Introduction to Probabilistic Methods in PDE
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Lecture 25
Summary of the Zaremba's cone condition

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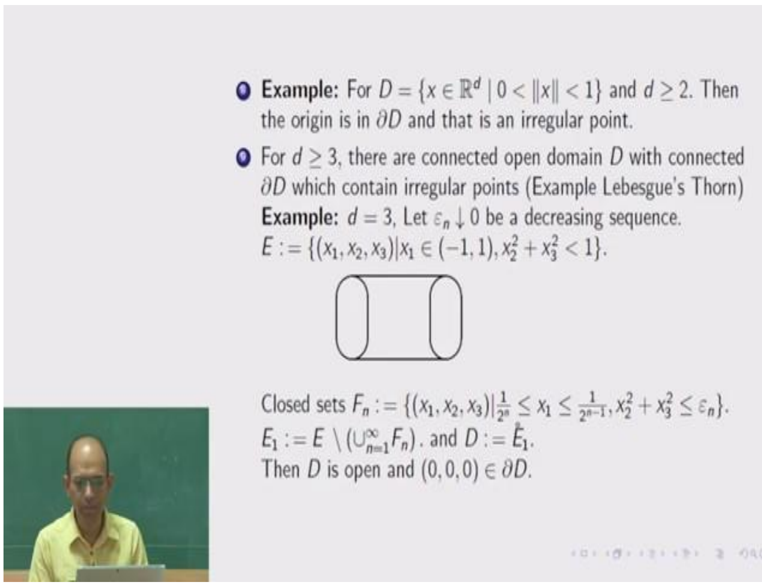

In the last lecture we have seen some examples of irregular points. So, here this is Fn, Fn is a closed set. This is basically, E is basically a cylinder and Fns are actually small cylinders of decreasing size. Epsilon n goes to 0. It is some small numbers, where this length is 1 over 2 to the power n and this is epsilon n is the radius. And then those are attached to a make union of Fn and then that is subtracted from E and then what you obtain is E1 and the interior of E1 is an open domain, open connected of domain that is D that looks like this type of domain.

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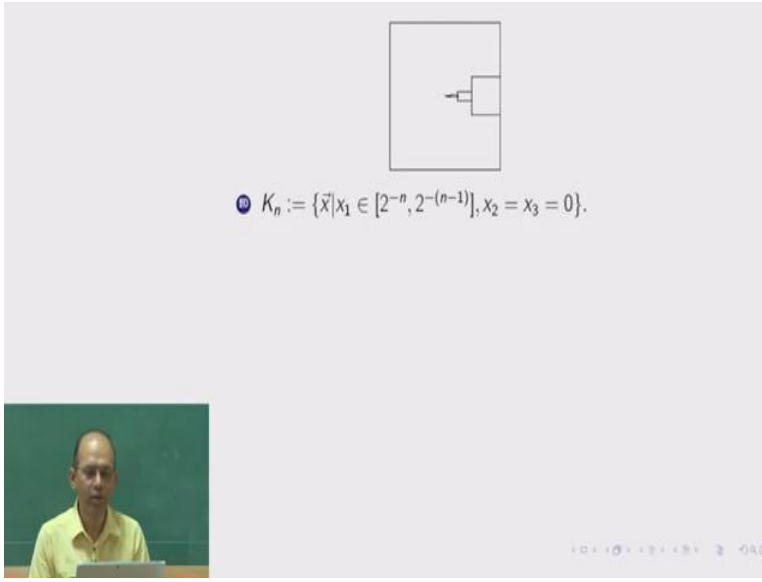

So, D is outside and this is, say, for example F1, this is F2, F3, F4, etc. So now we understand that here what is happening that no single cone can be attached here, so that that can stays completely inside the complement of D. So, if this is D, this part is the complement. Union of all these will be complement outside also. The whole thing is Rd. So, no cone can be attached.

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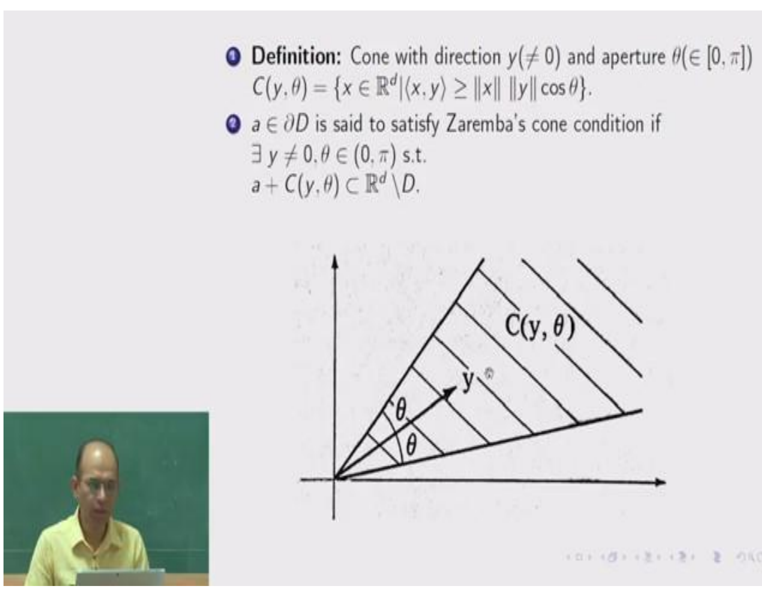

