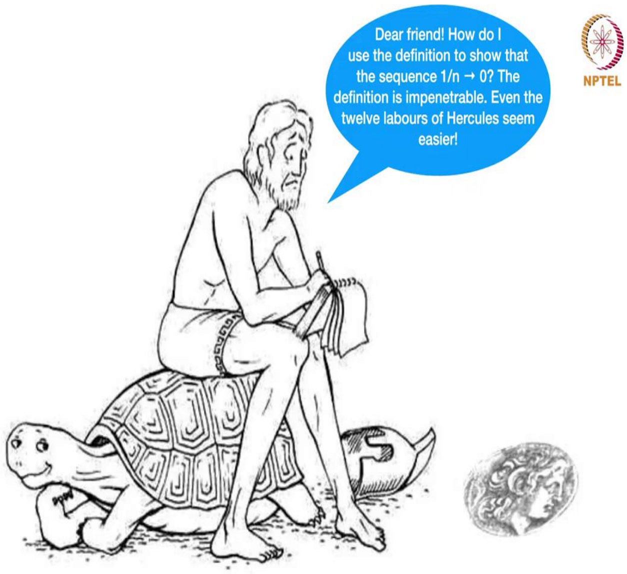
Real Analysis - I
Dr. Jaikrishnan J
Department of Mathematics
Indian Institute of Technology, Palakkad

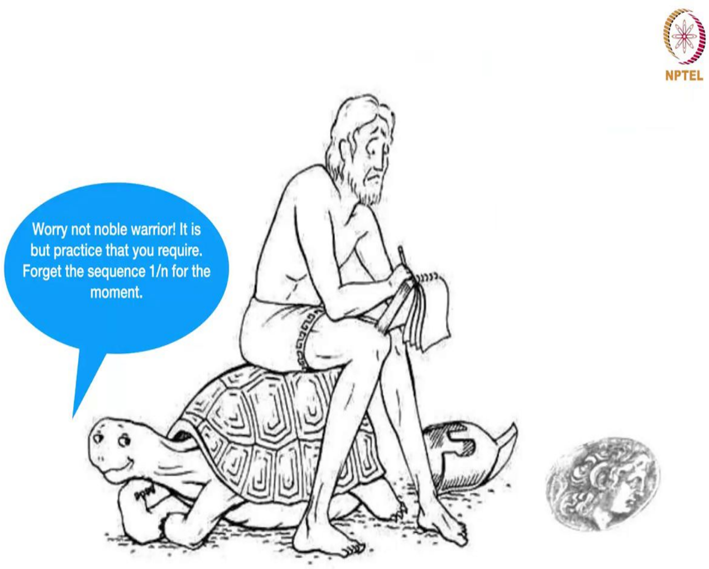
Lecture – 7.5
Achilles and the Tortoise

(Refer Slide Time: 00:13)



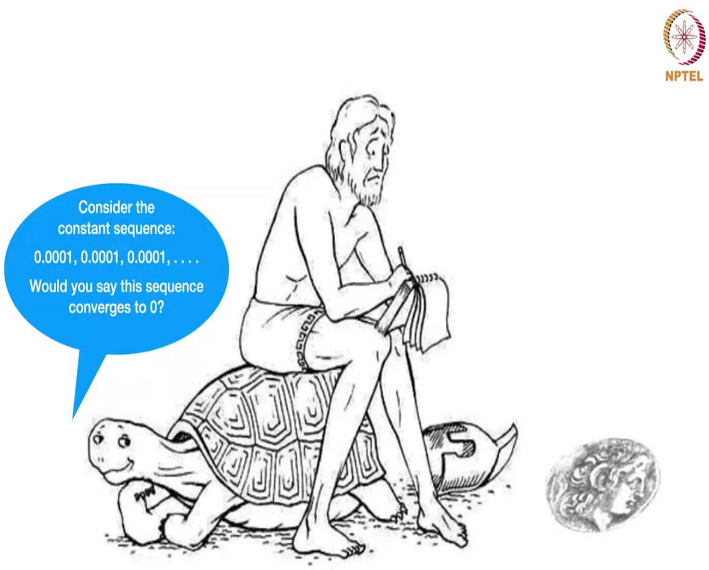
Dear friend, how do I use the definition to show that the sequence $\frac{1}{n}$ converges to 0? The definition is impenetrable. Even the twelve labours of Hercules seem easier.

