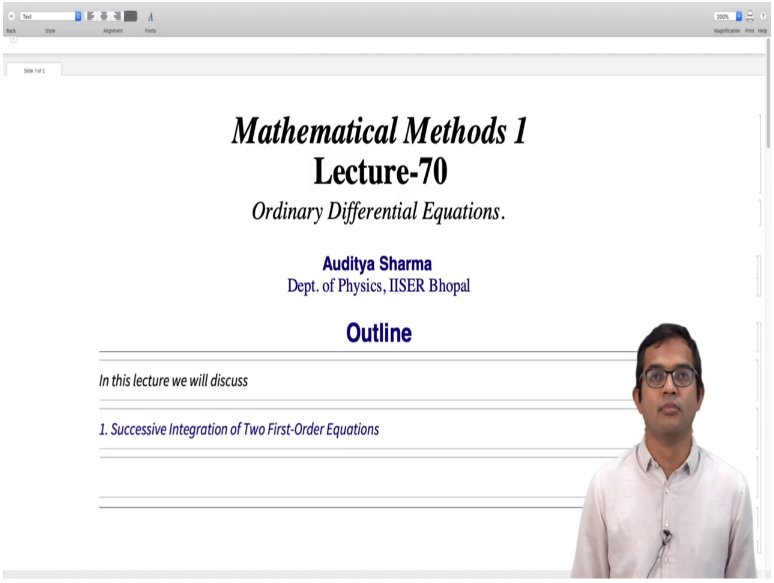

So we have seen how one can solve second Order Differential Equations which are linear and which have constant coefficients. We solved the homogeneous version, and then we looked at the inhomogeneous equation. So, in this lecture, we will look at a clever way of solving the same class of equations, but suppose we have forgotten about these methods right.

So, there is a way to just solve a second order differential equation simply from the knowledge of the first order differential equation. So, it's a, it ends up being two first order differential equations right provided you are working with linear differential equations, you know there is this prescription available. So, let us look at this clever approach in this lecture ok.

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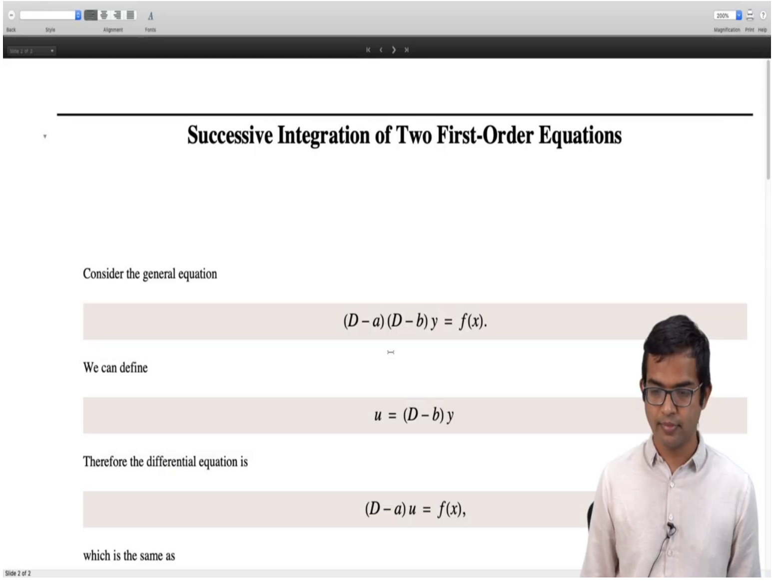

So, we are given you know this general equation D minus a times D minus b times y is equal to f of x right. So, you know the, you know our discussion started with you know putting f of x to be 0. We look at the homogeneous version from where we write down the solution for the homogeneous differential equation, and then we have to find a particular solution right.

We will look at how you know there are methods to solve to find the particular solution and so on right. Then we of course, if we can do this, then we have the solution. So, here we look at you know this differential equation D minus a times D minus b acting on y is equal to f of x as a two-step process. So, suppose u is equal to D minus b acting on y.

