Real Analysis - I
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Lecture – 15.6
Continuity of Square Root

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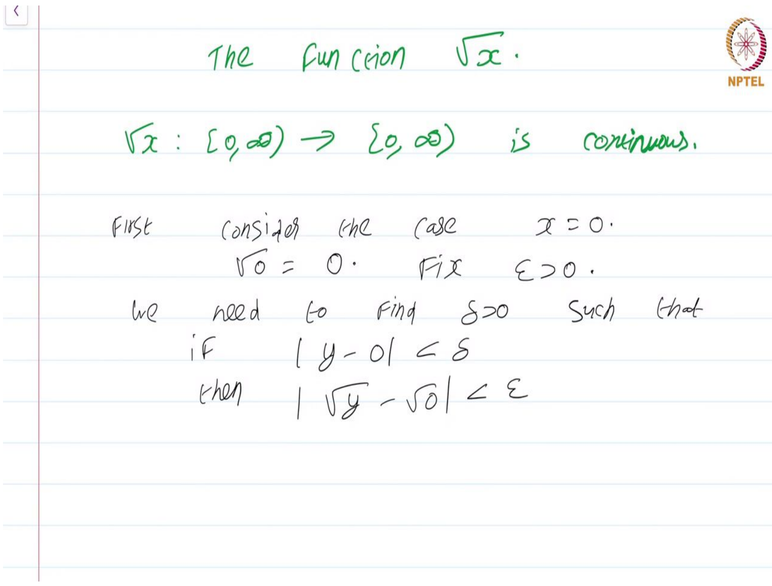

Let us now prove that the function $\sqrt{x} : [0, \infty)$ is continuous. Now, how do we show this? Well, we resort to the $\epsilon - \delta$ definition. This can be done using sequences also; but as an illustration, let me do it by $\epsilon - \delta$. First consider the case x = 0. Well, $\sqrt{0} = 0$. So, what we are asked to do is we have to somehow show that the $\epsilon - \delta$ definition is satisfied at the point 0. So, fix $\epsilon > 0$.

We need to find $\delta > 0$ such that if $|y - 0| < \delta$. Then, $|\sqrt{y} - \sqrt{0}| < \epsilon$. So, essentially we have to find a δ such that $|y| < \delta$ implies $\sqrt{y} < \epsilon$. I can ignore the modulus here also.

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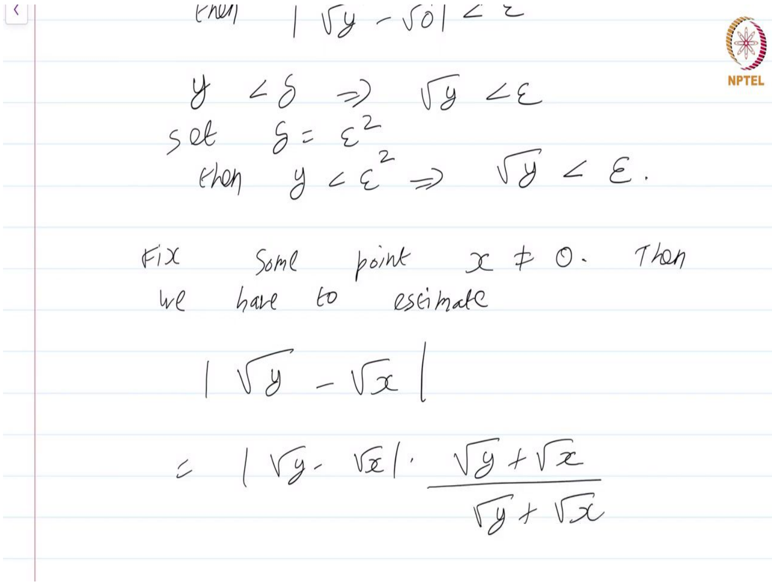

If I set $\delta = \epsilon^2$ then, $y < \epsilon^2$ implies $\sqrt{y} < \epsilon$. So, showing continuity at the point 0 turned out to be quite easy. Now, fix some point $x \neq 0$. Then, we have to estimate, $|\sqrt{y} - \sqrt{x}|$.

Now, we use the oldest trick in the book. This is equal to $|\sqrt{y} - \sqrt{x}| \cdot \frac{\sqrt{y} + \sqrt{x}}{\sqrt{y} + \sqrt{x}}$. I can divide and multiply by this. One reason for that is the denominator is not 0; $\sqrt{y} + \sqrt{x}$ is not 0 here. Now, what do we do? Well, we observe that this numerator is nothing but $|y - x|$.

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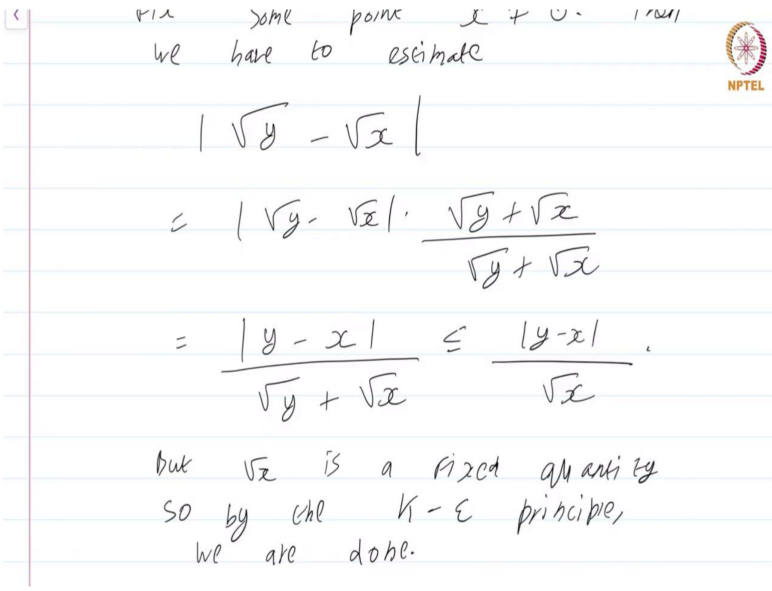

All I have done is $\sqrt{y}+\sqrt{x}$ is positive. I have put a modulus and taken the $\sqrt{y}+\sqrt{x}$ inside, multiplied and got $\frac{|y-x|}{\sqrt{y}+\sqrt{x}}$. Now, this is certainly going to be less than $\frac{|y-x|}{\sqrt{x}}$. Because the $\sqrt{x}$ would again be a non-negative quantity or it could be 0.

So, I am just getting rid of something that will make the denominator larger. So, I can write this inequality. But $\sqrt{x}$ is a fixed quantity. So, so, by the $K-\epsilon$ principle, we are done. That was fairly easy. Please as an exercise show the same thing, using sequences.

This is a course on Real Analysis and you have just watched the module on the function square root of x.