Algebra - I
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ALGEBRA I

1. Lecture 77: Smith Canonical Form for PID

In the last lecture, we saw how to reduce any matrix of entries are in Euclidean domain to Smith form. The same thing can be done over a principle ideal domain, but the algorithm is a little more involved.

So let us just focus on trying to clear out the first row, say there is some entry here a_{1j} in the first row and jth column that we want to clear out. And so, when we were in a Euclidean domain we were looking at the case of trying to clear out a_{1j} using a_{11}, and failing which we would somehow try to reduce this size of a_{11}using the Euclidean algorithm.

So, let me show you how to modify these steps when we do not have the Euclidean division algorithm. If a_{11} divides a_{1j}, then we can just clear out a_{1j}.

If a_{11} does not divide a_{1j}, then the ideal $(a_{11}, a_{1j}) = (r)$. So $r = a_{11}x + a_{1j}y$, and also $a_{11} = q_{11}r$ and $a_{1j} = q_{1j}r$.

So $1 = q_{11}x + q_{1j}y$. We construct matrix T where the $(1, 1)$ position is x and in the jth row y, and then 1s along the diagonal. The $(1, 1)$ entry of AT is $xa_{11} + a_{1j}y = r$. The ideal generated by r strictly contains the ideal generated by a_{11}.So we get an element in the $(1, 1)$ place whose size is strictly less than a_{11}.

We use the fact that a PID is a Noetherian ring cannot be enlarged indefinitely so the process must stop after finitely many steps.