Real Analysis - I
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Week - 2
Lecture – 4.1
Introduction

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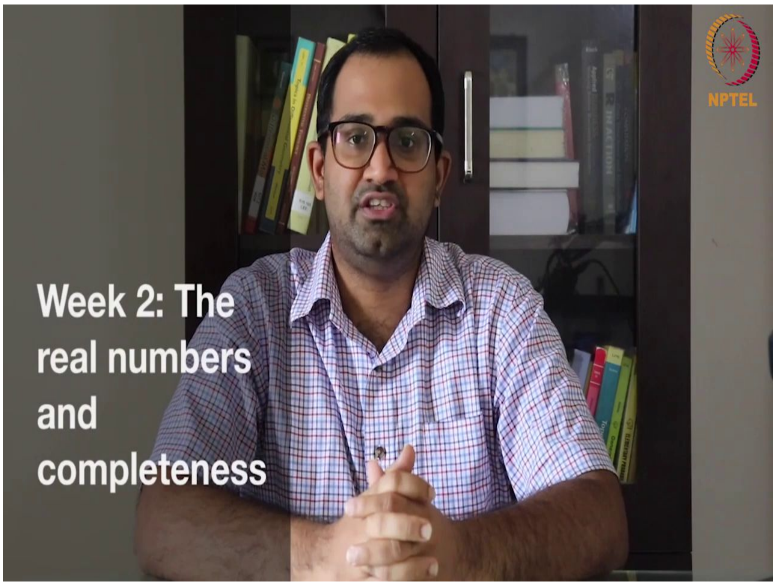

You are probably watching this on a mobile phone, a tablet or a computer. Maybe you are from the 22nd century and you are watching this inside a virtual classroom, irrespective of how you are watching. You can be sure that calculus was used in the device that is enabling you to watch this video. That is how ubiquitous calculus is. In whatever device you are watching this, there should be an option to speed up or slow down this video. You will notice that I speak slowly and deliberately with lots of pauses, this is intentional.

This course is targeted primarily to an Indian audience. Therefore, almost none of us has English as the mother tongue. In order to facilitate ease of understanding to all Indians, I am speaking slowly and deliberately. However, if you find that my pace is very annoyingly slow, you can always speed up the video to a speed that you find comfortable.

Welcome to Week 2 of this course on Real Analysis. We are going to study the real numbers in this week. Our treatment is going to be axiomatic. We list down all the familiar properties of the real numbers including things that might seem obvious and trivial like a + b = b+ a.

We will also prove stuff that seems completely obvious, like there is a unique number 0, such that a + 0 = a. Things that you have taken for granted from the time you learnt how to count, why do we do this? Well, I have already offered some justification in the motivational lecture in week 1 on why we need to undertake such a deep rigorous and nuts and bolts approach to calculus.

If you are still not convinced, I strongly recommend you read this wonderful blog post by the fields medalist Timothy Gowers link below. Whether you are convinced by me or by Timothy Gowers, I take it for granted that you now understand the need to carefully and rigorously study the real numbers. As I said, we are going to take an axiomatic approach. We simply list down all the properties of the real numbers that we need.

You might ask just because I list a set of properties, does not mean that there is something in the universe that satisfies the list of those properties. This is indeed a valid question, but to actually construct the real numbers starting from set theory is an arduous task.

Again, in the lectures, I have given a precise reference where this is done for those who are interested in seeing a construction. Once we list down the basic axioms of real numbers, we will spend a lot of time studying one particular axiom that you more surely have not seen before.

It is called completeness. It is this completeness that distinguishes the rational numbers from the real numbers and indeed, it is completeness that enables us to do analysis. Therefore, we will look at completeness from a number of angles. Unfortunately, the proofs of the many theorems quickly start to become quite challenging.

So, right at the outset, you are going to meet some challenging proofs. To make your burden somewhat lighter in all the proofs, I try to highlight what the key idea is. If there is a geometric significance, I try to highlight what the geometric significance is. So, when you read these proofs, study them carefully, look at why things are being done in the way they are done.

One tip that might prove to be useful is never try to commit the memory to any complicated proof. You have truly understood a proof only when you understand the crux of the proof. What is the crux of the proof? Well, it is the bare minimal amount that you need to remember in order to reproduce the proof by yourself. So, whenever you are studying this proof, keep a watch full eye on what the crux could be. The moment you identify the crux; a light bulb will flash above your head and you will know that you have understood.

All the best. Enjoy the content of this week.