So, here this is the definition of cone. So, this is of direction y. And then we have visited what is the Zaremba's cone condition. Zaremba's cone condition is in this slide. Imagine that your point a is on the boundary of the domain such that you can attach a cone like this, so that the entirety of the cone is outside the domain. That a plus c y theta, c y theta is the cone that started when, 0 is the vertex here. And a plus c y theta would be the cone attached at point a, would be subset of Rd minus D. Entirety of this should be on the complement.

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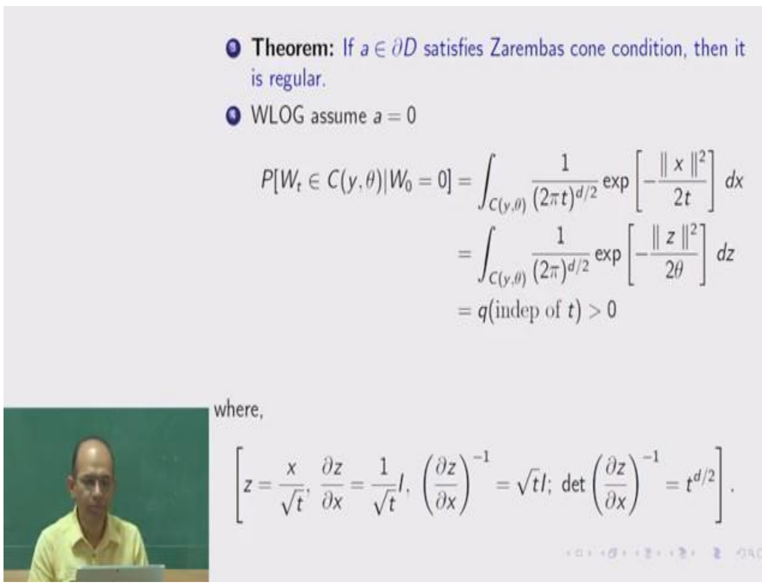

And then we have seen the statement of the theorem that if a point a is on the boundary of the domain and which satisfies Zaremba's cone condition, then it is regular. So, that proof also we have seen. We have seen this proof. There the idea of the proof is very simple, that to find out this probability and show that this is independent of t. So, it does not depend on t time. Some number q which is strictly positive that we could show due to this transformation.

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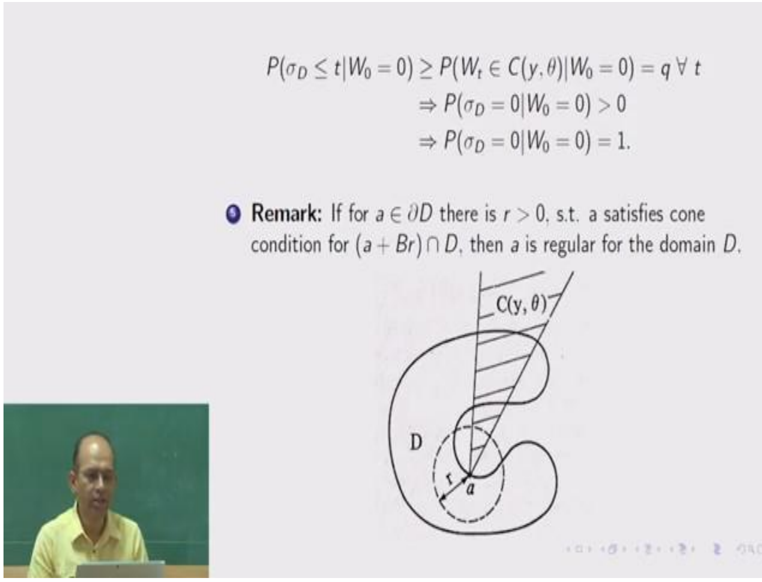

And then using that we could show that the probability of sigma D, sigma D is not exactly exiting time, but positive exiting time. That is strictly greater or equals to q and then we get that since it is true for all t, so sigma D is equal to 0 that is positive. And since we know that Blumenthal's zero-one law, by using that law we know that anything positive should be 1. So, here this probability, any number which is more than 0 should be 1.

And then this remark was just saying that, even if your domain looks like that where locally it is, you can attach a cone but globally not, then does not matter you can actually locally like the D intersection with an open ball and you can consider this moon-shaped region as your domain and then if you put this, then it is a regular domain.

That means here, if you look at this part, so if a continuous function on this you know boundary of this part, then you would get that solution of Dirichlet problem there you would see that the continuity there.