

Real Analysis - I
Dr. Jaikrishnan J
Department of Mathematics
Indian Institute of Technology, Palakkad

Lecture – 11.5
Resolving Zeno's Paradox

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The slide features handwritten notes in green and black ink on a white background with horizontal lines. At the top, the title 'Resolving Zeno's Paradox' is written in green. Below it, two points are listed in black ink: '1. Obviously Zeno is wrong from our experience.' and '2. Mathematical resolution using infinite series.' A rectangular box is drawn around the text 'An infinite number of events cannot take place in a finite amount of time.' Below the box, the question 'Why should I believe this?' is written. In the top right corner, there is a small circular logo with a star and the text 'NPTEL' below it.

We are now going to Resolve Zeno's Paradox using the ideas that we have seen of convergence of infinite series. If you do not recall this paradox please watch the module on Zeno's paradox way back in week 1, again.

Now depending on which philosopher you ask they might claim that Zeno's paradox is completely resolved to Zeno's paradox is nowhere close to being resolved. Such things happen in philosophy because in philosophy you want to analyze a situation completely from infinite depth as much as possible.

So, when you look at it as a philosophical problem you might get debate as to whether this paradox is resolved or not. However, I am not going to such philosophical depth and analyze this completely, you can please check the Stanford Encyclopedia of Philosophy for a detailed article on Zeno's paradox and its various controversies. Rather I am going to highlight that; if you are willing to take for granted what we have done in infinite series it is not hard to see where Zeno got it wrong.

Again I apologize if you are a philosopher and you get offended by my cursory treatment, but this is nevertheless a mathematics course and not a course on philosophy. So, recall that Zeno claims that Achilles will never be able to catch the tortoise, but in reality we all know that Zeno's paradox is certainly the situation, that is Achilles will not catch the tortoise is not going to happen.

Now there are two possible resolutions first, obviously Zeno is wrong, from our experience. Essentially what Zeno is saying is that it cannot happen that Achilles will ever catch the tortoise that certainly is not true in the real world. Moreover you can slightly modify Zeno's argument to show that motion itself is not possible that objects can never move, you can repeat this same argument by replacing Achilles by any object.

And to say that for the object to move one meter it must first move half meter then it must move one-fourth of a meter so on and so forth. So, it will never be able to move at all right. Clearly objects do move in the real world and therefore there is no paradox at all whatever reasoning Zeno's Zeno used is completely wrong, because it is simply not applicable in the real world.

Now, this is a very practical resolution of the paradox; however, it still does not satisfy us intellectually. It is sort of appealing though yeah this is not the case therefore we are done. So, this is not very appealing. So, let us try for a more mathematical resolution using infinite series.

Now, the question arises; what does it mean to resolve this paradox right? To resolve this paradox is to see and point out some flaw in the reasoning used by Zeno. If you think carefully about Zeno's paradox the central assumption that Zeno is making to derive this paradox is that an infinite number of events cannot take place in a finite amount of time.

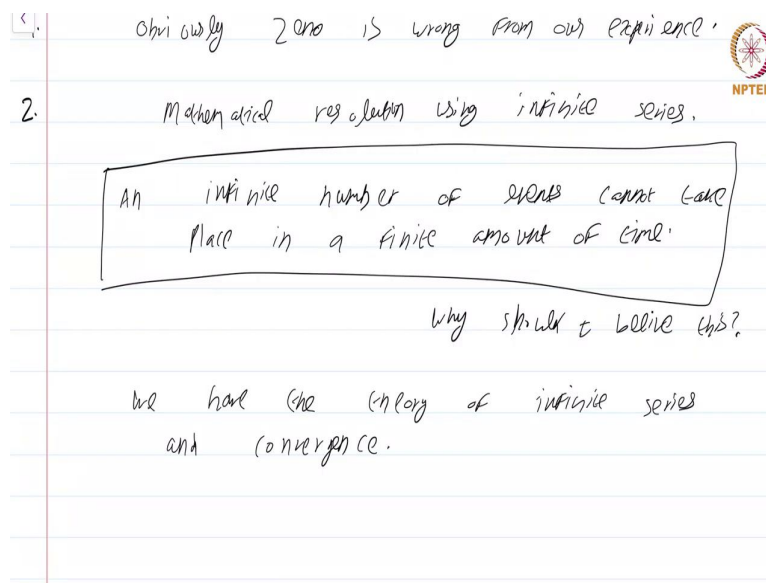
This is the central assumption behind Zeno's reasoning as to why Achilles will not be able to catch the tortoise. The infinite number of events is Achilles first covering half the distance to the tortoise, then covering half the remaining distance plus whatever little the tortoise has moved and so on and so forth right. So, an infinite number of events cannot take place in a finite amount of time.

The point is it is up to Zeno to devise an argument to say why should I believe this? Since the central argument that Zeno makes relies on this assumption the burden of proof is entirely on

Zeno to convince me that this assumption in the box is actually an appealing assumption to make.

Now, you might counter and say suppose there is a person who is ambiguous, he has not yet made up his mind whether this assumption is true or false, why should such a person not believe this assumption or rather why should such a person reject this assumption.

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Well, we have the theory of infinite series and convergence where we sum up infinitely many things and get a finite answer not only that we can manipulate infinite series.

We have seen algebraic properties of infinite series, we have proved non trivial theorems about infinite series, we have even seen a real world scenario with the chocolate wrapper example that shows that summing up infinite series comes up all the time in the real world. So, there is evidence to say that an infinite number of events can happen in a finite amount of time.

We have a mathematical model for the theory of infinite series that allows us to some infinitely many things and get a finite quantity. So, it is not the case that an infinite number of events cannot take place in a finite amount of time. It does not seem to be true or at any rate there is some evidence to reject this hypothesis and that evidence is this entire theory of infinite series.

So, it is up to Zeno and those philosophers who agree with Zeno to convince me or to convince anybody that this assumption is not really a bad assumption; they have to somehow justify that. Till that justification comes we can always say this paradox is not a paradox we have the theory of infinite series.

This is a course on Real Analysis and you have just watched the module on the resolution of Zeno's Paradox.