ALGEBRA I

1. Lecture 12: Dihedral Groups

The nth dihedral group as the symmetry group of the n cycle.

There are two types of symmetries of the n-cycle: rotations by $\frac{360}{n}$ degrees and reflections around an axis passing through the centre and the vertex 1. Let R_n be the subgroup of D_n generated by a rotation r by $\frac{360}{n}$. Then $R_n = \langle r \rangle$. Let s be the reflection, and note $s^2 = id$. Let $S = \langle s \rangle$ and thus $|S| = 2$. Thus $D_n = 2|R_n|$ and R_n is normal in D_n. In fact we have $D_n = R_n \rtimes S$. We have $R_n \cong C_n$ the nth cyclic group, and $S \cong C_2$ the cyclic group of order 2. So $D_n \cong C_n \rtimes C_2$.
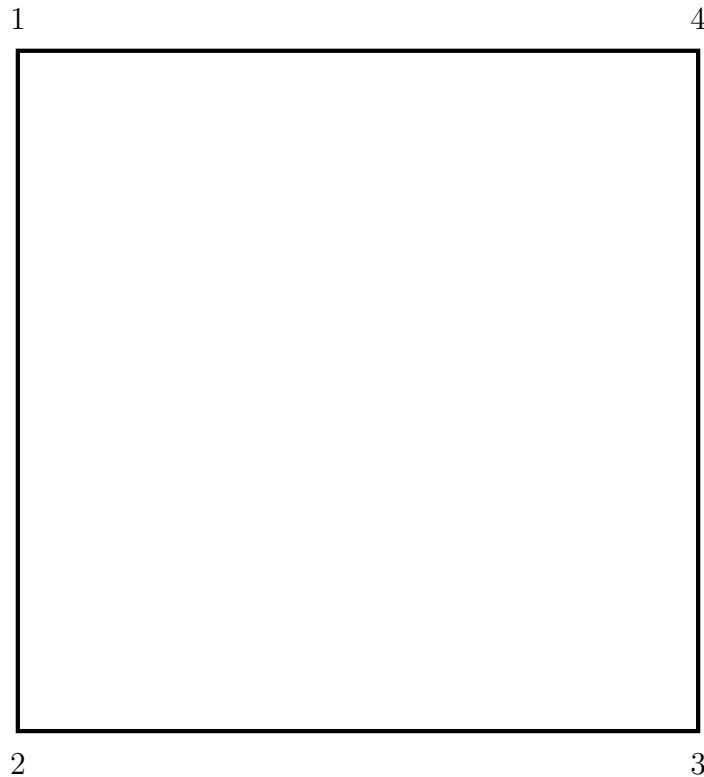

FIGURE 1. The 4-cycle, denoted Γ_4 (see lec6).