Real Analysis - I
Dr. Jaikrishnan J
Department of Mathematics
Indian Institute of Technology, Palakkad

Lecture - 1.4
Wason's Selection Task

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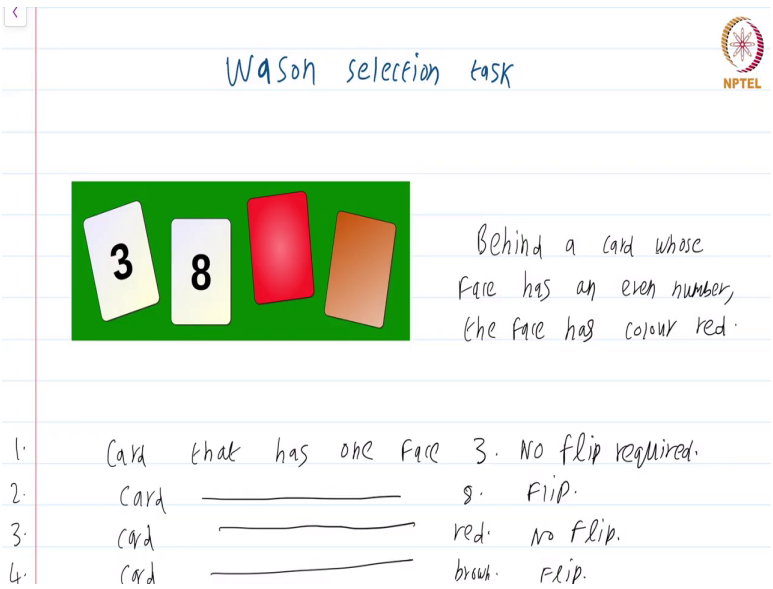

In the last module, we saw a few examples of results of calculus that seem geometrically obvious, but in fact, lurking underneath is a subtlety. One might think that such subtleties might arise only when complicated concepts such as continuity or derivatives are involved. But, actually even in simple logical puzzles sometimes human beings are not so good at identifying the key.

Here is a famous psychological puzzle called the Wason Selection Task. Before I explain the task let me first say that please do not feel bad if you are not able to solve this puzzle and do not feel too good if you are able to also. This is a puzzle that has been found to be unsolvable by most people who are attending colleges. So, let me describe the puzzle for you.

You see on the table in front of you four cards. Each card has two faces; one face has a number the other face has a colour. So, you see these four cards displayed in front of you on a table; obviously, you cannot see both faces, the face that is towards the table is not visible to you. So, here we see the cards with faces 3, 8, red and what I think is the

colour brown. Behind these cards of course, the first two cards there will be some colours behind the third and fourth card there will be some numbers.

Suppose you want to test the following statement; behind a card whose face has an even number the face has colour red; that is, whenever you see one face that has an even number then the opposite side must be red in colour. Suppose you are asked to test this proposition for the four cards that you see here; obviously, one easy way to check that is to flip over every single card. But, you do not need to flip over every single card.

What are the cards that you actually need to flip to check this proposition? I will pause for a while, think for a few minutes and try to figure this out, ok. Hopefully you have thought about this. Now, let us see the answer to this question, I am going to solve this question in excruciating detail step by step. Let us look at the card that says 3, that has one face 3. Is there any need to flip over this card to see whether the opposite side is red in colour? No. So, this card has no flip required.

Why is that? because if you look at the statement behind the card whose face as an even number the face has colour red; that means, the assertion the proposition is purely about those cards which have one face even number.

The card that has face 3 is not an even number 3 is not an even number therefore, it does not matter what colour is opposite this face. It is completely irrelevant to the truth of the given proposition ok. Second, card that has face 8; obviously, you need to flip this. Obviously, you need to flip this because you have to check that the opposite side is in fact, red. So, card that has one face 8 flip.

Now, third one, here is where some people make a mistake. Some people say you need to flip the card that has the face red, but you actually don't need; no need to do that. Why? Because there are only two possibilities for the opposite face either an odd number or an even number. If it is an odd number then regardless of whether this is red or brown or black or cyan it really does not matter because that proposition that we have does not assert anything about cards that have one face and odd number.

So, card that has one face red, no flip, it is completely irrelevant. Whether the opposite face is even number or not that is not what the proposition is saying. It is not saying that if one face is red the other face should be even. So, it is completely irrelevant. What

about the fourth one? Fourth one card that has one face brown, a flip is required. Why is a flip required? Because if the opposite side is odd, then it is not a problem, but the opposite face if it is actually an even number, then this particular last card is going to be violating the proposition that behind a card whose face as an even number the face has colour red, this would be violated ok.

So, I have now explain the solution in excruciating detail this puzzle is fairly straightforward once you write it down in this way. But most people when confronted with a puzzle do not take out a paper and a pencil and start writing down in such great detail. Consequently turns out that even this simple puzzle, a vast majority of college going students do not get it right. They believe that you have to flip the red card or not flip the brown card so on, various errors are possible.

What is funny is that this entire setup can be rephrased in terms of a social situation involving a bar and checking ID cards to ensure that the patents of the bar are all over the legal limit for the legal age limit for drinking. I urge you to look through the Wikipedia article on Wason selection task to know more about this.

So, what is the moral of this puzzle? The moral of this puzzle quoting the famous professor Scott Aaronson is, humans are designed to spear small animals not do abstract logical mathematics. So, we are in a revolutionary history, survival was the main goal. Even today it is not entirely clear how extreme abstract thinking is of benefit to survival. There are various possible theories given by various people how abstraction is very useful.

But, nevertheless it is not clear that survival and logical thinking are really mutually beneficial. So, in the rest of this course, we will be very careful when we write down proofs and be very detail. The reason being that is even if you cannot solve such simple puzzles without being detailed and rigorous, how are you going to be able to do calculus where there are a lot more subtleties.

So, see you in the next module where we will see yet another puzzle.