Real Analysis - I

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Week – 5 to 7
Lecture – 51
Introduction

Welcome to weeks 5 to 7 of this course on Real Analysis. In these weeks, we are going to be studying topology. What is topology? To answer this question, we must first ask ourselves what is mathematics? Mathematics comes from the Greek word mathema which means knowledge.

It is in ancient Greece that our story begins. So, warm summer evening in ancient Greece circa 600 BC, you have just finished your shopping in the local market are AGORA, and there you look up at the night sky and you see that some of the stars seem to move. So, you name them planet S or wanderer. 2200 years later Sir Isaac Newton finally figured out what makes these planet S move and how do they move.

Isaac Newton derived or proved Kepler's three laws of planetary motion. And to do this, he had to invent calculus. Almost contemporaneously Gottfried Leibniz also invented the tools of calculus. They later got into a bitter priority dispute, but that is not much relevant to this course. The issue is neither Newton nor Leibniz had a strong logically rigorous foundation to calculus. Both of them used the idea of a limit informally.

You have already seen limits in high school. What is the meaning of the function f(x) approaches or tends to or limits to a value l as x approaches a value a? This was probably left undefined in high school. We are now going to make this precise, and our approach is through topology.

Topology comes from the Greek word topos which means place. Topology is the abstract study of geometrical properties; in some sense topology is the study of space. Topology in the 20th century and the 21st century has exploded into a major branch of mathematics both pure and applied.

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Topology is extensively used in MRI machines; it is used in video games.

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The objects that topology studies are exotic objects like a one sided surface the mobius band.

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Coffee cups and donuts, and even the very shape of the universe are all studied in topology.

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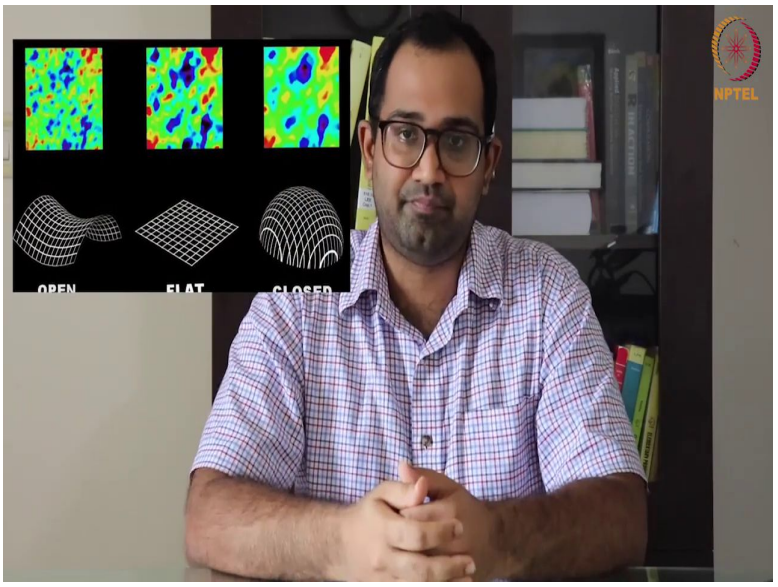

We will confine ourselves to those aspects of topology of the real line that are useful in analysis. Indeed one of the first English language texts in topology - General topology by Kelley, in its preface the author writes I wanted to name this book What Every Young Analyst Should Know, but unfortunately his publisher did not allow him to have such a fun title. We will be studying those aspects of topology that we will use in calculus. The key property that I want you to focus on is compactness.

Compactness is an abstract property that subsets of the real numbers can have that mimics closed and bounded intervals. Indeed, we will see that a number of famous properties of continuous functions that you are familiar with like continuous functions on a closed and bounded interval attained both its maxima and minima are actually a consequence of compactness.

We will also study some other properties of topology of the real numbers, namely connected sets and also perfect sets. These are less important because at least on the real line these properties are not so complicated. In a later course, if you get the chance, please study topology. You will encounter a lot of visual and geometric stuff that you ordinarily do not see in other branches of mathematics.

All the best for these 3 weeks.