Algebra - I
Prof. S. Viswanath & Prof. Amritanshu Prasad
Department of Mathematics
Indian Institute of Technology, Madras

ALGEBRA I

1. Lecture 13: Semidirect products

Given group G and normal subgroup N and another subgroup K with the property that every for every $g \in G$, $g = nk$, for some $n \in N$ and $k \in K$, and this product is unique. For the dihedral group $D_n = C_n \rtimes C_2$.

Given group G and normal subgroup N and another subgroup K with the above property with a group homomorphism $\psi : K \to \mathrm{Aut}(N)$, we can define the inner product by the rule:

$$(n_1, k_1) \cdot (n_2, k_2) = (n_1 \psi(k_1)(n_2), k_1 k_2).$$

Example 1.1. *We construct another group which will call the infinite dihedral group, D_∞. The infinite dihedral group is defined to be $\mathbb{Z} \rtimes \mathbb{Z}/2\mathbb{Z}$. Consider the graph L , graph with vertex set $\mathbb{Z}$ and edges of the form $(i, i+1)$ for each $i \in \mathbb{Z}$. D_∞ is isomorphic to automorphism group of this graph L.*