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NPTEL ONLINE COURSE

Transform Techniques for Engineers

Properties of Fourier Transform of a Periodic Signal

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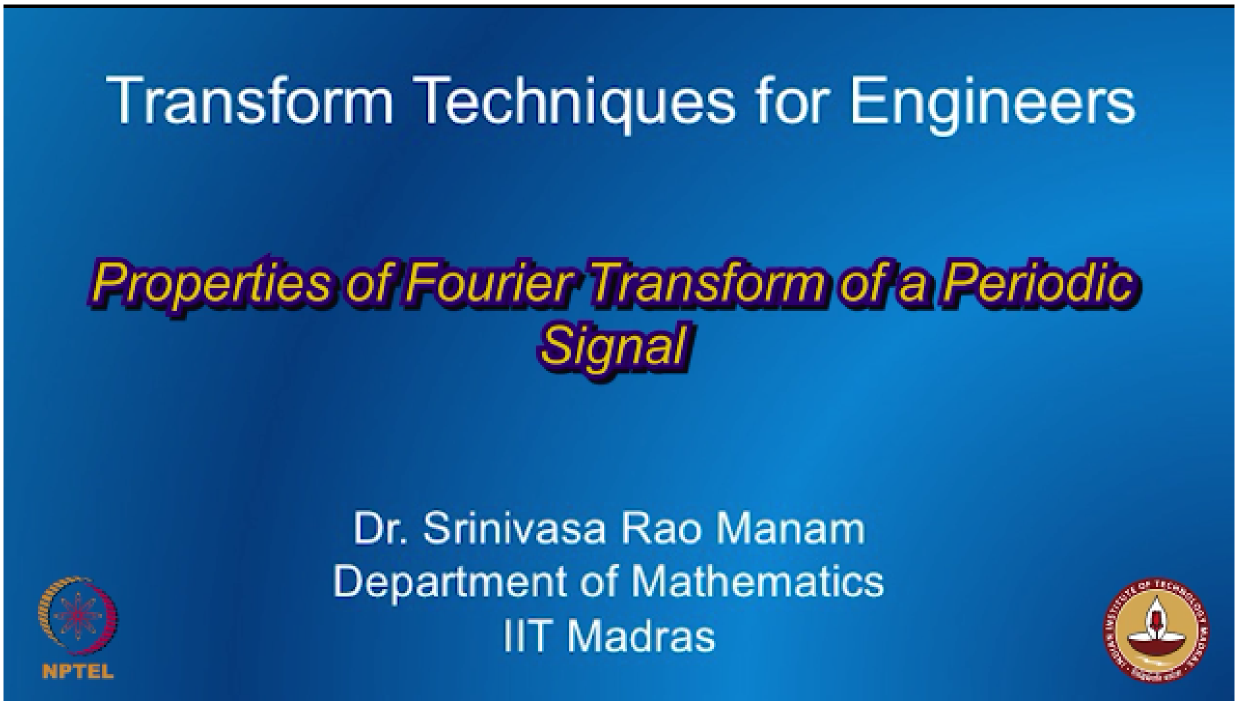
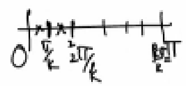
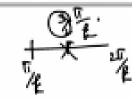


So in this video we will see certain properties of Fourier transforms of a periodic function, before we do that we will just have a recap of what we have done in last class, last video, that we have seen what is the Gibb's phenomena, what we have seen is peak, the first peak when you consider, when you try to estimate the Fourier series with few terms, what you see is there is an artifact coming into picture, so there is a first peak as many terms as so whatever may be your number of terms in your Fourier series, but that first peak around that discontinuous point either sides you will have a peak, when you calculate the value of that peak, and what you see is

$$= \frac{1}{\pi}\sum_{n=1}^{k}\frac{\pi}{k}\,\frac{\sin\left((2n-1)\frac{\pi}{2k}\right)}{(2n-1)\frac{\pi}{2k}}$$





$$\lim_{k\to\infty} S_k\left(\frac{\pm L}{4k}\right) = \pm\frac{1}{\pi}\lim_{k\to\infty}\sum_{n=1}^{k}\frac{\pi}{k}\,\frac{\sin\left((2n-1)\frac{\pi}{2k}\right)}{(2n-1)\frac{\pi}{2k}} .$$

$$= \pm\frac{1}{\pi}\int_0^{\pi}\frac{\sin x}{x}\,dx = \pm\frac{1}{\pi}(1.852) = \pm 0.589$$

0.5

0.089 is the error

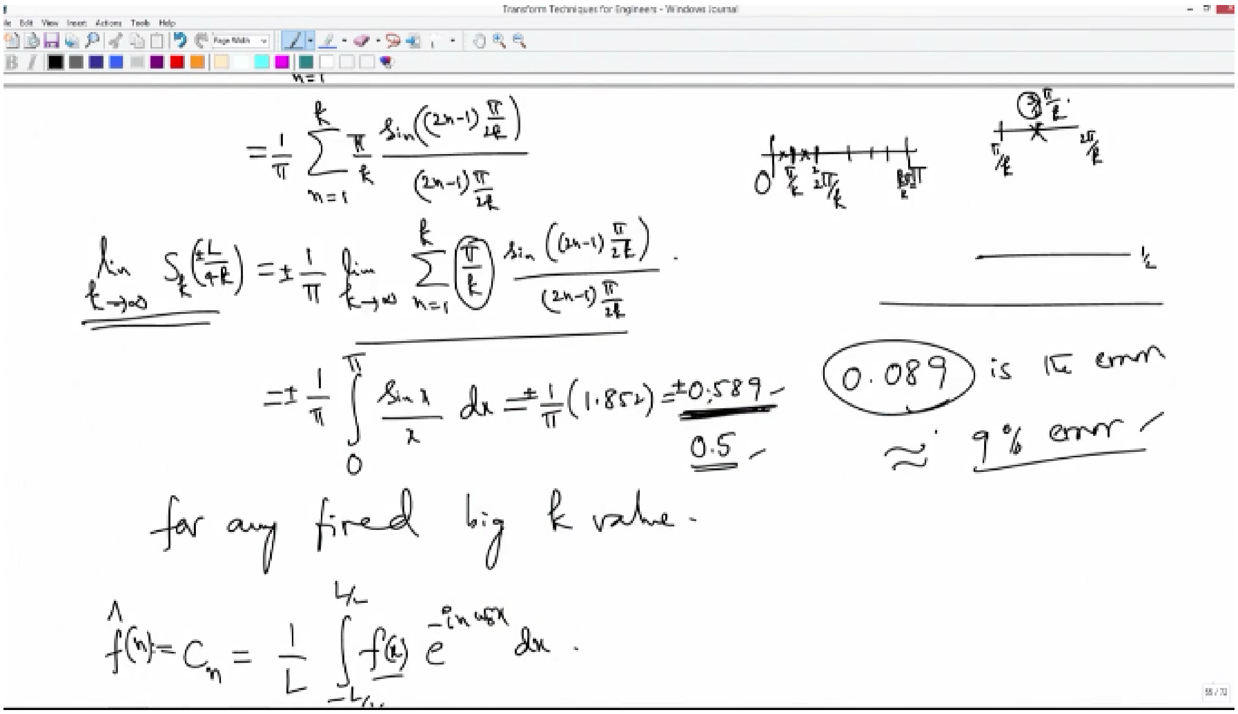
$\approx 9\%$ error

for any fixed big k value.

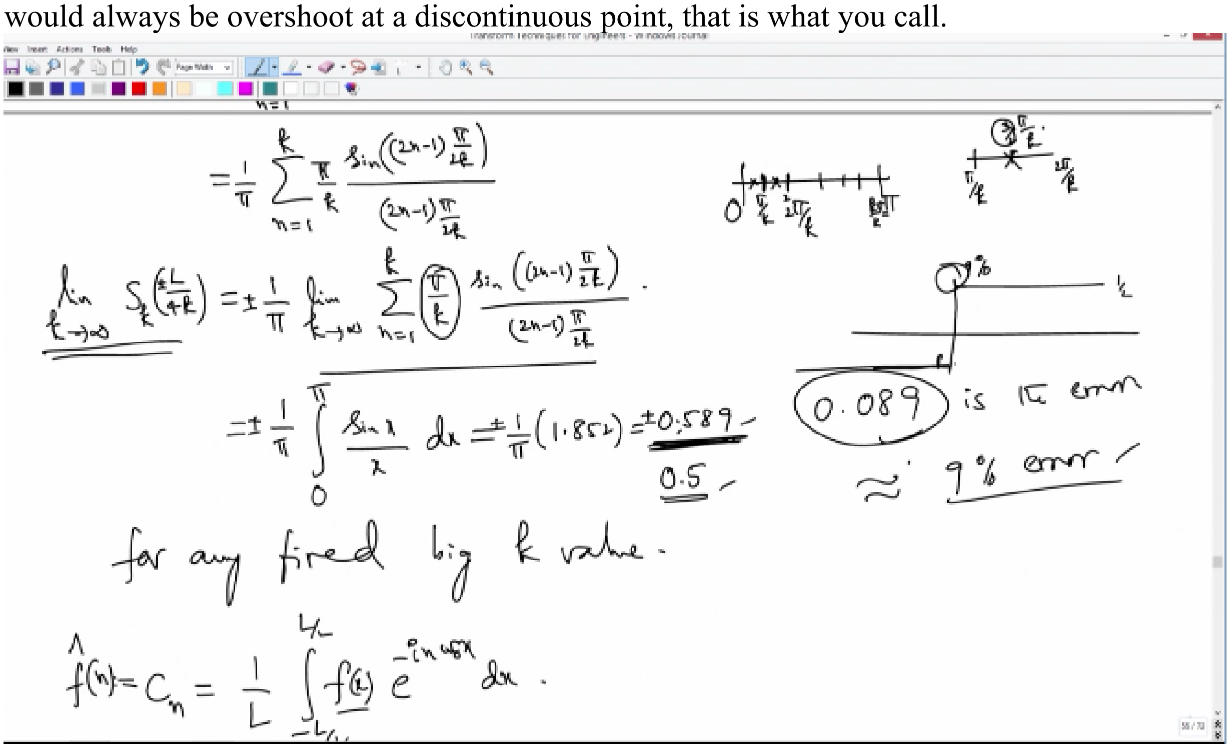
$$\hat{f}(n) = C_n = \frac{1}{L}\int_{-L/2}^{L/2} f(x)\, e^{-i n \omega_0 x}\, dx .$$

this SK, so this SK, this is the partial sums, if you take K terms and L/4K that is where you have the peak, SK(L/4K) that is the value of the peak.

