Algebra - I
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ALGEBRA I

1. Lecture 76: Solved problems

Exercise 1.1. *Find a 3 by 3 matrix over $\mathbb{Q}$ matrix A such that the associated module $M_A \equiv F[t]/(t^3 - 2t^2 + t - 3)$. Pick a basis for this module and then the matrix will just be however t acts on with respect to that basis. So, pick a basis $e_0 = 1, e_1 = t, e_2 = t^2$. They form a basis of this module and so let us figure out how t acts on this. So, $te_0 = e_1$, $te_1 = e_2$ and $te_2 = t^3 = 2e_2 - e_1 + 3e_0$. So the matrix is* $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -1 & 2 \end{pmatrix}$

Exercise 1.2. *Find a matrix A such that $M_A \equiv F[t]/(t^2) \oplus F[t]/(t)$. Pick a basis $e_1 = (1, 0)$, $e_2 = (t, 0)$ and $e_3 = (0, 1)$. Then let us figure out how t acts on these basis vectors $te_1 = e_2$ $te_2 = 0$ and $te_3 = 0$. So, the matrix A is just going to be* $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

Exercise 1.3. *Find A such that $M_A \equiv F[t]/(t-3)^3$. We take as basis $e_0 = 1, e_1 = (t-3), e_2 = (t-3)^2$ and $te_0 = e_1 + 3e_0$, $te_1 = e_2 + 3e_1$ and $te_2 = 3e_2$. So, then we get that the matrix of A is* $\begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$.

Exercise 1.4. *Find A such that $M_A \equiv F[t]/(t-a)^2 \oplus F[t]/(t-a)$, where a is some element of F. Take as basis $e_1 = (1, 0)$, $e_2 = (t - a, 0)$ and $e_3 = (0, 1)$ so that $te_1 = e_2 + ae_1$, $te_2 = ae_2$ and $te_3 = ae_3$ so that the matrix is* $\begin{pmatrix} a & 1 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$.