So, this is a first order operation acting on y is equal to u. And then so you see that D minus a times u is equal to f of x is a first order differential equation right. So, u is known, and f of x is known, a is just a constant. So, in fact, we should be able to solve for this u right.

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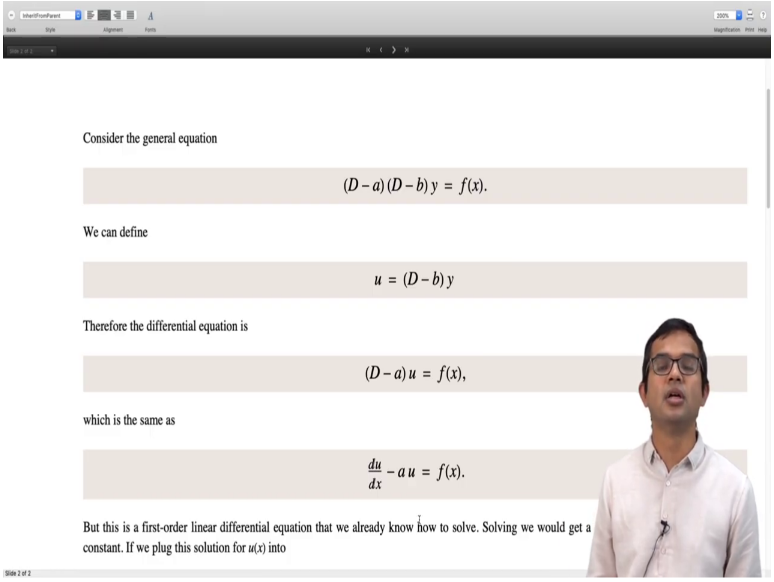

So, because this is nothing but D u by D x minus a u is equal to f of x. So, this is a first order differential equation. We have the prescription. We know how to find the integrating factor, multiply throughout by the integrating factor, then you have an exact differential equation. So, you write the left hand side as a total derivative. And then you just integrate, and then at least you have a formal solution for this right.

So, suppose you manage to find this formal solution right. Well, I mean it will depend on this function f of x how reasonable a function it is and so on the details you know one has to there may be struggle involved, but at least there is a principle you know a prescription available for this problem, and so that is what we are describing.

So, suppose you managed to find the solution u of x because it is a first order differential equation, and then we plug this back into our definition of u. So, but what is u? u is D minus b acting on y. y is unknown. So, we have D y by D x minus b y is equal to u of x. So, in fact, we can think of this u of x itself as like a forcing function for another first order differential equation right.

So, if we can solve this first order differential equation as well, so not only you know do would we like f of x to be a reasonable function, but it should yield for us another reasonable function u of x such that this equation also you know if we can integrate it out, and finally, we will have a solution y which will have two you know free constants right.

Because the first order equation will give us one free constant, and then when we integrate a second time that is going to give us another free constant. So, there will be two free constants which are characteristic of a second order linear differential equation right.

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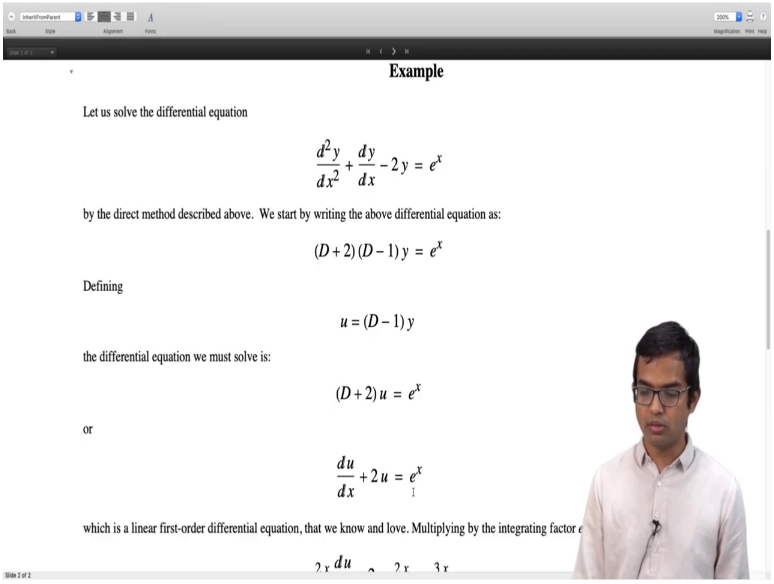