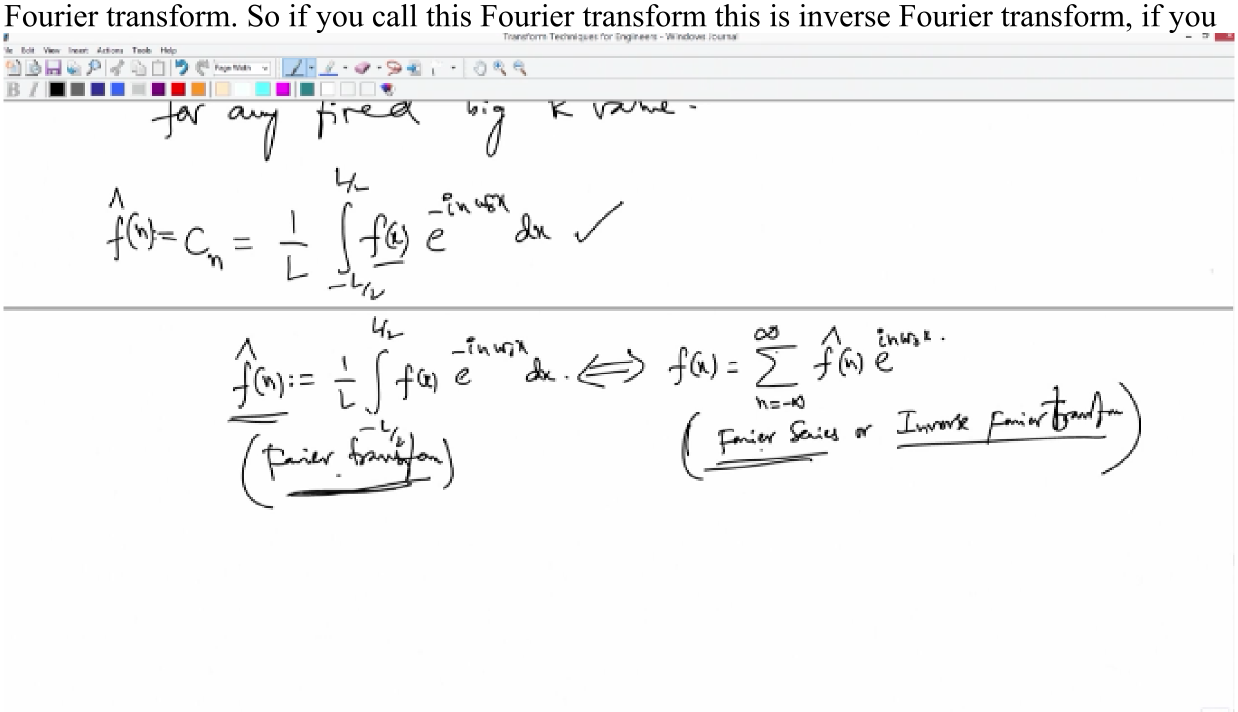
So that is what we have seen, that if this is a 1/2, this is your thing, so you get something like this, if you increase the number of terms these things slowly vanish, so what you see is you will now see that it's going to be valid, so as you increase K go bigger and bigger, if you increase further what you see is only, still the peak will be there and this width of this first peak, that is going to be very, very small, so very narrow, very narrow, but the peak, the value of the peak will be always, there will always be some peak, so as you increase say as you see mathematically, as K goes to infinity this limit is obviously this more than 0.589, but the actual value should be, as K goes to infinity, so they increase the number of terms, so you take the infinitely many terms in your Fourier series then only it will converges to the actual value, that is what the theorem says, okay.



At discontinuous point the theorem says if you take only infinitely many terms, infinite terms, a full Fourier series when you consider then that converges to point wise, so that at discontinuous point it is an average value of a function, so in this case, in this example the average value is -1/2 and +1/2 that is 0 divided by 2, so it's 0, but here whatever so you cannot take infinitely many values so this K goes to infinity means, K is bigger so it's never going to be infinity, so you're not considering all the terms in your Fourier series, so some big value there is always be a peak, okay, so that peak, the peak value is more than 0.5 so you have a 9% overshoot, there would always be overshoot at a discontinuous point, that is what you call.



So if this is what is discontinuous, jump discontinuity here so there will always be a overshoot of your Fourier series, okay, so at this point eventually you will see, you will always see if I remove this you will understand, so if I take really big number of terms, that many number of terms if you consider what you see is exactly matching somewhere here the small is going to be, there's a small, it's like this, so there's always be overshoot either here or here, so that is what is called a Gibb’s phenomena, that's what we have seen in the last video.
So this is a kind of artifact you see in the Fourier series, if you try to estimate with the finite number of terms of your Fourier series, so if you want really exact value at a discontinuous point it's only theoretically possible as you have to consider infinitely many terms from the Fourier series, so that means and so that converges to the average value of the jump, that's what theorem says that is how, so if you try to estimate from the Fourier series you have to consider only finitely many terms however big it may be, number of terms, but you'll always have a overshoot value at the discontinuous point that is what is the Gibb’s phenomena, it's not really important, it doesn't have any impact on the Fourier series just want to give you what happens, what is the Gibb’s phenomena by considering trying to estimate from the Fourier series of the function, so the kind of a reconstructing function from the Fourier series, if you try to do so it's not possible with discontinuous points, because there will always be an artifact, for example here is always 9% error and if you consider different example you may have a different error, so but that error is always fixed whatever may be number of terms is always so this will always be minimum of that, that error will always be there that’s what I want to convey from this Gibb’s phenomena.
So we have seen what is the Fourier coefficients and also called, we call them Fourier transform, and Fourier series is also called inverse Fourier transform, Fourier series or inverse Fourier transform. So if you call this Fourier transform this is inverse Fourier transform, if you

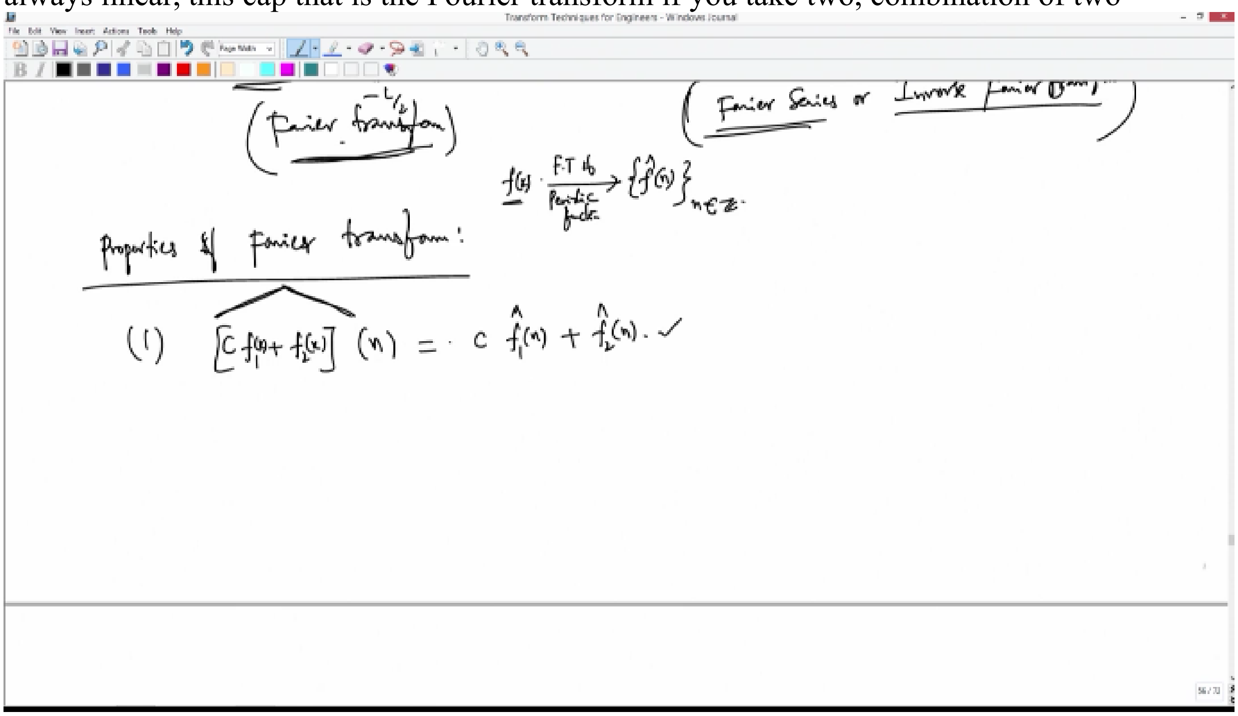


call this Fourier coefficient, then what you get is, you get back your signal by Fourier series, so kind of transforming and then you come back, that's inverse transform, so you try to see now certain properties of this definition, some properties of Fourier transform, so first one is a linear property as usual, so we will see even when you define Fourier transform over a full real line

Laplace transform and Z transform what you see, certain properties of these transforms, okay, it's a mapping from functions to function space, so you are actually defining, so if you look at this cap, cap is the operator acting on the function F what you end up is finally function of, so your integrals is a value, so that depends only on N, so you take the functions F(x) in some function space to some number F cap N, so some discrete sequence, basically you get a discrete sequence of numbers, this is what it is the Fourier transform of a periodic signal, okay, Fourier transform of periodic function.

