Algebra - I
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ALGEBRA I

1. Lecture 06: Group Actions

Let G be a group. A group action is a function from $G \times X \to X$ satisfying:

Identity axiom	$e \cdot x = x$
Compatibility	$gh \cdot x = g(h \cdot x)$

We use the symbol $G \circlearrowright X$.

Example 1.1. *The trivial action of a group on a set is defined to be* $g \cdot x = x$ *for all* $g \in G, x \in X$.

Example 1.2. *The most basic example is the (left) translation action of a group* G *on itself defined by* $g \cdot h = gh$.

Example 1.3. *The (right) translation action of a group* G *on itself is defined as* $g \cdot h = hg^{-1}$.

Example 1.4. *The permutation group* S_n *acts on* $[n]$ *by* $\sigma \cdot j = \sigma(j)$ *for all* $\sigma \in S_n, j \in [n]$.

Example 1.5. *Given a graph* Γ *on* n *vertices,* $Aut(\Gamma)$ *is a subgroup of* S_n *comprising permutations of the vertices preserve the structure of the graph.*

Two elements x, y of X lie in the same G-orbit of X if there exists an element $g \in G$ such that $y = g \cdot x$. This is an equivalence on elements of X. $G \backslash X$ denotes the set of orbits of X.

Example 1.6. *The automorphism group of this graph is generated by* (13) *and* (24). *The orbits are* $\{1, 3\}$ *and* $\{2, 4\}$.

The stabilizer of an element $x \in X$, denoted G_x, is a subgroup of G comprising all $g \in G$ such that $gx = x$.

Theorem 1.7. *For an element* x *of a* G*-set* X, *let* $\mathcal{O}_x$ *denote the orbit of* x. *Then*

$$|G| = |G_x||\mathcal{O}_x|.$$

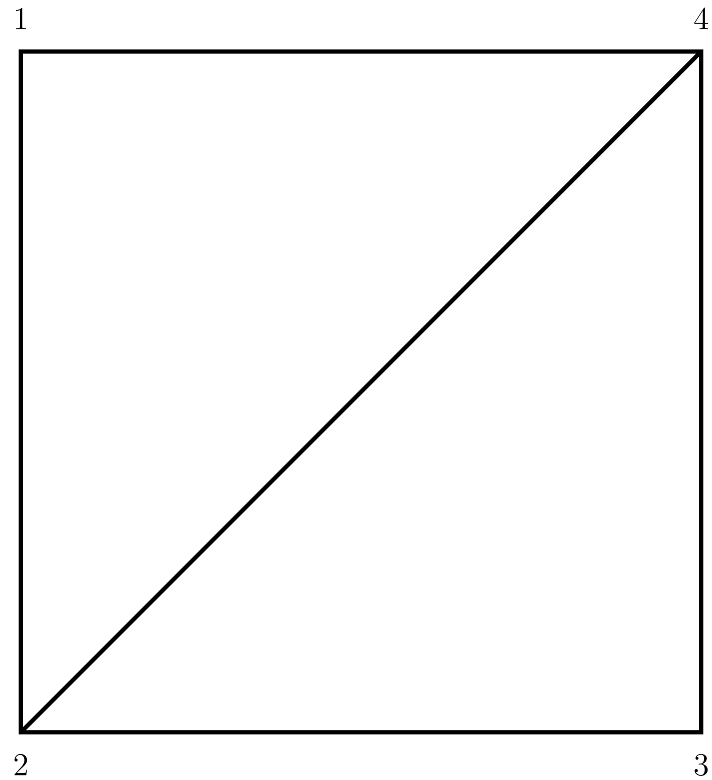

FIGURE 1. The graph Γ.