

Algebra - I
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ALGEBRA I

1. LECTURE 10: QUOTIENT GROUPS

Given a group G and normal subgroup N , we have seen that the left cosets and right cosets of N are the same. Define a binary operation on G/N as follows: $xN \cdot yN = xyN$.

Theorem 1.1. *Given group G and normal subgroup N , G/N is a group under the binary operation defined above.*

Example 1.2. *Take G to be the group of integers and N to be the group of integers that are multiples of some fixed integer n . Then $\mathbb{Z}/n\mathbb{Z}$ is a group.*

Example 1.3. *Let G to be the dihedral group D_n and take $N = \langle r \rangle$. Then $D_n/N \cong \mathbb{Z}/n\mathbb{Z}$.*

Theorem 1.4 (First isomorphism theorem). *Given a surjective group homomorphism $\phi : G \rightarrow H$ we have $G/\ker(\phi) \cong H$.*