Real Analysis - I
Dr. Jaikrishnan J
Department of Mathematics
Indian Institute of Technology, Palakkad

Week - 01
Lecture - 1.1
Introduction

The 1st week of this course is about motivating why we need to study real analysis. We will see that taking a naive approach to calculus can lead to all sorts of difficulties. This is summarized in the lecture, why study real analysis. After that, we will see some interesting puzzles where, your initial solution will turn out to be wrong. And a careful study later will give you the correct answer. If you have solved assignment 0 you would have encountered similar such questions.

Once I have justified and convinced you that, there is a necessity to carefully rigorously and logically study calculus; we will proceed with the prerequisites. The prerequisites are rudimentary set theory and basic logic. You are already familiar with this subject from high school we will revisit it at breakneck pace, I will quickly summarize what set theory is, what the basic notions of set theory are and the basic constructions of set theory including the important notion of an equivalence relation.

We will also learnt, some aspects of logic and proofs, this will mostly serve as a motivation to the several proofs that we will see in this course. Talking briefly about what a proof really is, is only going to be there. I shall not cover this in great depth, I urge you to check the references that I have provided if you are interested in a more in depth study. Finally, we will study the somewhat unintuitive concept of cardinality.

We will see that you can count the elements of a set, even if it is infinite and it will turn out that not all infinite sets have the same number of elements; there is a hierarchy of infinities. This is the work of Cantor, and really this is the work that push set theory as the foundation of mathematics. We will also for cultural purposes take a brief treatment of axiomatic set theory as I have emphasized it is very important to be logically precise and utterly rigorous to make sure everything is error free.

However, in this course we shall take a naive approach to set theory. The reason is that axiomatic set theory is quite difficult and requires a lot of study and is usually a master's

level topic. Nevertheless, for cultural purposes in one module I will summarize the axioms and talk briefly about how these axioms are used in real analysis and elsewhere. Enjoy this week's content.

(Refer Slide Time: 03:14)

How to study for this course? For many of you this might be your first serious exposure to studying and more importantly writing proofs. You have grappled with proof writing and studying in 9th and 10th standards where you must have studied geometry, namely things like similar triangles, circles etcetera.

Curiously, these aspects of mathematics disappeared in your 11th and 12th standard where you focused on calculus, though real analysis is supposed to be the theoretical underpinning of calculus; this course will be more similar to the courses in geometry that you studied in 9th and 10th standard, rather than the course on calculus that you had in your 11th and 12th standard.

Proof writing is a skill that can only be learnt through practice. Therefore, in the lectures I will not be giving complete proofs for all the theorems, many many things will be left for you. The notes that are provided along with the lectures will have a little bit of more detail.

Nevertheless, you will still have to write many many proofs on your own. As the course develops even further more and more things will be left for you, towards the end of the

course entire theorems might be left for you. Apart from these fill in the blank type things you will also have to solve a lot of exercises; I recommend that you solve every single exercise in the accompanying notes in detail, you will not be required to submit any of these as part of your evaluation simply because it is not practical for me to correct every single assignment.

On the other hand, the assignments that are present in the NPTEL website will be used for evaluation. They will not emphasize the proving aspects that much, but there will be one or two questions where you will have to fill in the blanks in certain proofs.

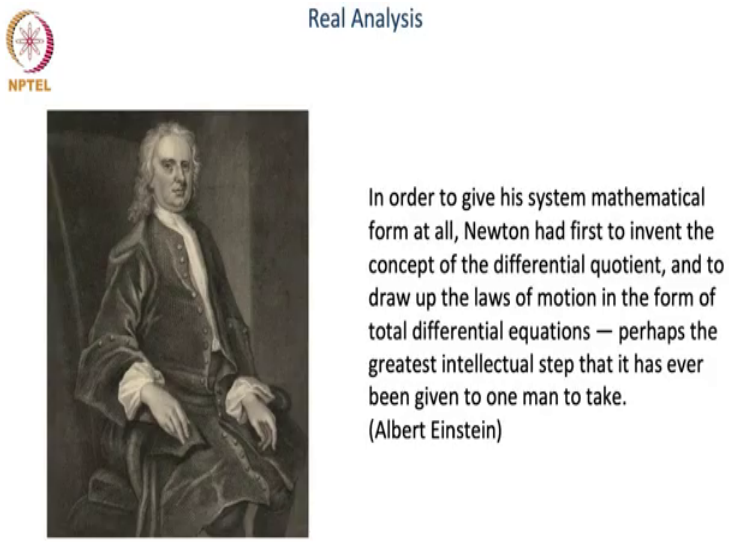
(Refer Slide Time: 05:32)

Learning any complicated subject like mathematics can never happen in a linear fashion; it is never the case that you understand one topic completely then move on to the next, then move on to the next that simply not possible.

In the real world understanding happens progressively, as you learn and learn your knowledge becomes more clear and more refined; your clarity increases your perception becomes deeper, to reflect this many many modules in this course require you to revisit and watch that module again after you proceed a bit further. I strongly suggest that whenever I say this module is supposed to be watched twice or thrice you actually watch it twice or thrice.

This will reinforce and make your understanding really strong. All the best for this course and remember practice is platinum.

(Refer Slide Time: 06:35)



Welcome to this course on real analysis. On the left of the screen you see a portrait of Sir Isaac Newton and on the right a quote by Sir Albert Einstein. In order to give his system mathematical form at all, Newton had first to invent the concept of the differential quotient.

And to draw up the laws of motion in the form of total differential equations — perhaps the greatest intellectual step that it has ever been given to one man to take.