Real Analysis - 1
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Lecture - 14.3
Negating Continuity

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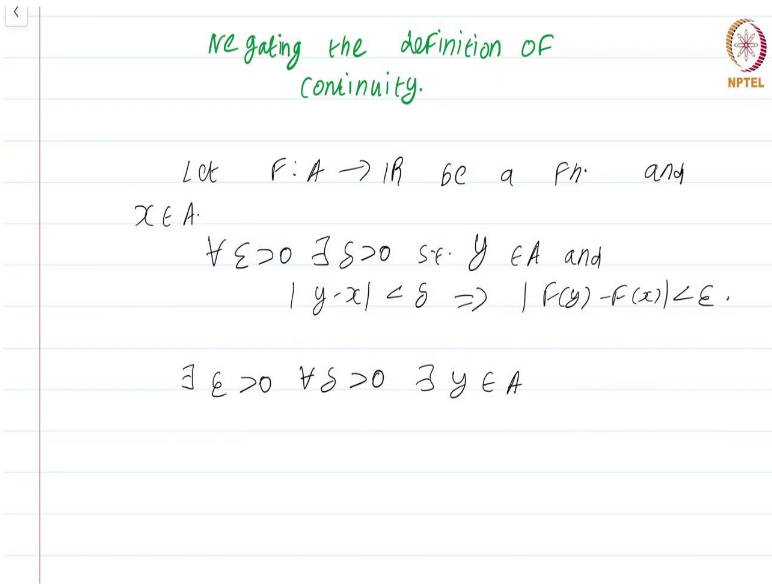

Since the definition of continuity is central to this course. It might be a good idea to learn how to negate the definition properly. If not for anything to use in proofs. Let us write down the definition of continuity in logical notation. So, again the setup is

Let $F : A \longrightarrow \mathbb{R}$ be a function and $x \in A$.

Now, we say that F is continuous at x if for each $\epsilon > 0$, $\exists\ \delta > 0$ such that if $y \in A$ and $|y - x| < \delta \implies |F(y) - F(x)| < \epsilon$. So, this is the logical notation for the definition of continuity at least the $\epsilon - \delta$ version.

Now, how would you negate this? Well, it is pretty simple. This for all ϵ becomes there exists $\epsilon > 0$ and the negation sign moves inside. Thus, there exists $\delta > 0$ becomes $\forall \delta > 0$

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For all $\delta > 0$ such that $y \in A$ and $|y - x| < \delta \implies |F(y) - F(x)| < \epsilon$ means there exists $\epsilon > 0$, $\forall \delta > 0$, $\exists y \in A$. So, to understand why 'there exists' comes let me change this such that and make it for all y.

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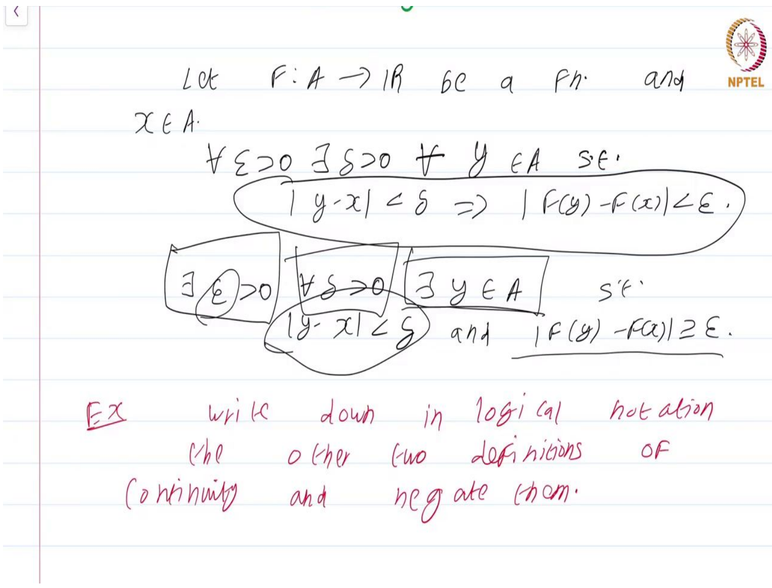

For all $y \in A$ such that $|y - x| < \delta \implies |F(y) - F(x) < \epsilon$. You will understand why I phrased it this way; it will become very clear because only then do you see that there is a there exists y in A such that $|y - x| < \delta$.

So, how do you negate an implication? Well, the antecedent should be true, but the consequence should be false such that $|y - x| < \delta$ and $|F(y) - F(x)| \geq \epsilon$.

So, what is this definition saying there is some $\epsilon > 0$ such that no matter what δ you choose, there will be some element $y \in A$ that satisfies $|y - x| < \delta$, but nevertheless $|F(y) - F(x)|$ is greater than or equal to ϵ.

So, this choice of ϵ plays spoilsport. What this is saying is no matter what δ you choose that. In other words no matter how close you get to the point x there is always some other y that is closer to x such that $|F(y) - F(x)|$ cannot bring an ϵ close.

So, this is the formal way of negating the definition of continuity. So, I am going to leave it as exercises for you.

To write down in logical notation, the other two definitions of continuity and negate them.

This is a course on Real Analysis and you have just watched the module on Negating the Definition of Continuity.