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ALGEBRA I

1. Lecture 73: General Linear Group over a Commutative Ring

Denote by $M_n(R)$ the ring of all n by n matrices with entries in R. The General Linear Group over a Commutative Ring R is denoted $GL_n(R)$ is the set of matrices in $M_n(R)$ such that the determinant is a unit in R. It is a group under the multiplication of matrices. The unit element is the identity matrix and the inverse exists since the determinant is a unit.

Example 1.1. *The shear matrix $u_{ij}(x)$ for $i \neq j$ has 1s along the diagonal and 0s in every other location except the (i, j) location- which is $-x$.*

Example 1.2. *An axis dilation $\delta_i(a)$ has 0s at every nondiagonal element and is a at the (i, i) position and 1 at every other diagonal position.*

Example 1.3. *The axis transposition is just σ_{ij} for $i \neq j$ is the identity matrix except that the ith and jth rows are exchanged.*