(Refer Slide Time: 00:26)

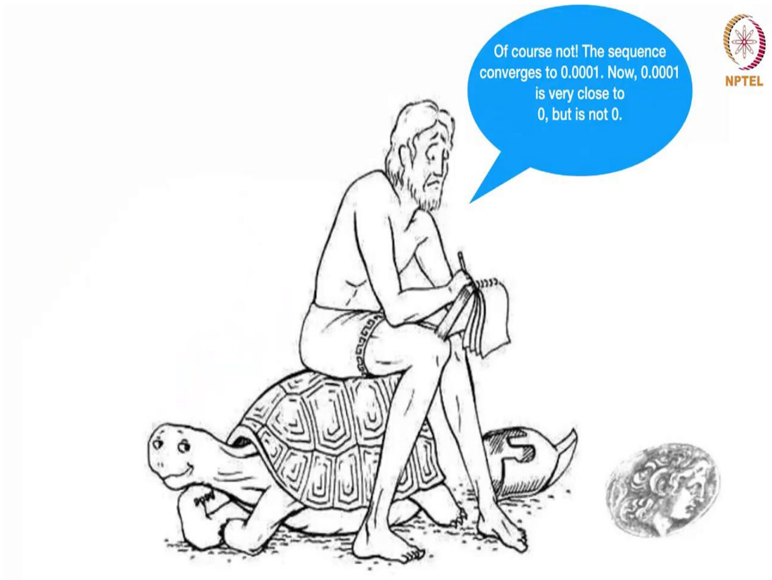


Worry not noble warrior. It is, but practice that you require. Forget the sequence $\frac{1}{n}$ for the moment. Consider the constant sequence 0.0001, 0.0001, 0.0001 and so on. Would you say this sequence converges to 0?

(Refer Slide Time: 00:34)

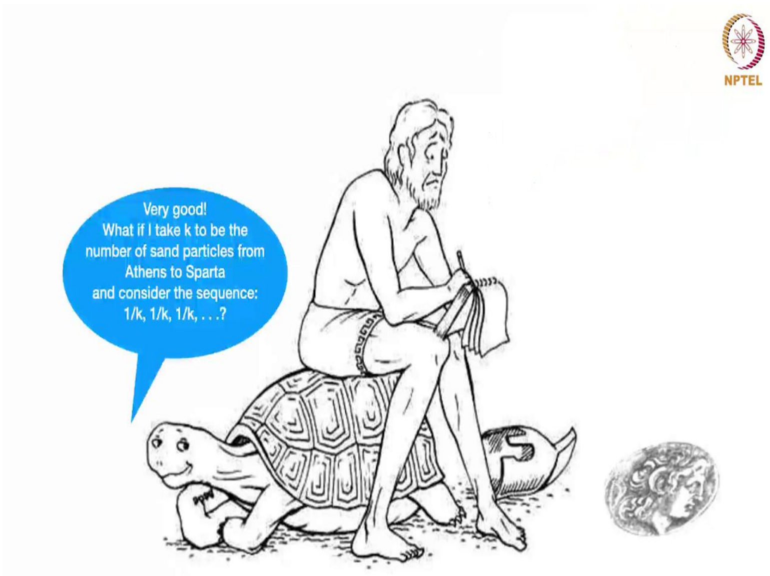


(Refer Slide Time: 00:48)



Of course not. The sequence converges to 0.0001. Now, 0.0001 is very close to 0, but it is not 0.

