Real Analysis - I
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Lecture - 16.4
Limits of Polynomials

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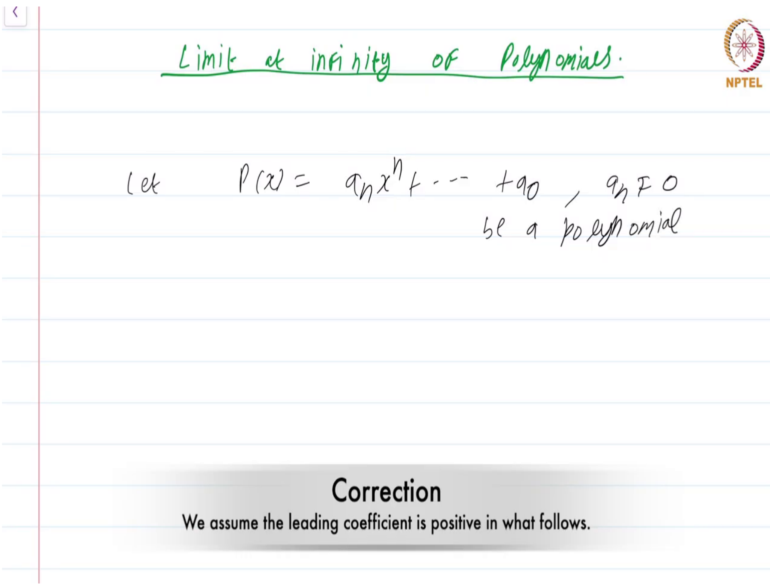

In this short module, let us see what happens to polynomials as x goes to infinity.

Let $P(x) = a_n x^n + ... + a_0,\ a_n \neq 0$ be a polynomial that is non constant. So, we are not obviously considering the constant polynomials. We want to study what $\lim\limits_{x \to \infty} P(x)$. This is actually going to be equal to ∞.

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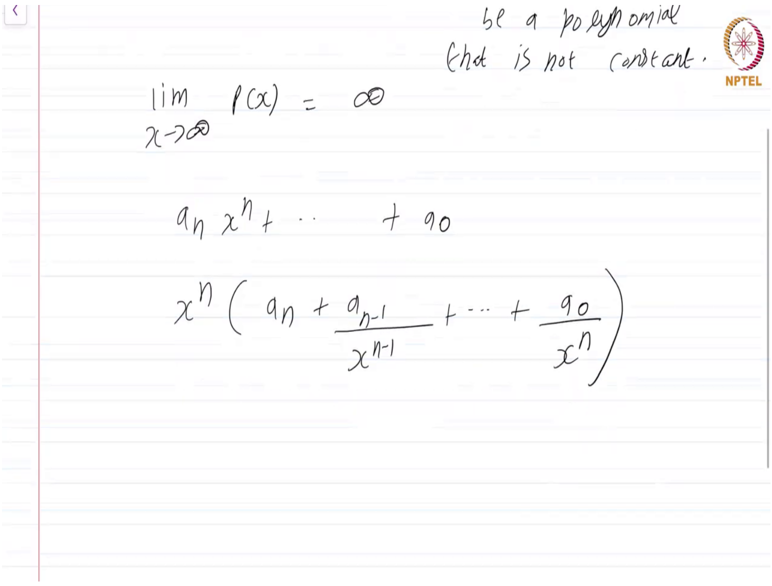

Let us see why this is the case. So, you have this $a_n x^n + \ldots . + a_0$. What you do is you take x^n outside, you get $x^n(a_n + \frac{a_{n-1}}{x^n} + \ldots . + \frac{a_0}{x^n})$. Now, we already know that $\lim_{x \to \infty} \frac{1}{x} = 0$. In fact, $\lim_{x \to \infty} \frac{1}{x^k} = 0$.

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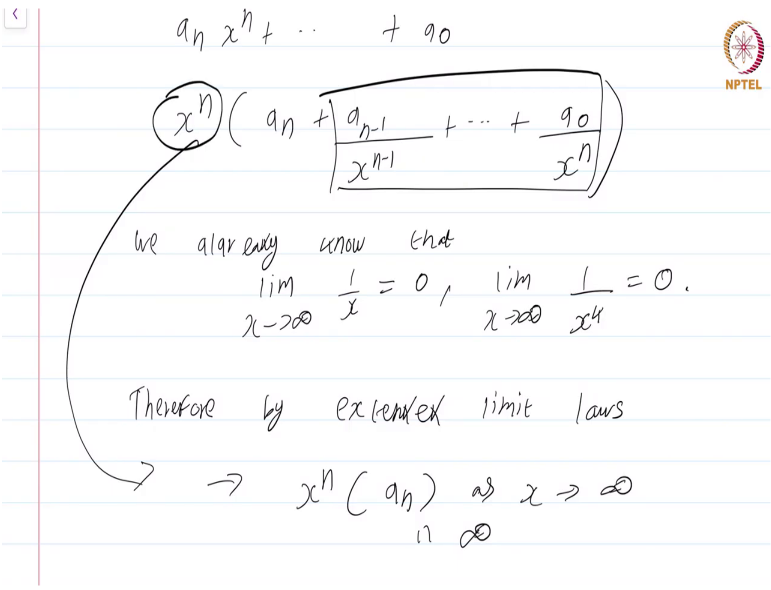

Therefore, by extended limit laws which is part of the exercises; by extended limit laws, we get that this just converges to $x^n a_n$ as x goes to ∞. So, essentially what happens is all these terms become really small and this term the leading coefficient x^n dominates the entire polynomial and as x goes to infinity, this goes to infinity. So, once you solve the exercises, this proof will become rather trivial. So, the upshot of all this is P(x) goes to infinity as x goes to infinity.

