Introduction to Probabilistic Method in PDE

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Example of a Dirichlet problem

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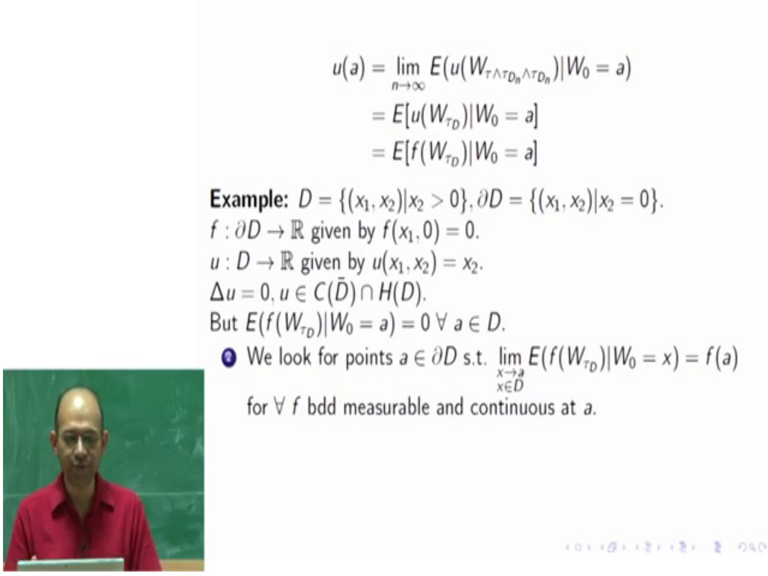

Now, let us see this example, which clarifies many of the issues I was talking before. Consider this D two-dimensional space, d-dimensional Euclidean space but we restrict x2 to be non-negative that means open half plane. x2 is the vertical axis, x1 is on the horizontal axis, x2 positive means upper half plane.

Is that open domain yes that is an open domain, connect it open domain unbounded and what are the boundaries. What is the boundary of this? Boundary is a x-axis. X-axis is boundary, x2 is equal to zero. Now, we consider function f, which is just zero. So, x1, 0 whatever value of x1 doesn't matter its value is zero, so on the boundary on this it is zero.

Now, D is given f is given so that means Dirichlet problem is given. How many solutions does this Dirichlet problem have? It has infinite many solutions. I mean one trivial solution is zero solution, constant zero because that satisfies the boundary condition also the Laplacian of zero is here. On the other hand, if u of x1, x2 is equal to x2.

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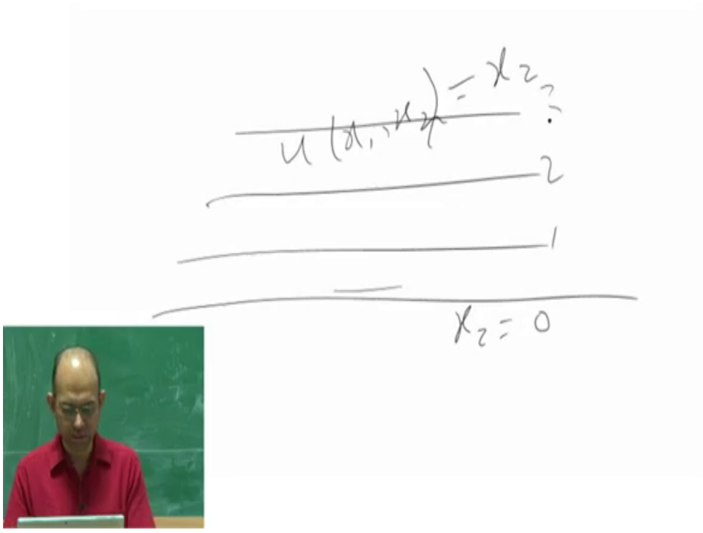

That means that this is for example the x-axis, x2 is equal to zero and then the function is just u of x1, x2 is equal to x2. That means the level surfaces are like this, it is in increasing like this. And zero here and at one which is 1 here 2 here 3 here et cetera.

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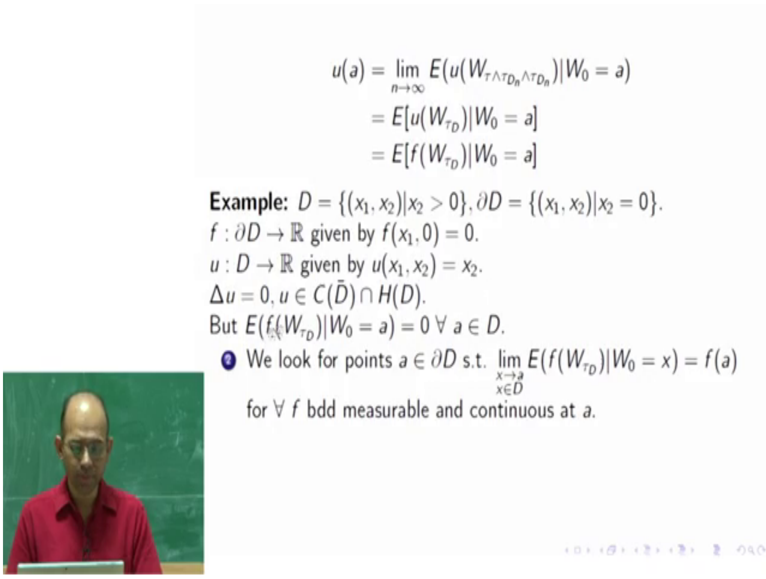

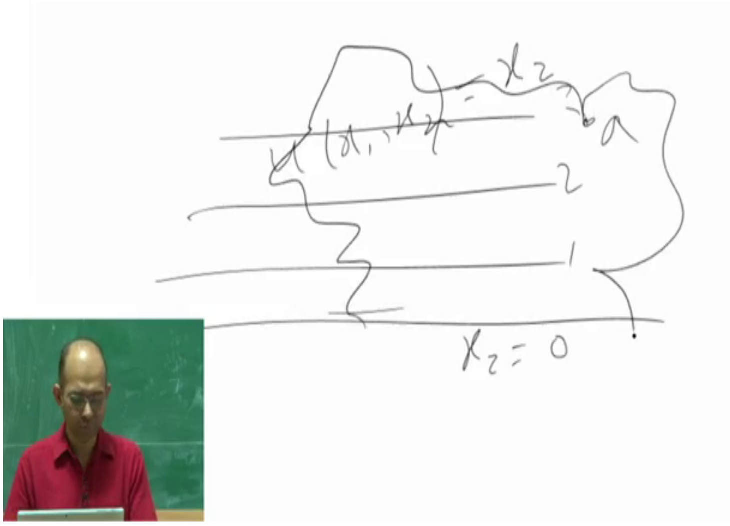

So, then the partial derivative of u, there is a twice like Laplacian would be zero here and then it satisfies therefore the boundary condition and the harmonicity. So, this is also a solution of the Dirichlet problem. Actually, one can take any constant multiple of x2 that would also be the solution of this problem.

It has infinitely many solutions but here expectation of f of W tau D given W zero is equal to a if you compute it, that for any a in D that means we start any f a from here and run Brownian motion and compute you know expectation here, expectation f of W tau D, we would see that ok expectation of this thing would be a zero.

Why is it so? Because f is itself zero for you know conditional expectation of zero function is always zero. Ok, fine so here this is one example that the bounded solution there is a trivial solution is actually you know coinciding with the stochastic representation conditional expectation presentation of the solution. So, This is the point which would actually discuss in the next class that what if we do not have this constraint. So, then how much general we can talk about, thank you very much.