(Refer Slide Time: 01:00)

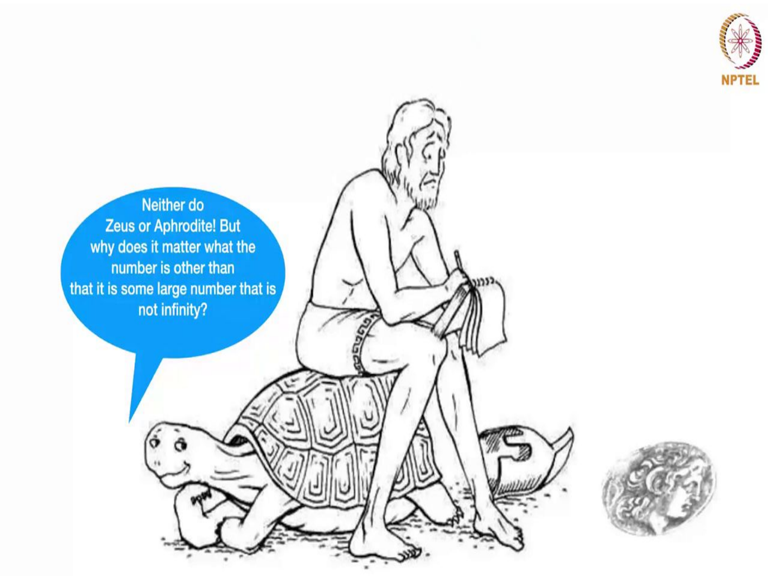


Very good. What if I take k to be the number of sand particles from Athens to Sparta and consider the sequence $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}, \cdots$ and so on?

(Refer Slide Time: 01:13)

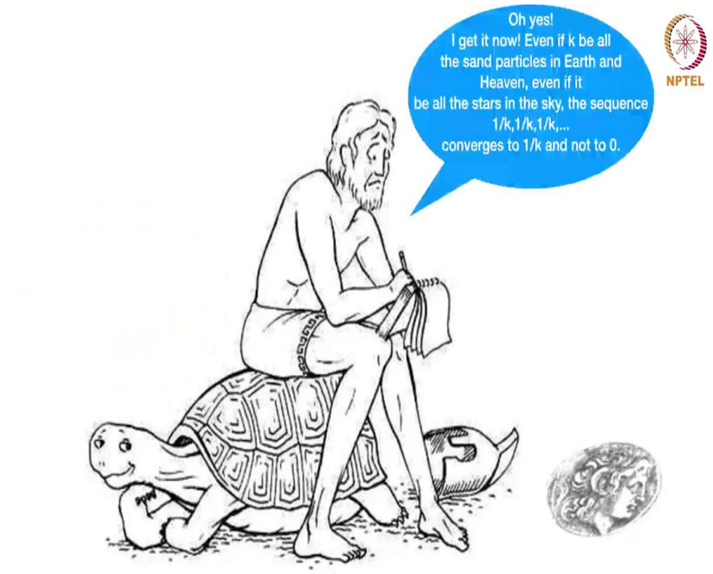
Oh wise friend. I know not the number k.

(Refer Slide Time: 01:17)



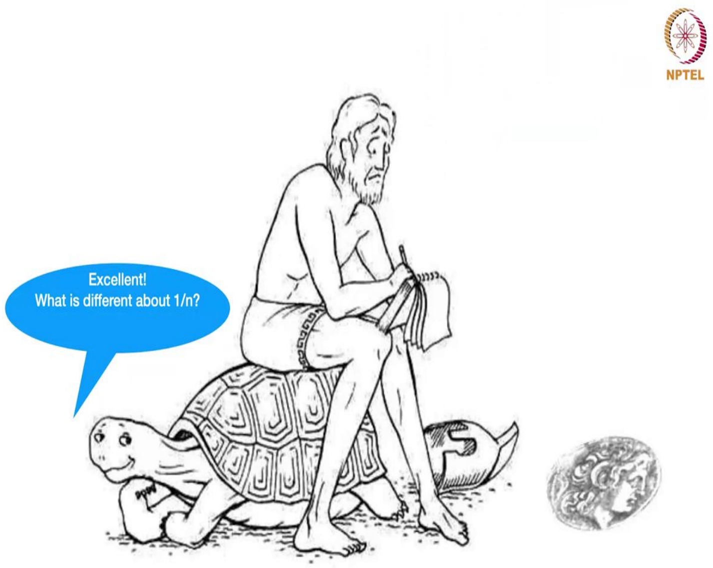
Neither do Zeus or Aphrodite. But why does it matter what the number is other than that it is some large number that is not infinity?