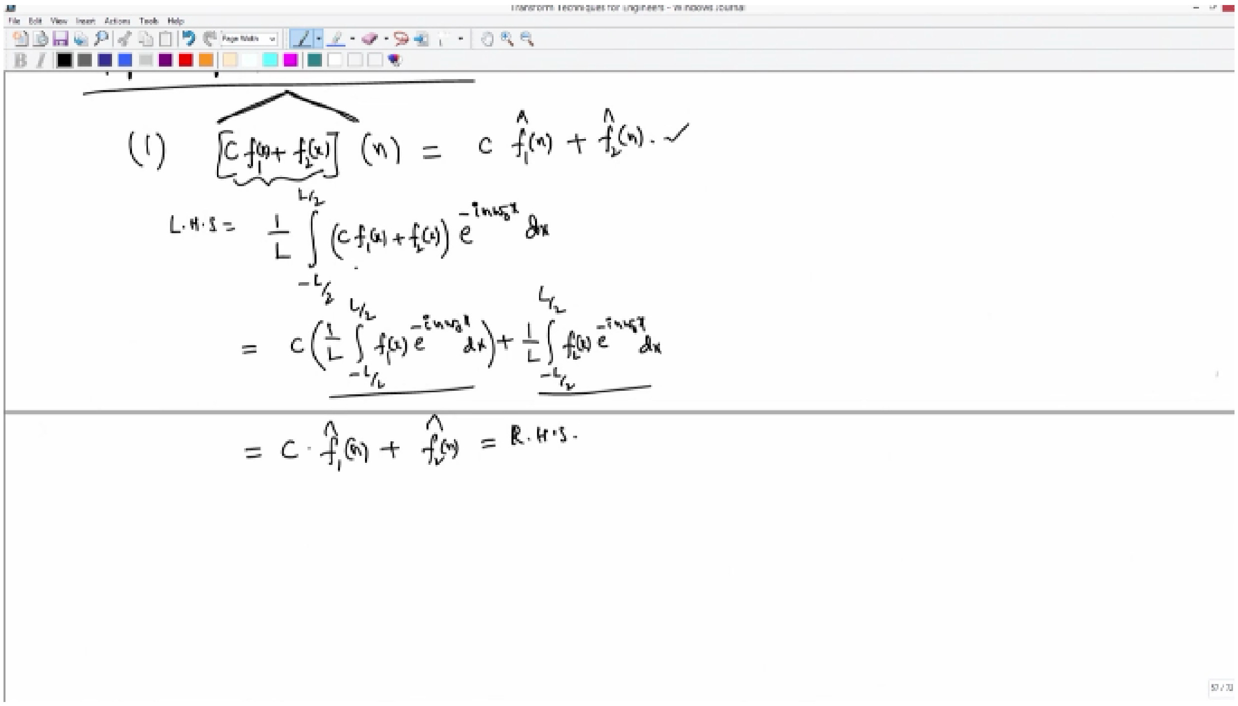
So even if you define it over full real line, so you take F(x), and what you end up is here, for each N, for each F you have a different N values, you have this F cap(N), so there's a sequence of numbers, N is running from, N belongs to Z, okay, that's what it means.

So look at the first property that is called a linear property, so what it tells you is that you consider instead of this function F if you consider some C constant times F1(x) + F2(x) so you consider this function, for this you consider that transform, transform of this, this is the meaning, since this N = C times F1 cap(N) + F2 cap (N), so this is what the results, this is always linear, this cap that is the Fourier transform if you take two, combination of two



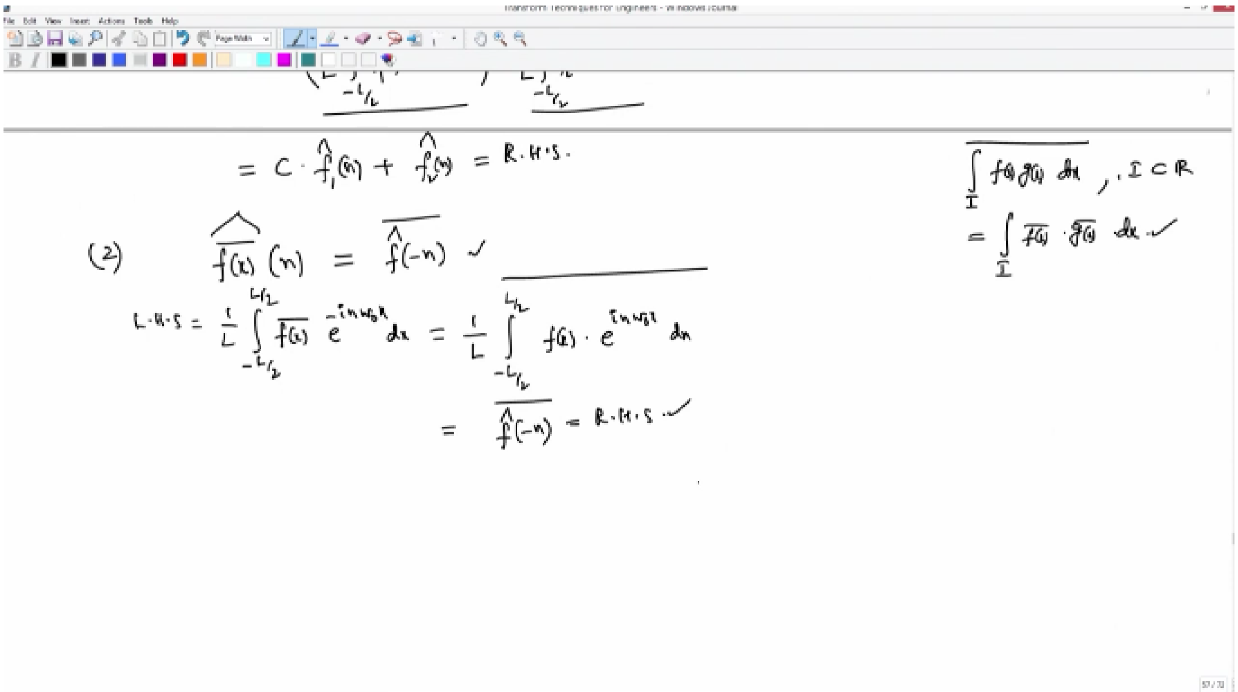
functions with a coefficient, the coefficients comes out and this is just a linear, so cap you can linearly distribute it, so you can easily see this just by the definition LHS, left hand side if you consider, what is this one? So by definition this is going to be 1/L –L/2 to L/2, now instead of this function F you have C times F1(x) + F2(x) times E power -IN omega naught X DX.

Now what is this one? So because it's a linear function so, so that's clear right so this is 1/L –L/2 to L/2 F1(x) E power – IN omega naught X DX, you can split this integral into two parts, when it's constant other one is 1/L integral –L/2 to L/2 F2(x) E power - IN omega naught X DX, of course you have to consider all the functions combination F1, F2 are both functions with period L, then this transform will be a definition is, Fourier transform is this, and this is nothing but a C times, this is exactly Fourier transform of F1, F1 cap(N) + this is F2 cap(N), so this is exactly your RHS, so this is obviously this is true.



Second property that we have is, we'll just consider F(x), you take a signal, you take this signal can be in complex valued, so take its conjugate and you have a conjugate function so as some G(x), and for which if you take the Fourier transform there's a function of N and that should be equal to F cap(-N) and then you take a conjugate for that, it's a complex number on which you had a conjugation, so this is also true, so again you start with LHS 1/L –L/2 to L/2 F(x) bar into E power - IN Omega naught X DX. So at the right hand side you have a bar comes up, so we'll try to take the full bar outside, L is a constant real number so –L/2 to L/2, so when you take the full bar so what happens, when you take the full bar so this is F(x) and then you have, when you take this bar, bar, so when you take the bar, when you take the bar so you have F(x) G(x) DX for example, for this bar is actually integral F(x) bar G(x) bar DX that's what it means, because if it's a real, a real interval I belongs to, I is a real interval so this is what is the case. So using this you can write F(x) and this E power – I, when you take the conjugate because bar you're imposing bar so you have 2 bars, right, you have 2 bars and what you have is this, E power - IN omega naught X DX.