So, let us look at an example to illustrate how this works out. So, suppose we want to solve this differential equation d squared y by d x squared plus d y by d x minus 2 y is equal to e to the x right. We know how to solve this equation by the other method we have already described. We would find the homogeneous equation and find its two roots. Write down the complementary function, then look for a particular solution for this right.

So, first we will based on the complementary function, we can you know make the right ansatz for the particular solution, find the, you know coefficient method of undetermined coefficients we plugged it in plug it in. And extract the coefficients, we find a particular solution, and then we just simply add it to the complementary function. But here let us do it by the direct method that we just described.

So, we have D plus 2 times D minus 1 times y is equal to e to the x right. So, here well we could have you know factorized in either way could have thought of it as D minus 1 times D plus 2 times y equal e to the x which is which I will leave it to you as homework to work it out in the other direction. So, by the way I mean you see that the roots here are minus 2 and 1.

So, probably you would run into difficulties if you try an ansatz of the form e to the x, you will have to try x times e to the x right. So, this is if you are doing it by the other way which is to find a particular solution. But anyway all of that is bypassed here in this approach. So, we have D plus 2 times D minus 1 times y is equal to e to the x.

So, we define u is equal to D minus 1 times y. And so we must solve the differential equation D plus 2 acting on u is equal to e to the x, so that is du by dx plus 2 times u is equal to e to the x.

So, we have to multiply throughout by this integrating factor which is e to the integral it is just 2x in this case integral of 2 dx will be 2x. So, you have to multiply throughout with e to the 2x. And we have e to the 2x times du by dx you know there would be some constant which does not matter.

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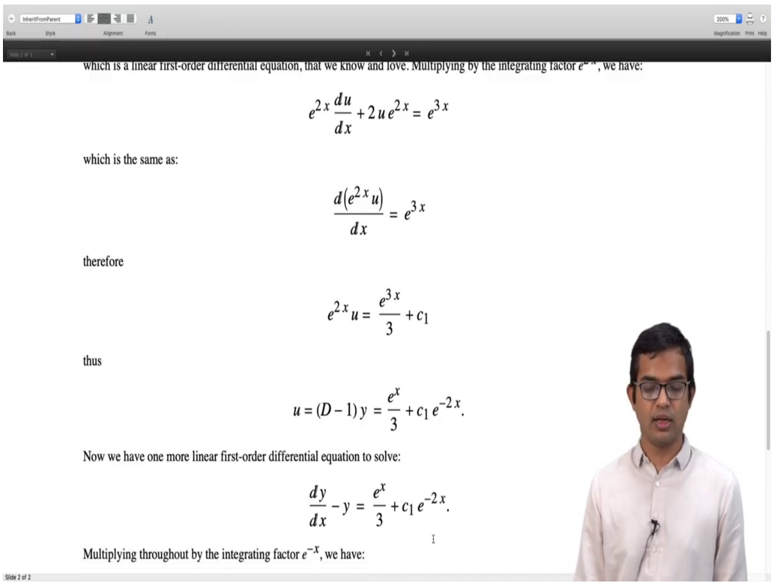

So, we have e to the 2x times du by dx plus 2 u e to the 2x is equal to e to the 3 x which is the same as saying the total derivative of the function e to the 2x times u as you can verify here is equal to e to the 3 x. Now, it is simply a matter of integrating both sides which yields for us e to the 2x times u is equal to e to the 3 x by 3 plus some constant. So, we get the first free constant here.

