Algebra - I
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ALGEBRA I

1. Lecture 07: Cosets

Given a group G and subgroup H, the right coset $Hg := \{hg \mid h \in H\}$. Similarly we may define the left coset.

Example 1.1. *Let $G = \mathbb{Z}$ and $H = 5\mathbb{Z}$. Then $H3 = \{h + 3 \mid h \in H\} = \{\dots, 3, 8, 13, \dots\}$.*

Example 1.2. *Take G to be the group S_n and H be the subgroup of permutations that take n to n. Given $w_0 \in S_n$, the coset w_0H is all permutations $w \in S_N$ such that $w_0(n) = w(n)$. H (which is isomorphic to S_{n-1} has n left cosets in S_n.*

Right cosets are orbits of left multiplication on G. G is a disjoint union of left cosets or right cosets of H, and each of these cosets has the same size namely the cardinality of H and there are $|G|/|H|$ cosets of H.