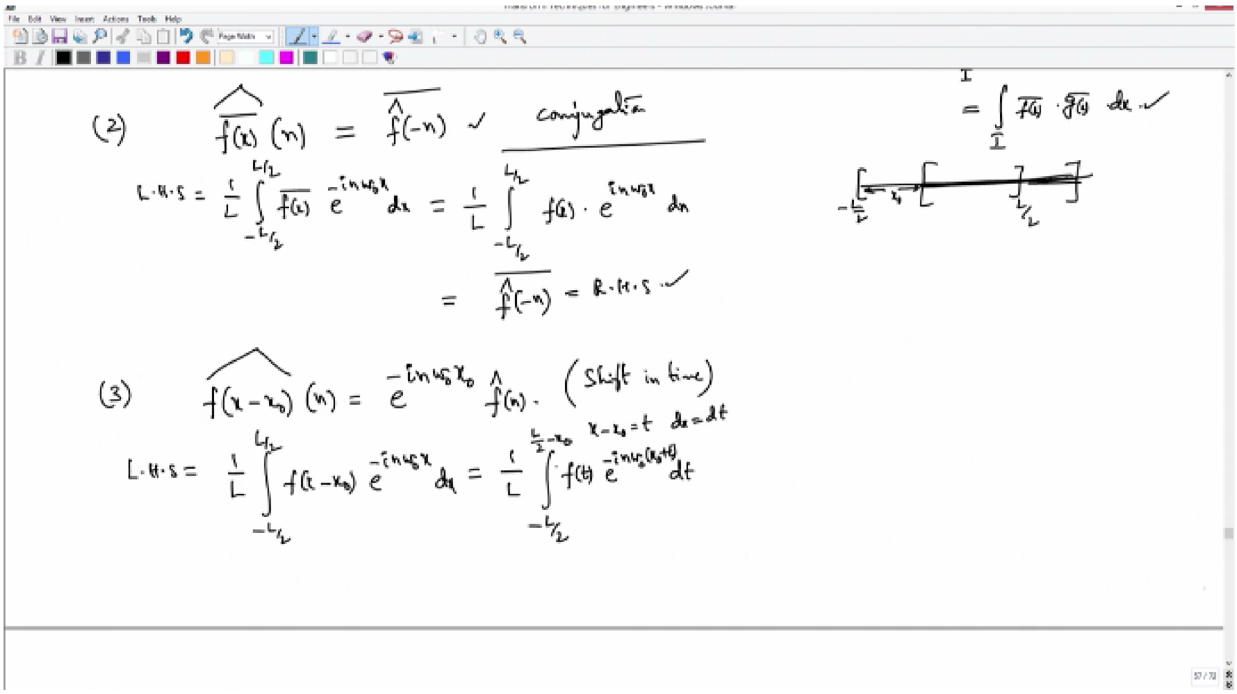
Now first this bar you try to put it here, bar bar goes, and this bar over this and DX is as such, so here but first because of this you have a plus, so you have this one, so this is exactly equal to this is FN, F cap(-N) for this whole bar, so this is exactly your right hand side, so these are trivial properties which you can see sometimes it'll be useful okay, so I'll just do it, it's called



conjugation, linearity first one is called linearity, a linear property this is called conjugation, so if you shift in time, so time variable we are considering as X, if you shift in time and also what is the effect in the Fourier transform that you can see that F(x-x naught) you shift with X naught, okay. And then for this if you take the Fourier transform and this is exactly equal to what comes out is exponential term, so IN omega naught X naught, and you have F cap(N) so this is shift in time, so time is X variable here, so X is time.

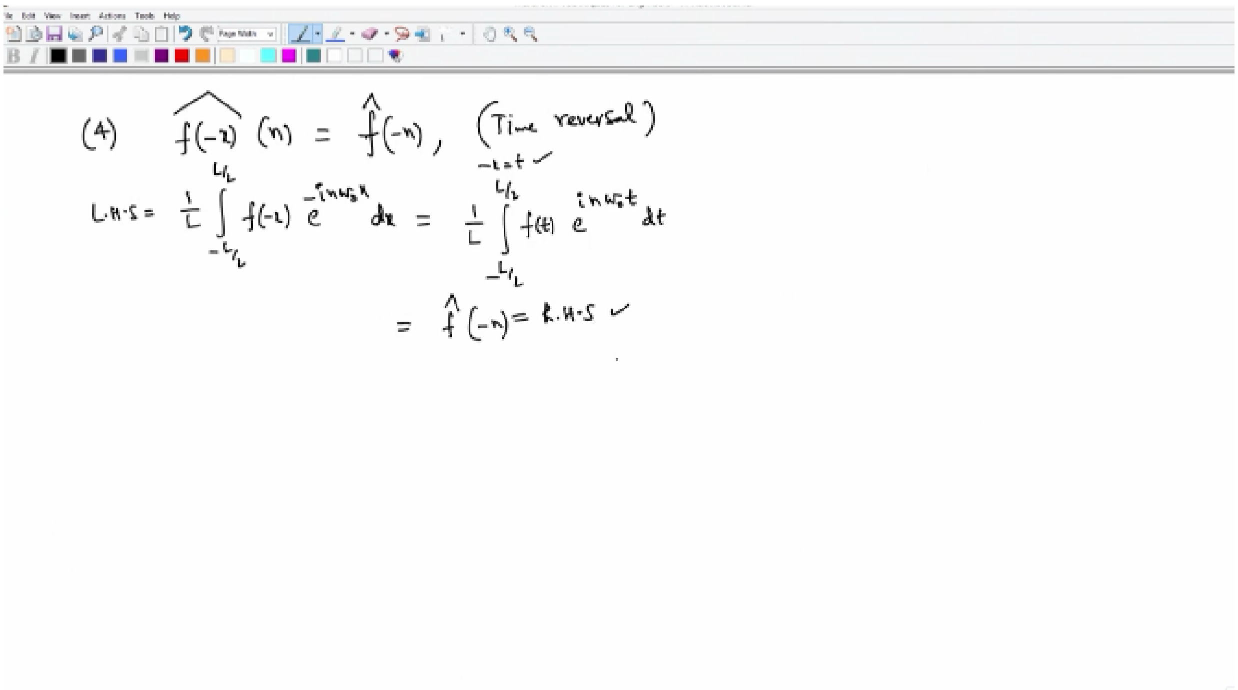
So again so LHS is proof is easy, that's why I'm just doing it here itself by considering LHS = RHS, 1/L by definition this cap means the definition –L/2 to L/2 F(x-x naught) E power - IN omega naught X DX, this is equal to 1/L.

Now what you do is you consider X-X naught = T, 1/L if you consider X - X naught, if you put X-X naught = T, so DX = DT, so you have DT F(t) E power – IN omega naught, X is X naught + T, okay. And what happens to your limits? Limits will be, when you put X = -L/2 - X naught, that's your T, L/2 -X naught, because it's a periodic function F and also exponential function is also periodic, okay, so the integrant is periodic so that implies with period L so this is also having a period L, so you have already seen earlier, so it's the same as considering L/2 to L/2 is same as you consider –L/2 + some X naught this is the length of X naught, this to the same length you go out so this is same, same values will be repeated here, so this is because of



periodicity you can replace this as –L/2 to L/2, so this is exactly, so you remove this E power - IN omega naught X naught, because it's the constant that comes out what is left is simply F cap(N), this is your RHS, that's the third property.

And we have the fourth one that is a time reversal you can just go back, time reversal property there is F of, you can go back to the time, so – X for this cap if you take the transform, what is the result? So this is F cap(-N), so this is called time reversal, this is called time reversal property, this is also easy to show so that I consider LHS 1/L –L/2 to L/2, F(-x) now and E power - IN omega naught X DX, and you want the Fourier transform of F, okay, so F(x) basically so that means you put -X = T, so that will give –L L/2 to L/2, -L/2 because after substituting the limits, and F(t) E power +, E power plus is - IN omega naught T, okay, that is T and DX is – DT, so that makes it this if you minus this you can replace, you can interchange them, you can exchange the limits, that's what it is, so you have –L/2 to L/2, this is exactly your F cap(-N) you need - off – N that is, so you have minus here, this is RHS, so you can easily see these properties, a simple properties if you're working if you are doing algebra with this transforms to the signals that will be useful.