(Refer Slide Time: 01:29)



Oh yes. I get it now. Even if k be all the sand particles in Earth and Heaven, even if it be all the stars in the sky, the sequence $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}, \cdots$ converges to $\frac{1}{k}$ and not 0.

(Refer Slide Time: 01:47)

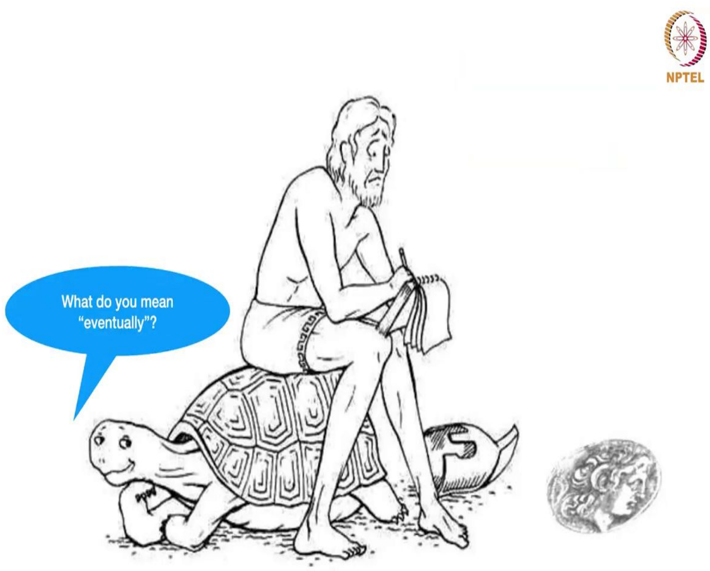


Excellent. What is different about $\frac{1}{n}$?

(Refer Slide Time: 01:52)

Well, it eventually gets smaller than $\frac{1}{k}$.

(Refer Slide Time: 01:56)

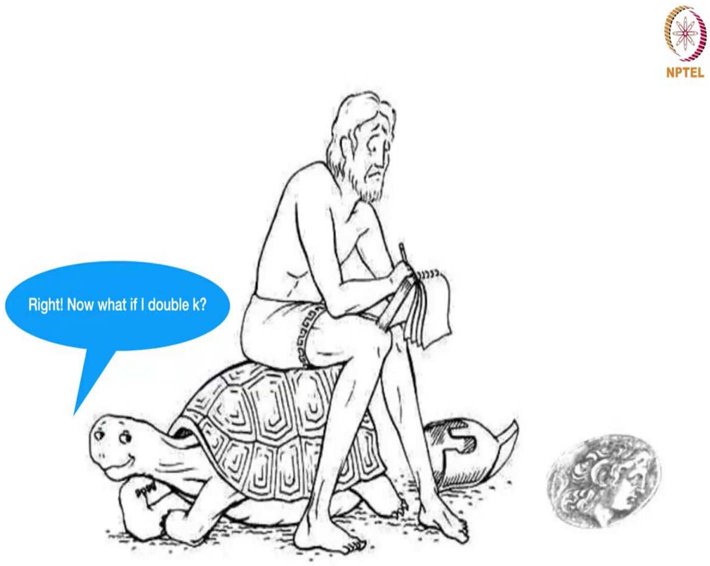


What do you mean "eventually"?