Now, we are only halfway through. So, we have to solve the other first order differential equation which is the definition of u, u is equal to D minus 1 acting on y. So, which y is unknown, so u is equal to D minus 1 times y. And since u is from here we have already solved for u which is e to the x divided by 3 plus c 1 times e to the minus 2x.

So, this is the second differential equation that we must solve first order linear differential equation. So, we have dy by dx minus y is equal to e to the x by 3 plus c 1 times e to the minus 2x. We know how to solve this. Again we have to integrate to find the integrating factor.

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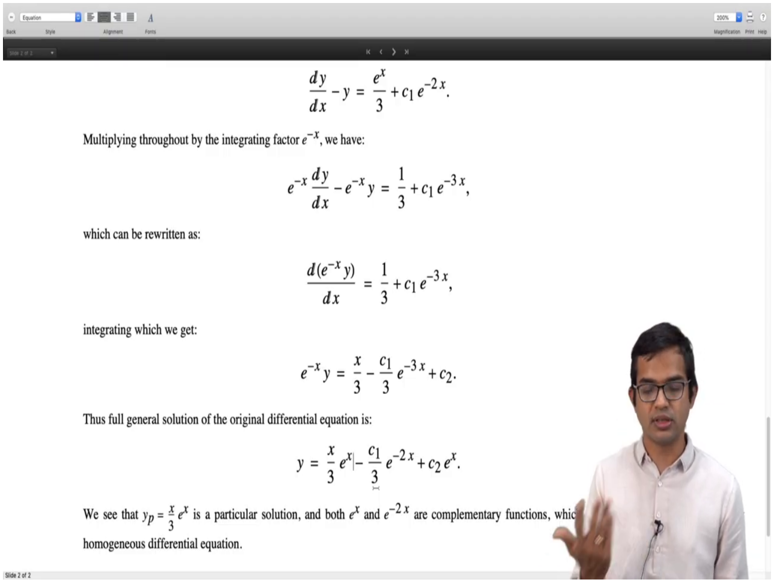

So, which in this case is just e to the minus x multiply throughout with e to the minus x. So, we have you know the right hand side simplifies you have e to the x into the e 2 minus x will become just 1 over 3 plus c 1 times e to the minus 3x. Now, the left hand side can be written as a total derivative its d by dx of e to the minus x y is equal to one-third plus c 1 times e to the minus 3 x integrate both sides, we get e to the minus x times y is equal to x by 3 minus c 1 by 3 times e to the minus 3x plus another free constant c 2.

So, the full solution of the general equation now we are able to write down as y is equal to x over 3 times e to the x right, we divide throughout by e to the minus x. So, we get e to the x here, and minus c 1 by 3 e to the minus 2x plus c 2 times e to the x, so that is a full solution right. So, now looking at this full general solution, we can actually you know see how we would have perhaps got the same answer if we had used the other method right.

So, we see that y p is equal to x over 3 times e to the x is in fact you know the particular solution it is a particular solution and then you know this minus c 1 by 3 does not matter right we could have just called this minus c 1 by 3 as some constant c 1 e to the minus 2x and e to the x right these are the you know two solutions of your homogeneous differential equation which we could have directly got from here D plus 2 into D minus 1. So, the roots would be minus 2 and plus 1.

So, the solutions would be e to the minus 2x and e to the x right, so which is what we see here. So, there is x by 3 times e to the x which is a particular solution plus some constant times e to the minus 2x plus another free constant times e to the x right.

So, we have seen how there is this alternate method in which we could adopt brute force for a second order differential equation and then solve two consecutive first order differential equations and get to the answer. That is all for this lecture.

Thank you.