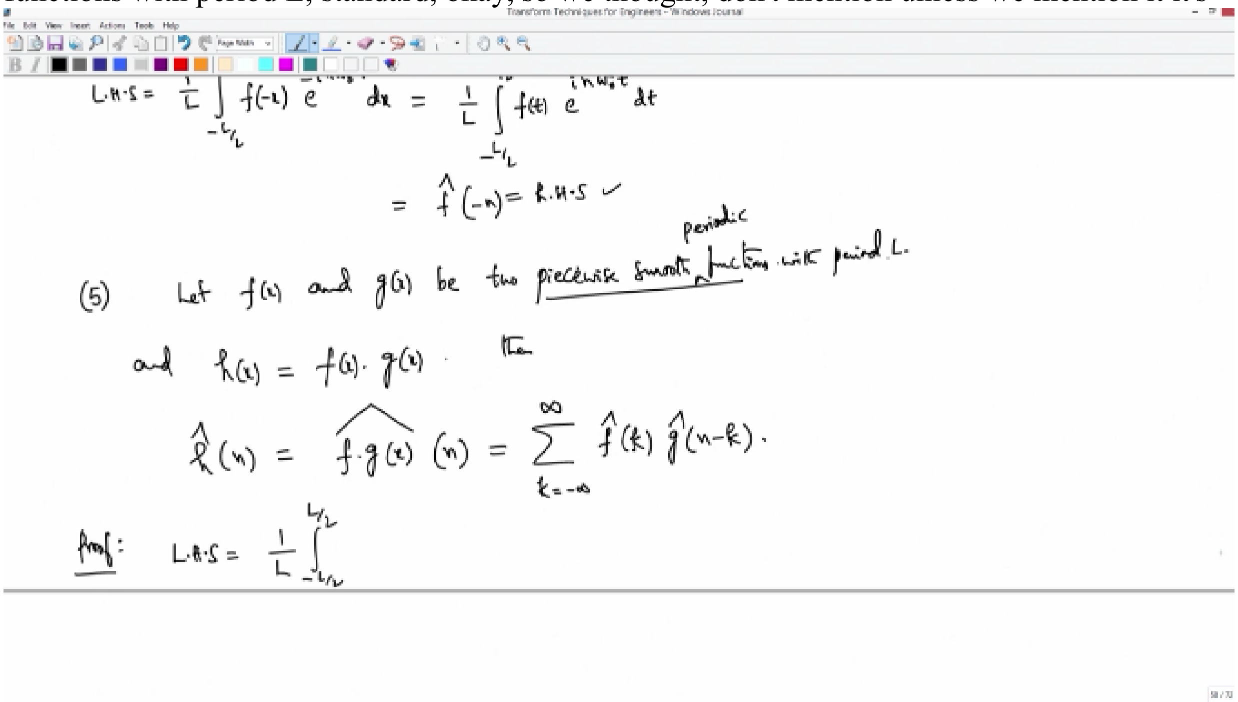


And some other properties, non-trivial properties, not so easy properties or if I define you take the product of two signals, so if you take two smooth signals so let F(x) and G(x) be two piecewise smooth, smooth means smooth functions, what is the smooth function? We define what is called piecewise differentiable functions, so piecewise differentiable function means F is piecewise differentiable, piecewise differentiable function means F dash is piecewise continuous, and you take any N derivatives if piecewise differential for every N, that means any N+1 derivative is piecewise continuous, right, so if F of nth derivative of FN is a piecewise differentiable function for every N, then F is piecewise smooth function, piecewise smooth function, okay.

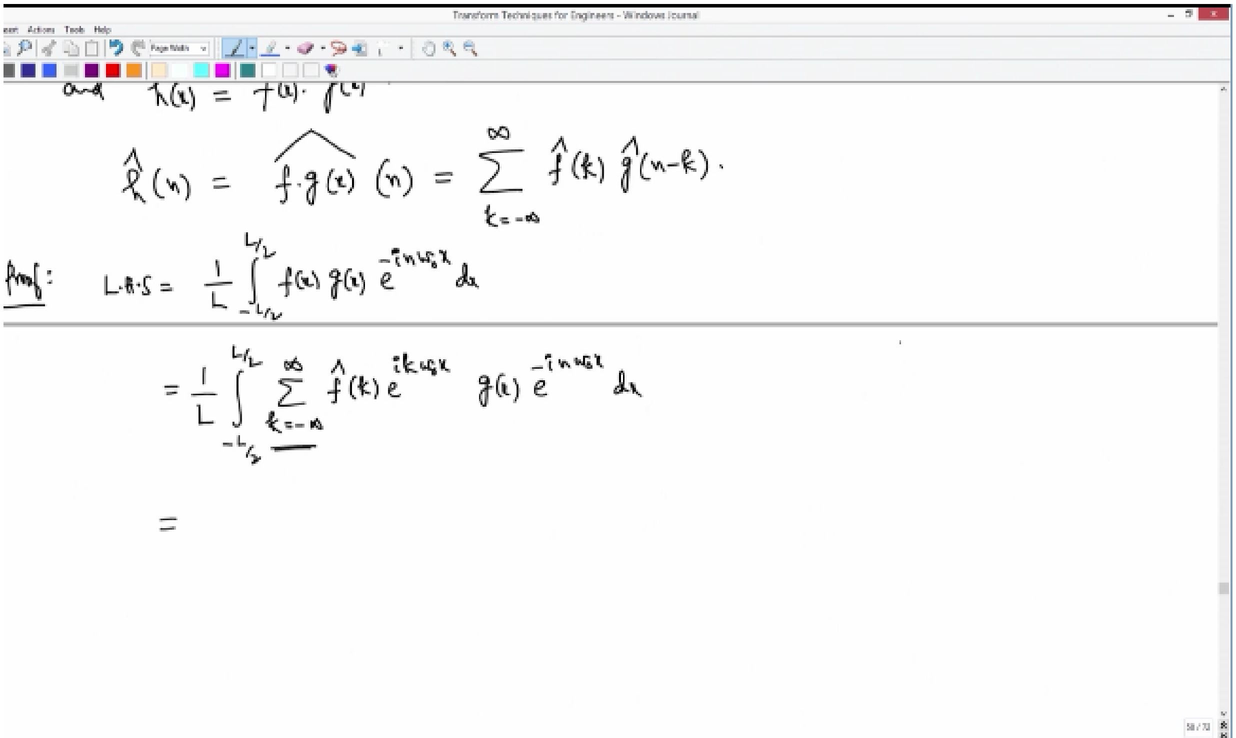
So that means you take next derivative that is piecewise continuous function, so this is true for every N, so then you say that it is piecewise smooth function so such a piecewise smooth functions all the examples that we consider with elementary functions, for example this one this is derivative is 0 here, 0 here, but it's not defined at this point 0, so except at that point is 0, 0 function, so again next level what is that? Except that point, 0 only again its differentiable any number of times.

Similarly you consider this and this, anything with the elementary functions if you represent your signal there is actually piecewise smooth function, such thing if you have then H(x) that is and let, and let H(x) = F(x) into G(x), take the product of that then H cap(x), H cap(n) that is F into G(x) for this you take the Fourier transform (n), okay, this should be equal to, you get a series, K is from minus infinity to infinity, and you have F cap(k) into G cap(n-k), that's what is the result.

So this you can prove by simply considering the left hand side again, this will be 1/L integral – L/2 to L/2 they're all piecewise smooth functions, both periodic functions okay, periodic functions with period L, standard, okay, so we thought, don't mention unless we mention it it's



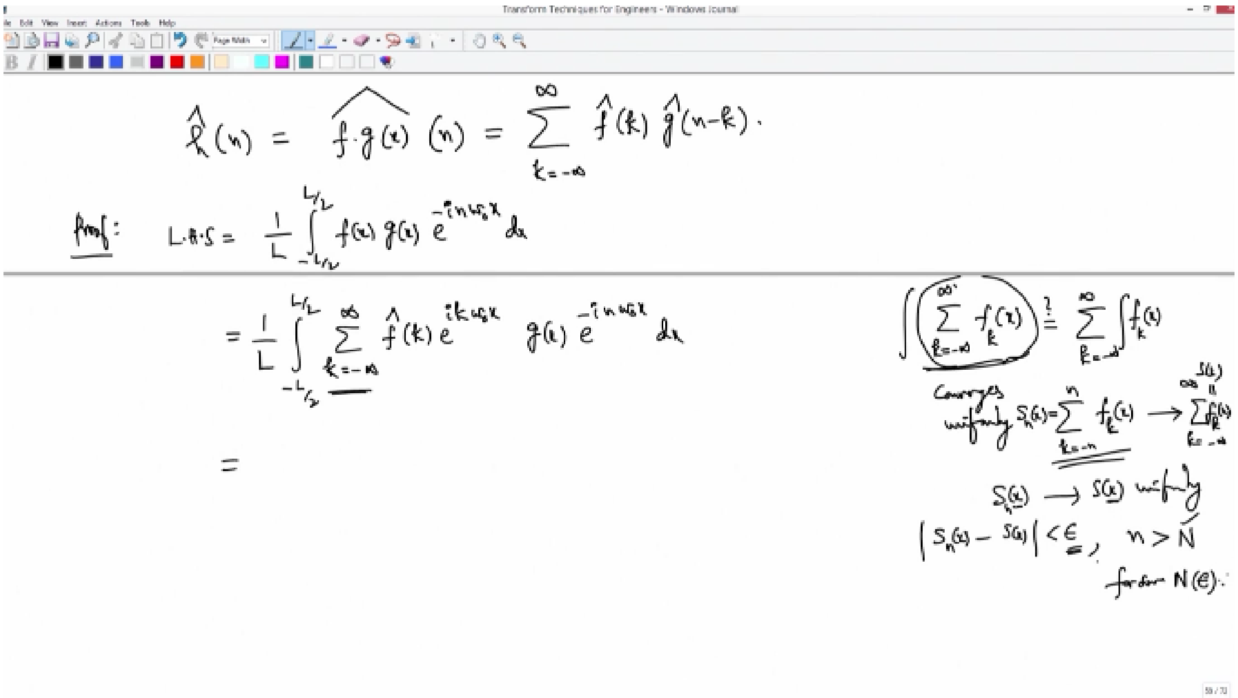
always a periodic function with period L, so this into H(x) that is F(x) into G(x) into E power – IN omega naught X DX, okay.
Now what you do is, so you have, because F is piecewise smooth function, piecewise differentiable function, and you have a Fourier series for F, so let me write this Fourier series – L/2 to L/2, F(x) I have a Fourier series that is N is from minus infinity, so K is from minus infinity to infinity, F(x) is sigma, K is from minus infinity to infinity F cap(k) into E power IK omega naught X this is your F(x) into G(x) E power – IN omega naught X DX, I just substituted what is F, you can also substitute, for G you will have a different thing, so F and G can be interchanged in this summation, if you replace G as a Fourier series you will get F and G are interchanged in this final result, okay, so we'll see what you get.