(Refer Slide Time: 02:00)

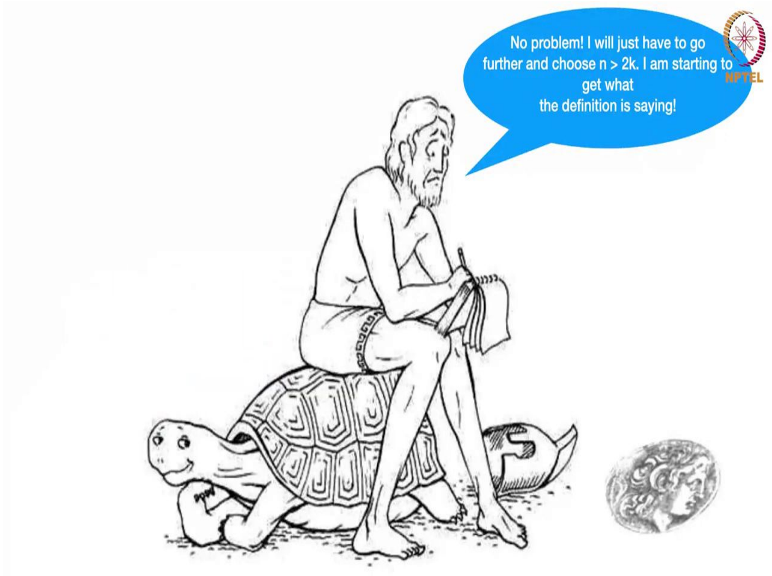
Well, eventually means after some appropriate point in the sequence. If $n > k$, then $\frac{1}{n} < \frac{1}{k}$.

(Refer Slide Time: 02:11)



Right. Now, what if I double k?

(Refer Slide Time: 02:15)



No problem. I will just have to go further and choose $n > 2k$. I am starting to get what the definition is saying.

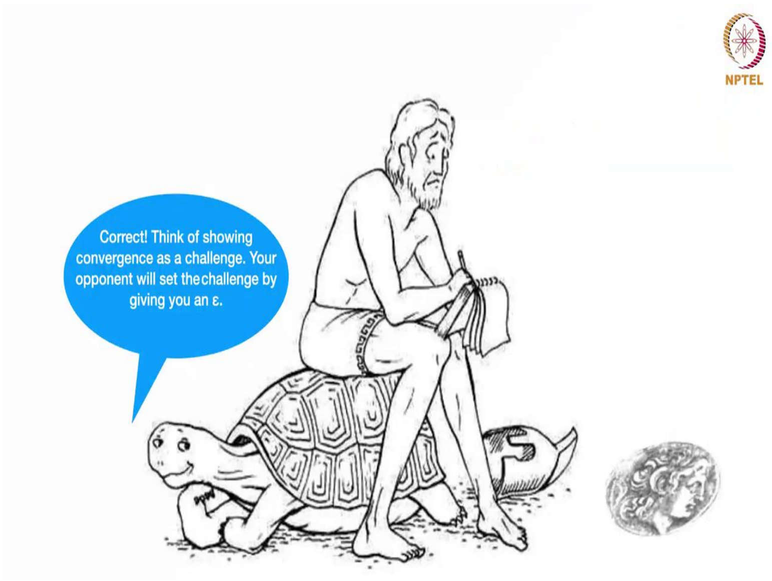
(Refer Slide Time: 02:24)

Yes Achilles. The definition requires you to construct a function N that tells you precisely what "eventually" is for any given choice of $\varepsilon > 0$.

I understand. I can take $N(\varepsilon)$ to be any integer greater than $\frac{1}{\varepsilon}$.

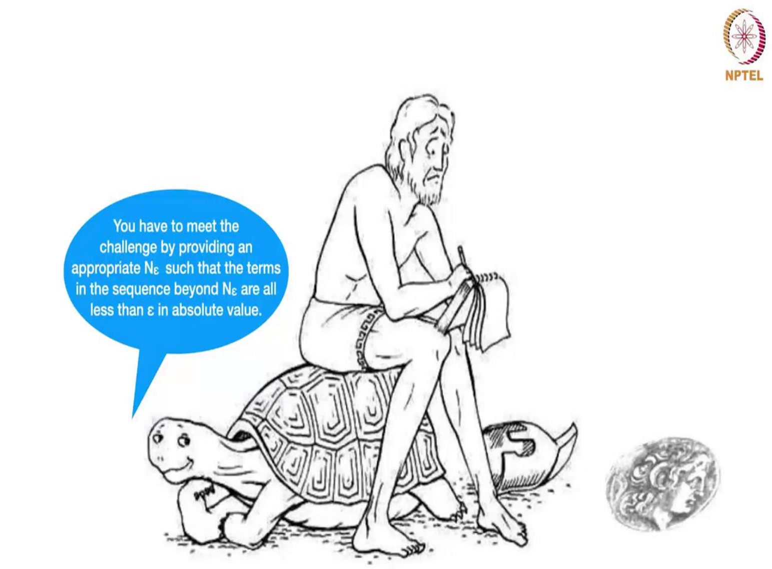
(Refer Slide Time: 02:38)

