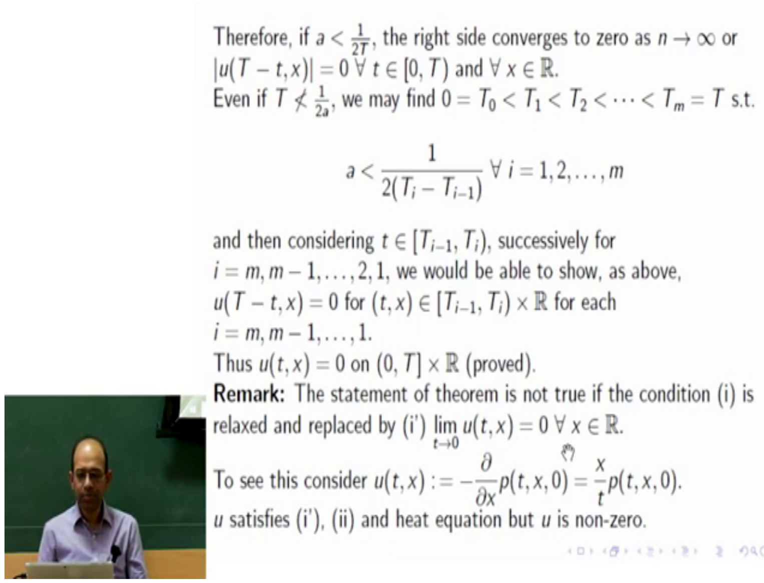
Introduction to Probabilistic Methods in PDE
Professor Dr. Anindya Goswami
Department of Mathematics
Indian Institute of Science Education and Research, Pune
Lecture 32
Remark on Tychonoff's Theorem

(Refer Slide Time: 0:18)



So, here a there is a one very important remark, we must talk about, so this says that, the statement of the theorem is not true if the condition 1, which is saying that, that the double limit is 0, that the initial condition written in terms of double limit. Instead of that if one replace that by 1 prime, that is just take x and limit t tends to 0, u, t x that is 0.

So, that is also one condition, this is just saying that okay if we approach to the initial line vertically, so you do not move the x, but only one direction and there if you find 0, that is the condition given, then that is equation has any non trivial solution or not? The answer is yes, in that case there could be a non-trivial solution. This is an example that small p is the probability density function for normal random variable, what we have already introduced in earlier lecture.

So, if you take that probability density function, where the mean is x, variance is t, and evaluated at point 0. So, that function if you take partial derivative with respect to x you get another function of t and x and you consider negative of that function as u, and you check that whether this u satisfies the heat equation and also this condition. Surely, this is a non trivial function, however this value is x by t del p I mean p, t, x, 0.

And then we show that, okay is second derivative would also be like x by t whole square p, t, x 0 plus p, t, x 0 divided by t. So, that will be their expression and if you do appropriate manipulation of these variables and del p, del t, you compute and then you see that, okay that would also satisfy that heat equation. Actually we have seen that p satisfies heat equation earlier. If p satisfies heat equation del p, del x, will also satisfy, because it is a linear PDE and if you know this switching of the partial derivatives are allowed, then you can put this del p del del x there.

So, inside the domain p is a C infinite function, so you do that and then, we actually get a contradiction because this function satisfies this condition that as t tends to 0 this is 0. We can see it very clearly, if x is some fixed number and you are finding out the probability density of at the value 0, when variance is t. Now, if t decreases to 0, then you know that the density function becomes more concentrated at x and beyond the neighborhood of x the value decreases to 0 and as t tends to 0 this value because x is not 0.

So, I had a 0 this p t would be 0 and here the same thing also happens for del p del x, because I mean we would also like just like flat, So, del p del x derivatives also become 0. So, this is one very important remark, that how to talk about initial condition. I told that, okay when we talk about heat equation and initial condition, if the function is defined only in the interior and the initial line is not inside the domain of definition of the solution, then we write down limit but the limit should not be the single limit but should be the double limit and double limits there.

So, if we take double limit that to be some function f, then for that problem you get a unique solution. So, this actually also clarifies some way of writing you know when you specify a heat equation how are you going to write down that. Thank you, this much I want you to say, thank you very much.