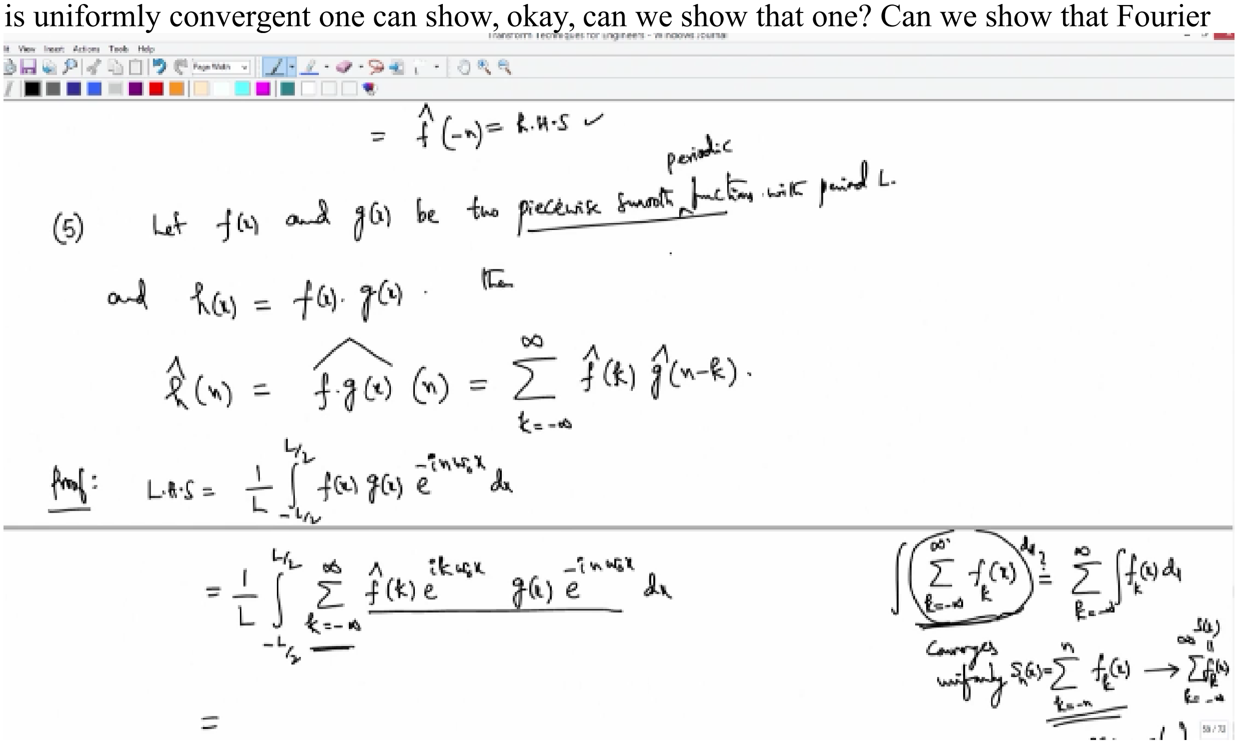
So what is the next thing is, I'll take this, I'll bring this summation or rather I take this integral inside this infinite series, when is this possible? You can do, you can do this Fourier series, you can do any series for that matter, any function series FN of, FK, let us say FK(x), and K is running from minus infinity to infinity, if you have an integral this is same as K is from minus infinity to infinity, integral FK(x), this is a questionable, if it is a finite sum this is always true, because it's infinite sum, when you can do this? If this series converges, converges uniformly then you can do, then you can do this, okay, if this series is converging that means you take the partial sums of the series that is K is from -N to N FK(x), if this converges to some function, this actually converges to the series, as a sequence this converges to FK(x), this sequence of functions if you call this SN(x), and this is your S(x), and this SN is a sequence of functions this converges to S(x) uniformly, then you can do this.

What is the meaning of uniformly? It means this converges you give an epsilon, so that means it doesn't depend on the domain, so that means if you know the definition of convergence basically this is SN's will be close to S(x), how close? Let me put some, however small number you give your epsilon there exist some number N beyond which, some number big N, okay, for some N, for some N this is the meaning you fix X, SNX converges to SX, okay, point wise that is point wise because you're fixing X, so given epsilon, if I give epsilon there is a N, for some N that depends on epsilon and also X because that fixed it depends on X, every time you change your X value there will be different N, that depends on epsilon and, not only epsilon but also on X.

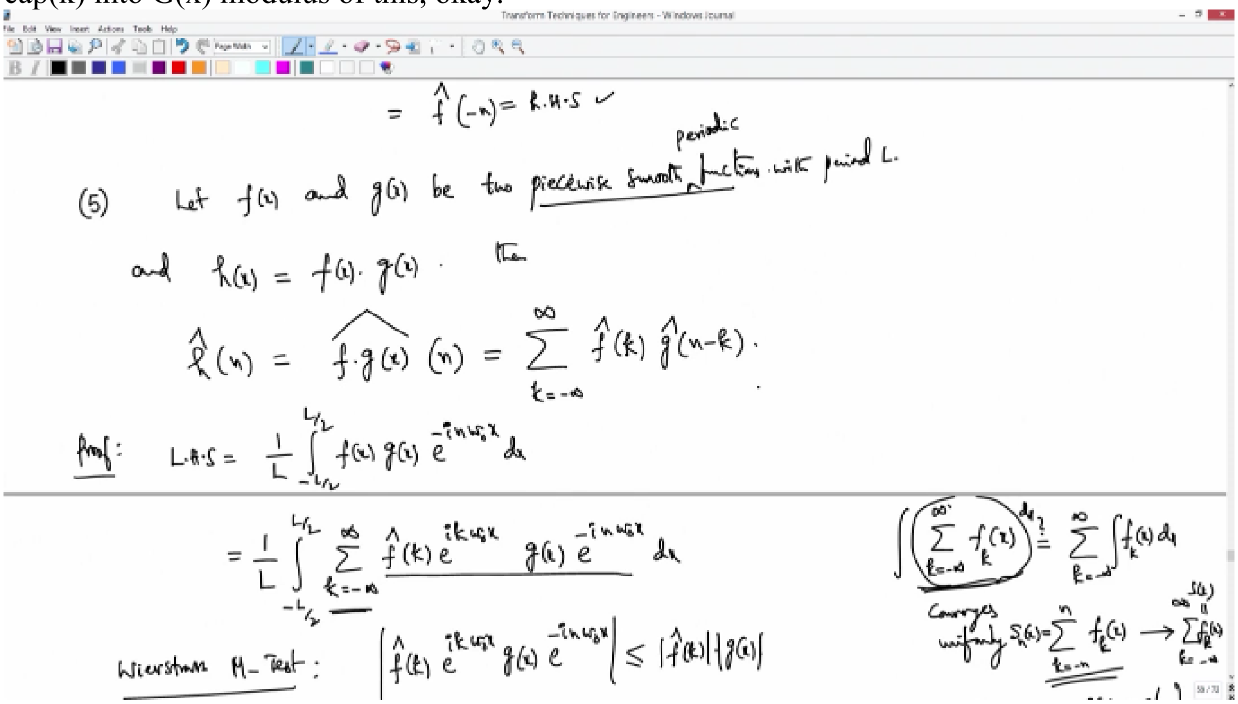
If you can get some N that depends only on epsilon that means independent of X it works for all there is a N that works for every, for some N that depends only on epsilon, that means it works for every X in the domain, okay, X belongs to actually here –L/2 to L/2, so if that is true then it is called this uniformly convergence, okay.



So you have the Fourier series basically what is this, what you have is a Fourier series including the Fourier series F cap(k) into E power IK this is a constant times E power IK omega naught X that is a function G(x) function and this is another function so you have this, this is your function, this is a function series that is your FK, your FK is actually complete whatever is inside here, okay. So this integral DX is same as integral, so that is what it means.
So is this convergence, so Fourier series if it is uniformly, if it is a piecewise smooth function it is uniformly convergent one can show, okay, can we show that one? Can we show that Fourier

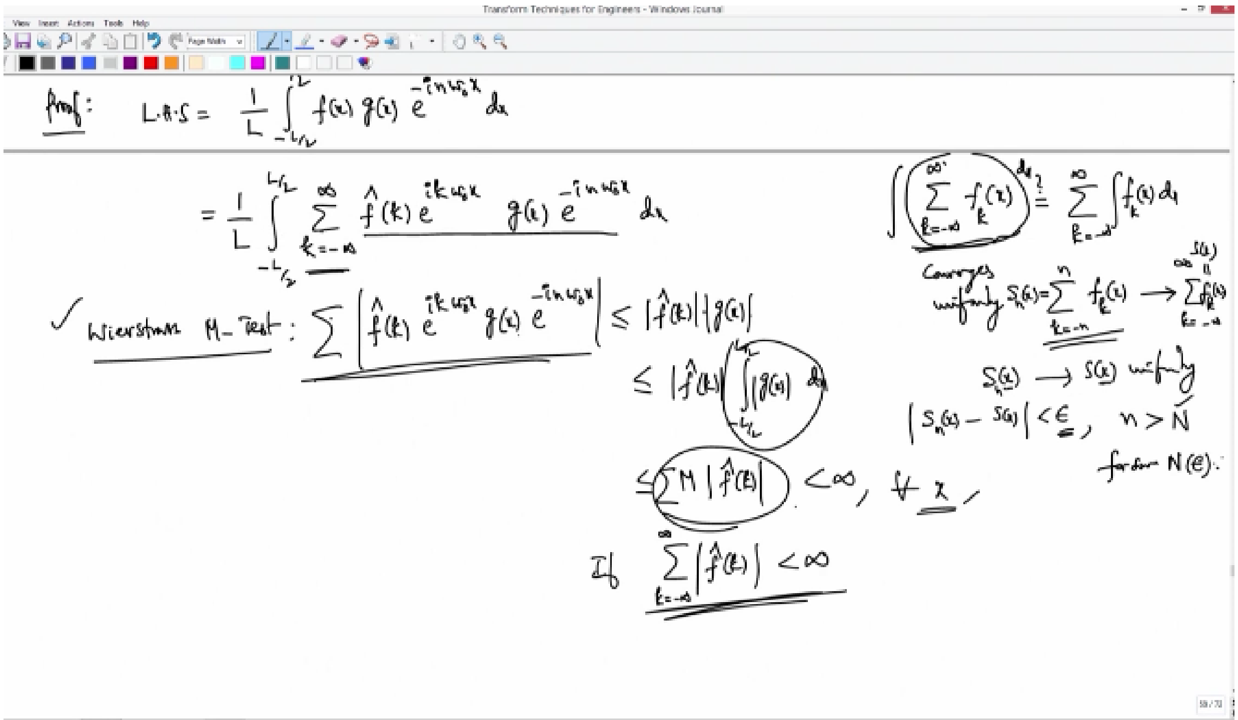


series is converging uniformly? Yeah, so we will show this series, how it is uniformly convergent, if you look at that Weierstrass test. Weierstrass M test, if you know Weierstrass M test that tells you that this sum, in this series this part F cap(k) there is a constant and E power IK omega naught X, G(x) and E power -IN omega naught X this summation, K is from minus infinity to infinity, and if you apply the M test each of this part, this is less than or equal to F cap(k) into G(x) modulus of this, okay.