(Refer Slide Time: 02:46)



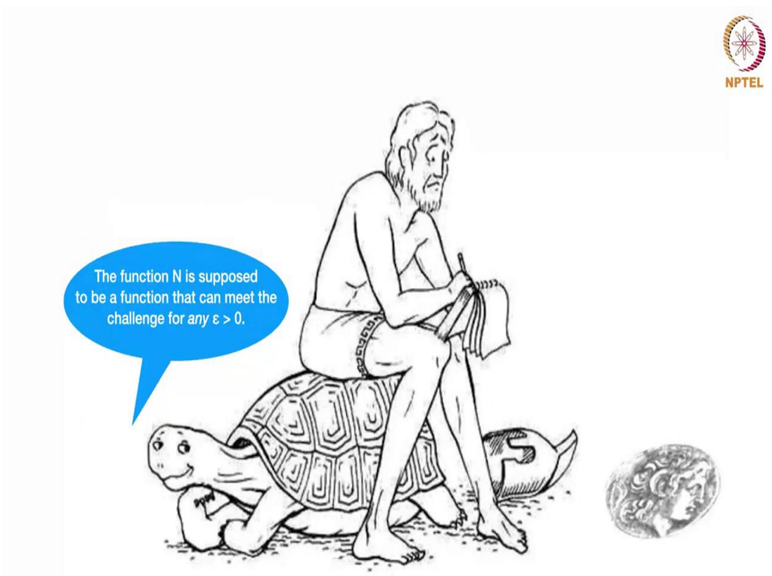
Correct. Think of showing convergence as a challenge. Your opponent will set the challenge by giving you an epsilon.

(Refer Slide Time: 02:55)



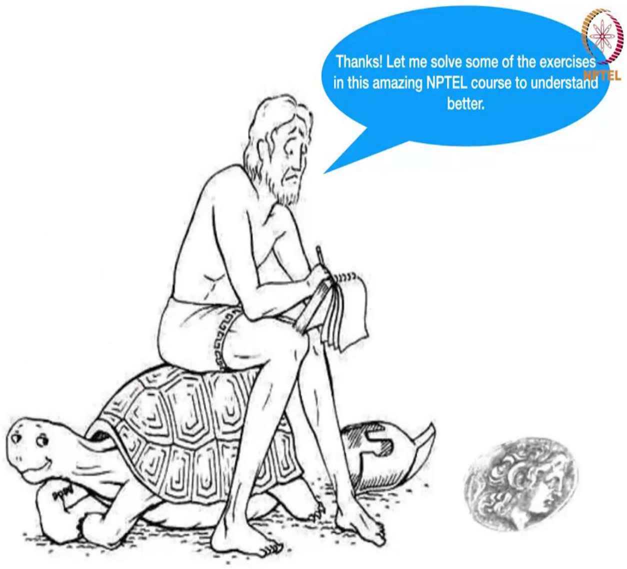
You have to meet the challenge by providing an appropriate $N(\varepsilon)$ such that the terms in the sequence beyond $N(\varepsilon)$ are all less than ε in absolute value.

(Refer Slide Time: 03:08)



The function N is supposed to be a function that can meet the challenge for any $\varepsilon > 0$.

(Refer Slide Time: 03:17)



Thanks. Let me solve some of the exercises in this amazing NPTEL course.