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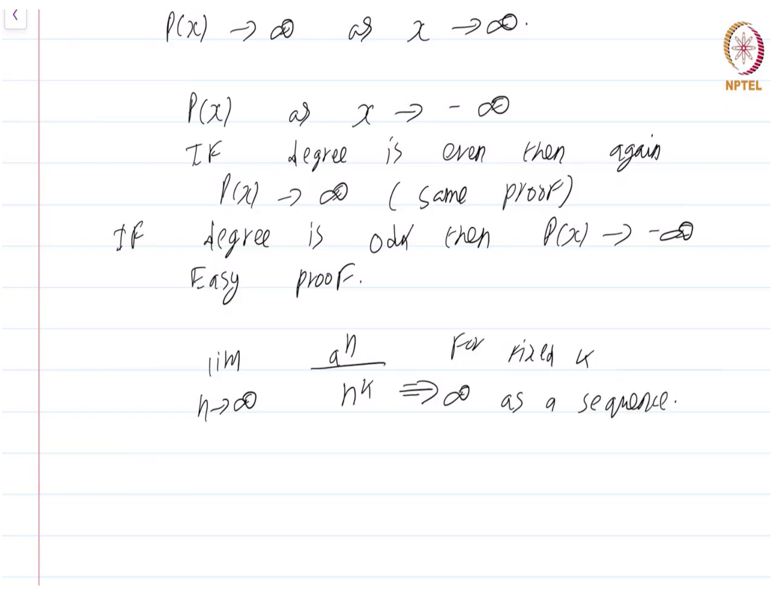

Now, what about P(x) as x goes to $-\infty$? Well, it depends on the degree. If degree is even, then again P(x) converges to infinity. Same proof will work if the degree is odd, then P(x) converges to $-\infty$. This is just again easy proof. What happens is when the degree is odd, the powers will still continue to be negative power will still be negative and you will get that P(x) converges to $-\infty$.

So, we have seen now that polynomials behave in an unbounded way as you approach infinity. Now, one more comment I would like to make, we have already seen that $\lim_{n\to\infty} \frac{a^n}{n^k}$, for fixed k this goes to infinity as a sequence. Now, by using the same idea and by using the definitions of limit going to infinity, you can solve this easy exercise.

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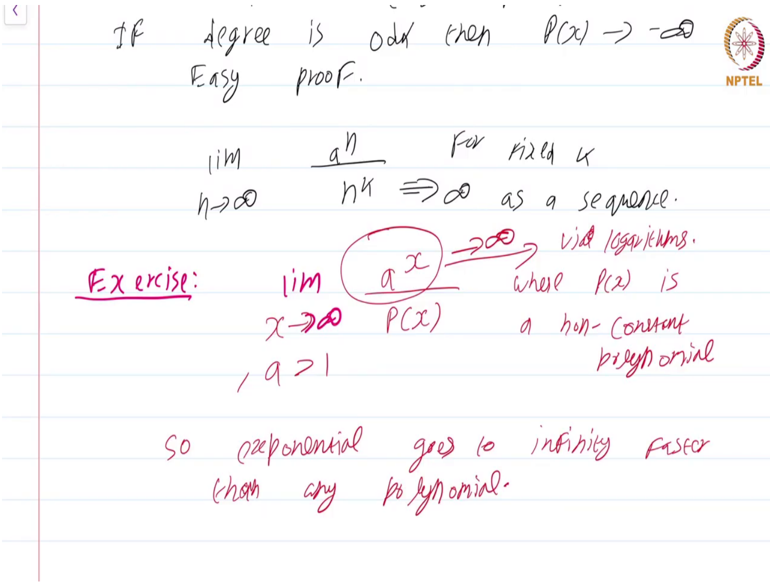

So, $\lim_{x\to\infty} \frac{a^x}{P(x)} = \infty$, where P(x) is a non-constant polynomial, a> 1.

So, exponential goes to infinity faster than any polynomial. So, the definition of a^x, if you recall, is via logarithms which we have not yet which we have not yet seen in rigorous detail. But your high school and even lesser, middle school knowledge of logarithm should be more than enough to tackle this exercise.

So, let me just repeat. For this, you just need to use the various properties of infinite limits combined with what you have already seen for sequences. So, please prove this. This is very important. It illustrates the power of exponential growth; exponentials quickly dominate any polynomial.

This is a course on Real Analysis and you have just watched Limits at infinity of Polynomials.