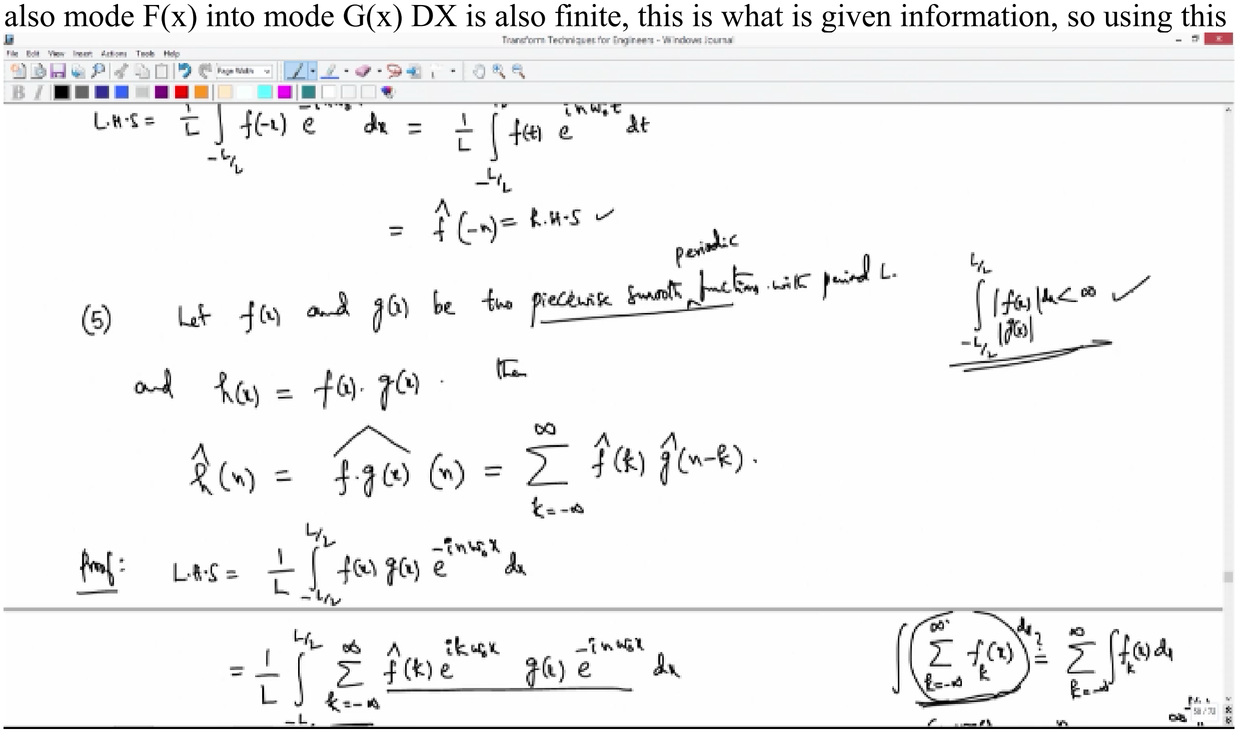


Now since G is integrable piecewise periodic smooth function, periodic smooth periodic function, so you have this, this is obviously bigger than F cap(k) into this integral G(x), so L/2 to L/2 DX this is anyway finite, this is the finite quantity so this is something like some, because G is piecewise smooth, so M times F cap(k) okay, so this is what it means.

Now this series is always finite for every X if this series is finite, if you know that this series is finite that is by M test, so since sigma F cap(k) this series K is from minus infinity to infinity this should be finite, do we know this one? If this is true then we can say that this is true, this is finite for every X that means it's uniformly convergent, so you can do this integral you can take it inside this sum, that's what it means.

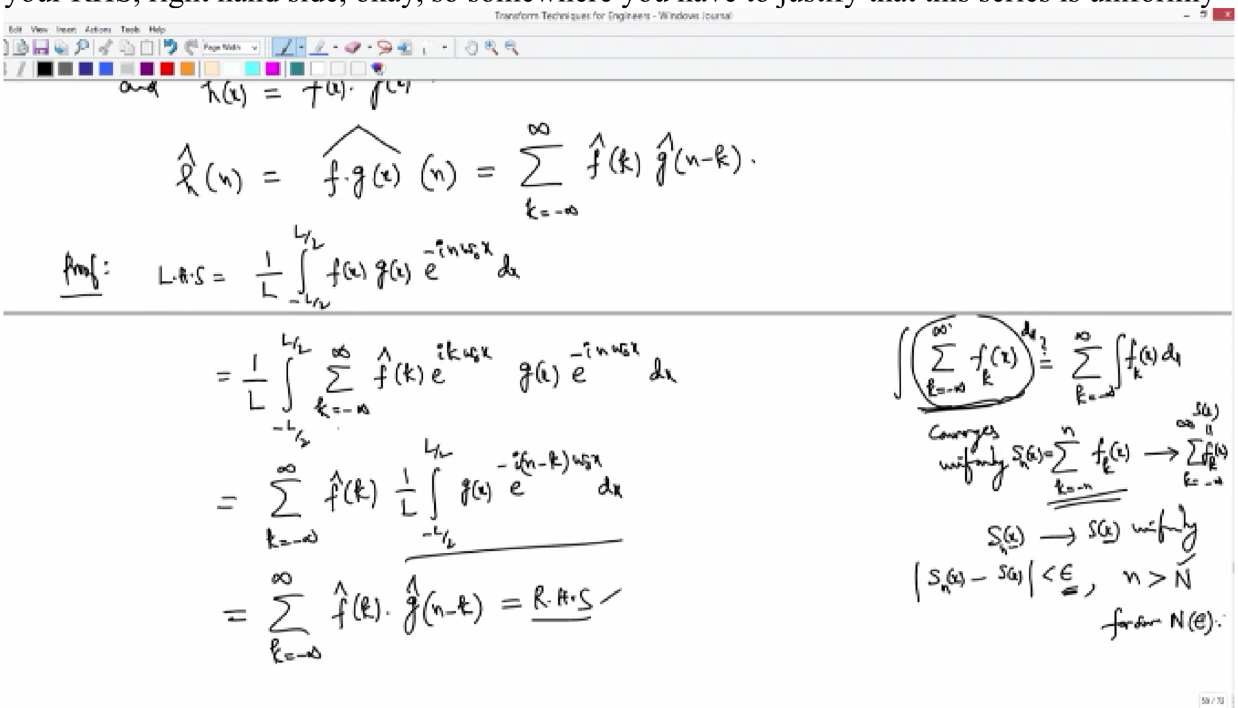


So let us not get into this, so assume that this integral we will try to justify later, so as of now so let's assume for some reason so what is given information is that F1G be piecewise smooth periodic function so the product is also piecewise smooth periodic function, so it is obviously it is, absolutely integrable so F is a piecewise integral means it's obviously what we know this information is –L/2 to L/2 mode F(x) is finite. DX is finite and also G(x) implies, the product is also mode F(x) into mode G(x) DX is also finite, this is what is given information, so using this



