Real Analysis - I
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Lecture – 28.3
Notation for Taylor Polynomials

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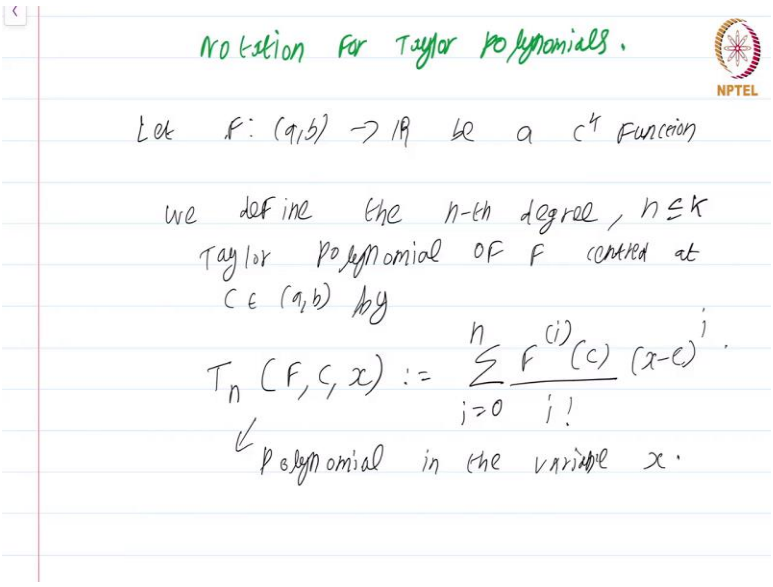

The purpose of this really short module is to set up some convenient notation for Taylor polynomials. Let $f: [a, b] \longrightarrow R$ be a C^k function. We define the nth degree Taylor polynomial of f centred at c in $[a, b]$ by, I must mention nth degree $n \leq k$ of course. $T_n(f, c, x) := \sum_{i=0}^{n} \frac{f^{(i)}}{i!}(x - c)^i$, ok. So, you can view this as a polynomial in the variable x.

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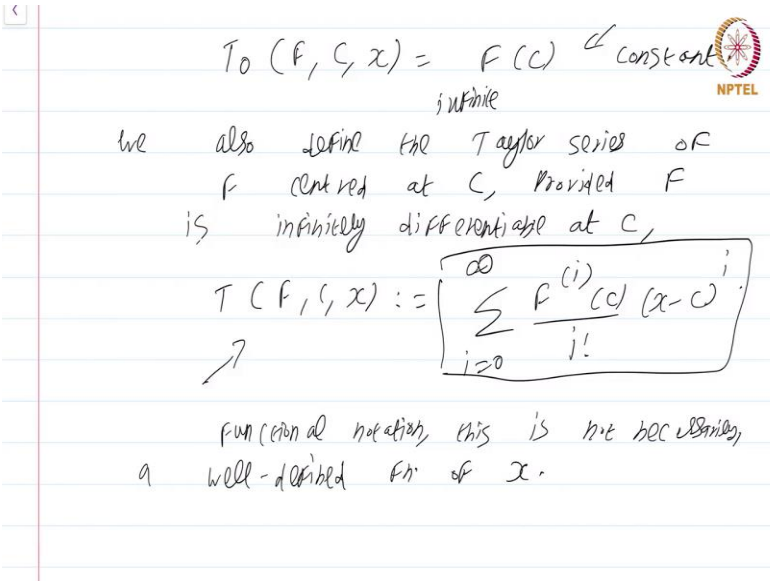

Notice also that $T_0(f, c, x) = f(c)$; this is a constant, ok. Now, we also define the Taylor series or I should write the infinite Taylor series of f centred at c, provided f is infinitely differentiable at c.

You will see more about such functions which are known as smooth functions in the next module, ok. Say this just means that you can repeatedly take derivatives of f at c, $f'(c)$ exists, $f''(c)$ exists, $f'''(c)$ so on and so forth all of them exist.

We define T(f, c, x):=$\sum_{i=0}^{\infty}\frac{f^{(i)}}{i!}(x-c)^i$. Now, notice even though I have used functional notation, even though I have used functional notation ok; this is not necessarily a well defined function of x, this is not necessarily a well defined function of x, ok. It can happen that this series does not converge; it is very possible that our series does not converge at all.

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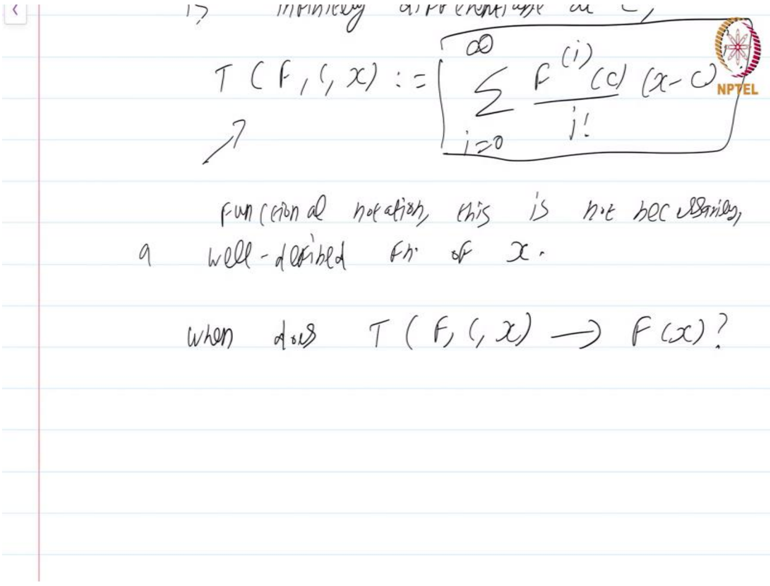

So, one interesting question is when does $T(f, c, x)$ converge to $f(x)$; when does this happen? And in the next module you will be in for a shocking surprise. This is a course on real analysis and you have just watched the module on a Notation for Taylor's for Taylor polynomials.