Real Analysis - I

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Week - 08 and 09
Lecture- 85
Introduction

Welcome to weeks 8 and 9 of this course on Real Analysis. We are now past the halfway mark; I am glad that you could make it this far, weeks 8 and 9 is the core of this course.

In week 8, we study differentiation and in week 9, we study integration; because you have already seen much of this material in high school, we are going to go really fast. One thing you will realize is that, with this new technology of epsilon, deltas, convergence and so on; most of the proofs in both differential and integral calculus become quite easy.

Some of them might be long, but nevertheless there is nothing really difficult about these proofs. In the section on differentiation, I want you to carefully note one thing; we are going to interpret the derivative not only geometrically as the slope of the tangent, but also algebraically as the best linear approximation of a given map.

This is in some sense the new perspective that you are going to see now. Linear algebra is the study of linear maps and in recent times because of the popularity of both data science and quantum computing, it has come to the forefront of science.

Linear algebra is very well understood, you know a ton of stuff about linear mappings. Therefore, having a linear approximation to a complicated map will greatly simplify the situation for you; because now you have at your disposal all the tools of linear algebra to tackle the problem at hand, that is why this new emphasis is very important. So, make sure you understand in what sense is the derivative the best linear approximation. Apart from this subtle point, there is nothing much difficult in this section.

The central theorem is Lagrange's mean value theorem that you have no doubt spend some time in school; we will prove more or less everything using Lagrange's mean value theorem. I also want you to focus on the version of Taylor's theorem that I have given in this particular week.

Taylor's theorem is a really important, simply because it allows us to approximate functions. In the real world, more often than not you can never compute functions exactly; you need good approximations and Taylor's theorem is very very useful in such scenarios.

Now on to the week 8 sorry, week 9 which is on integration. Now, integration as you all know in high school is first treated as the opposite of the differentiation or taking anti derivatives.

And later you make the connection precise by defining the Riemann integral and then saying that the fundamental theorem of calculus is what makes integration the reverse of differentiation. The approach I take in this particular week is quite different; we directly begin with the Riemann integral. And the Riemann integral is motivated by the physical problem of calculating areas.

So, we briefly discuss what we mean by area. Unfortunately we cannot do a full in just one week; in fact it is not even possible to do a full treatment in 6 weeks, so I give you an appropriate reference where you can read more.

Having made the connection between areas and integration; we prove the fundamental theorem of calculus first, only then do we proceed and prove various stuffs about the integral. In this week, there will be one really challenging theorem; that is the Riemann Lebesgue criteria for when function is Riemann integrable.

We have to introduce a concept called sets of measure 0 and give a condition. Now, this is usually a graduate level topic, but I have chosen to include it in this undergraduate level course; simply because it is this theorem that makes all the proofs of the various basics properties of the Riemann integral transparent.

So, for this reason of deeper understanding, I have chosen to include a difficult theorem. Please spend a lot of time on this theorem; because once you learn this theorem, everything else in integration will become more or less trivial.

One more thing you would have noticed is that, I hardly emphasize calculating integrals. Now, the reason for this is twofold; number 1 that is not the aim of this course, the aim of this course is not to make you a really fast calculator of integrals, number 2 that is not no longer necessary.

We have now tools like wolfram alpha that can compute really complicated integrals faster than most human beings. So, there is no necessity to involve oneself in several months of studying some dozen techniques of integrals and knowing what to substitute in which integral, that is completely unnecessary for the time being.

Therefore, I am not focused on the computational aspects. But if you are interested on the computational aspects; there are dozens and dozens of calculus books which deal with this extensively. So, all the best for weeks 8 and 9, do well; concentrate on the theorems that I have told you to concentrate on and you will have a deep understanding, all the best.