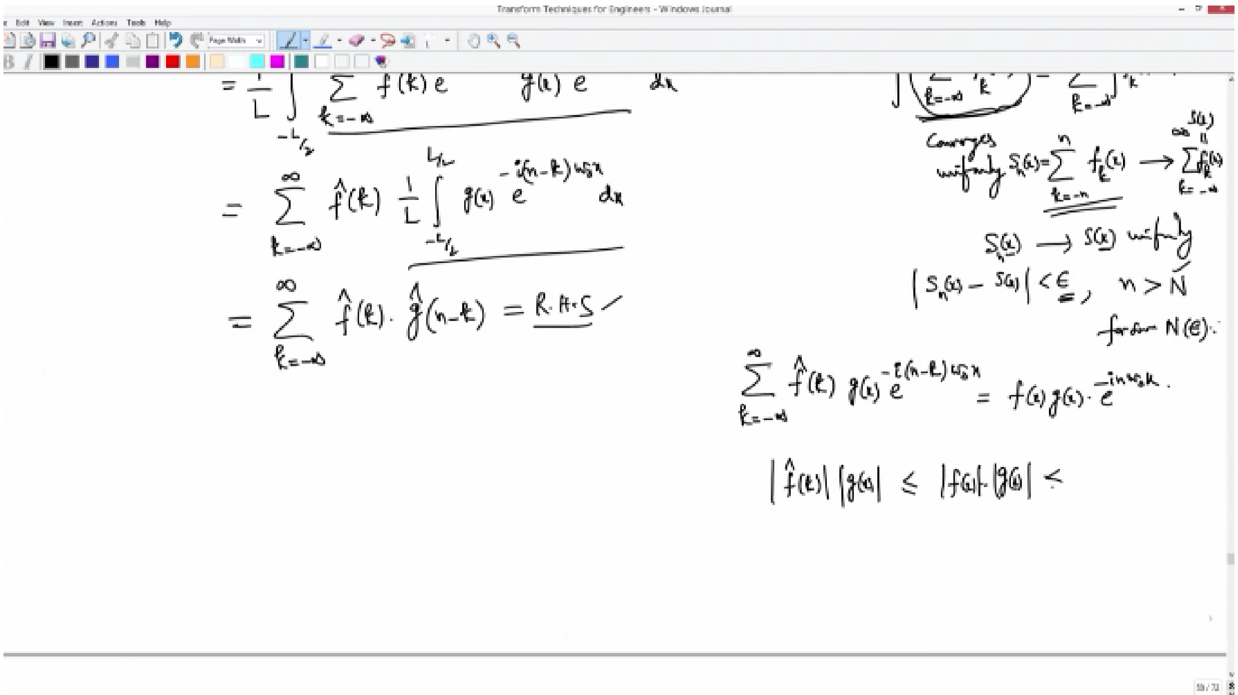
one can actually show that this series converges uniformly, so through M test or by some other means we will see later, okay, so this is one way of showing that uniform convergence, so let me not dwell on this.

So assume that you can do that, this is possible to do so you can take this inside and then what you get is then we get K is from minus infinity to infinity, F cap(k) so what is left with is 1/L – L/2 to L/2 G(x) E power – I N-K omega naught X DX, so this is exactly equal to K is from minus infinity to infinity, F cap(k) into, this is nothing but G cap (n-k) at N-K, so this is exactly your RHS, right hand side, okay, so somewhere you have to justify that this series is uniformly



convergent, okay, so what is given information is that if F product, the product function mode H(x) okay, this is absolutely integrable function, F into G is absolutely integrable function, so that implies you can see that it is going to be uniformly convergent, that's clear right so and you can, why is this so? That is not difficult to see because yeah so I think it's easy.

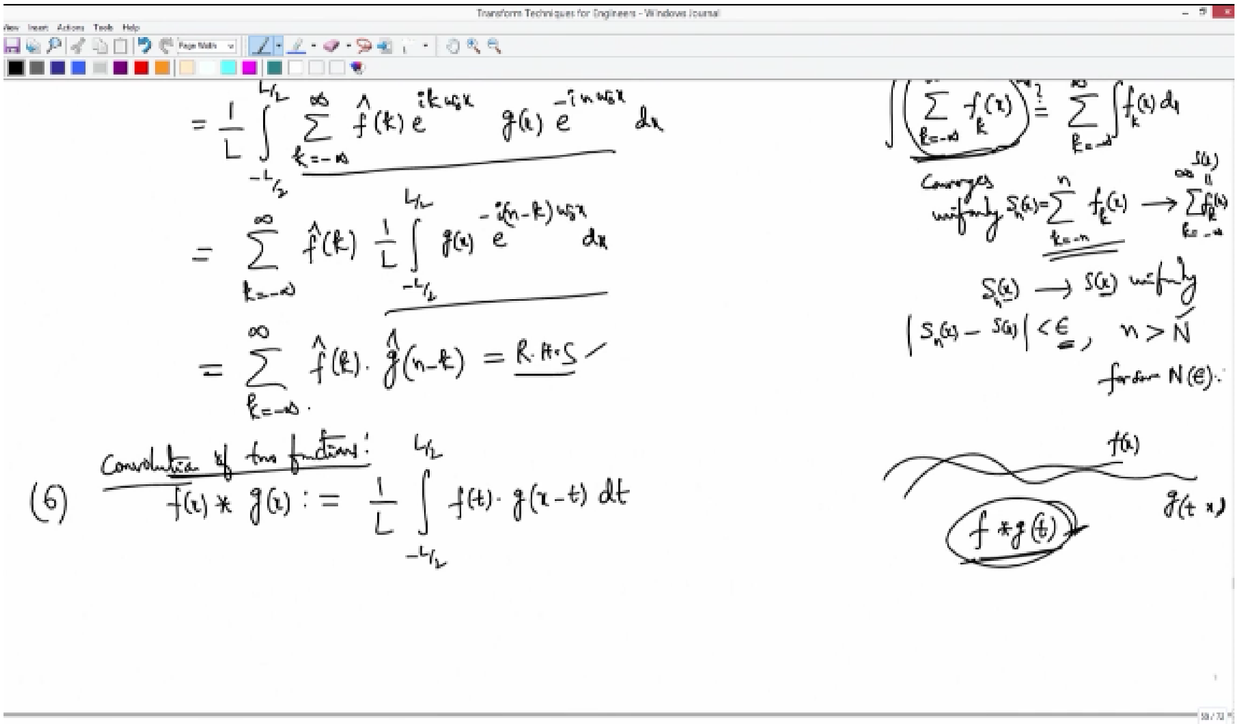
So how we will justify this? We justify this step, okay, by showing that the series is uniformly convergent, so how do I do? I consider F(x) into G(x) okay modulus of this, okay, this is less than or equal to the F(x) G(x), no wait, we can't start this, K is from minus infinity to infinity, F cap(k) into E power into G(x) E power – I (N-K) W naught X this is what, this series which is equal to F(x) G(x) into E power - IN Omega naught X, okay, so this is exactly same as this one, this integral, each of this term when you take with modulus F cap(k) into G(x) modulus exponential modulus is 1 is obviously less than or equal to, it should be less than or equal to this modulus that is F(x) into G(x), this modulus is 1, right, so the integrand is this, and this will



be less than or equal to, so let's assume this result let's try to just, we will try to justify this later okay.

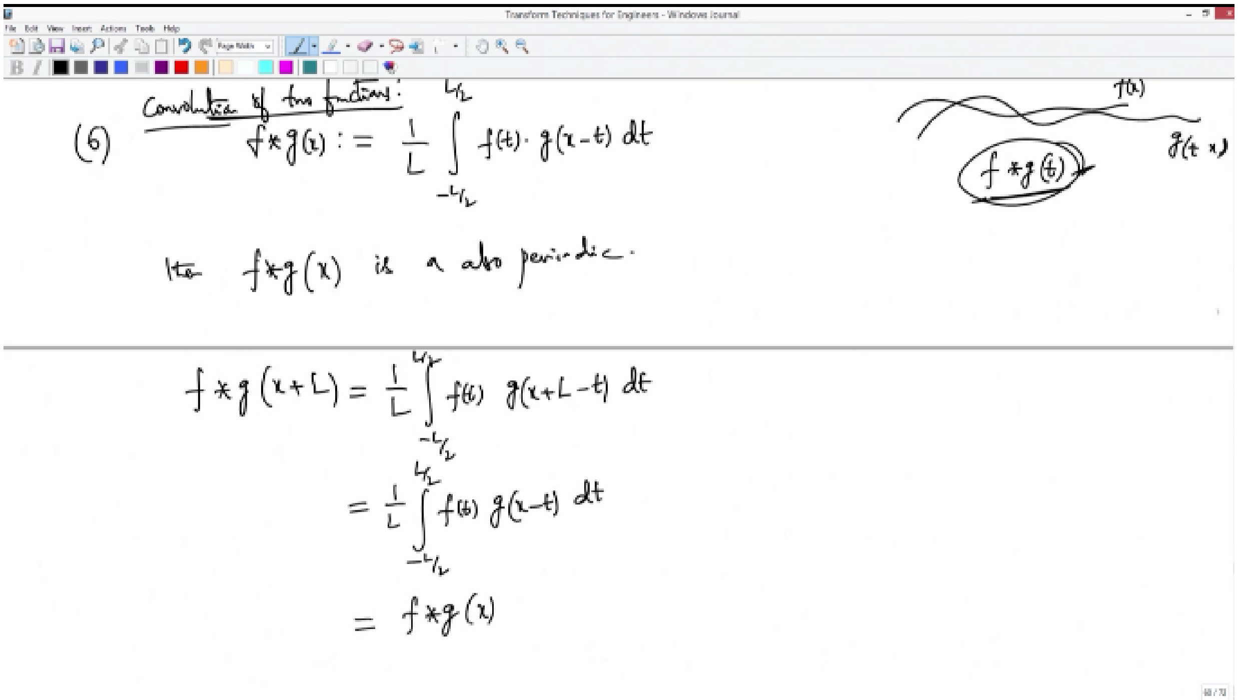
So I think I'm not getting exactly or somewhere you have to use, because of this integrability of the product of this function we will see that, we can show that this series is convergent that implies you can take this integral inside this sum, that will give this right hand side, so let's move on so I'll try to justify this in the next video, so we will see some other property, another non-trivial property that we have is a convolution of two functions, if you have a 2 functions F(x) and G(x) you can convolve them, so this is the notation for this, that is you just, that is some integral of, that is a definition of this L/2 to L/2 over the same integral, so these are periodic function with period L, so that is –L/2 to L/2, so you convolve with, convolving means you consider F(t) into product of G(x-t) DT and then you take the integration, so that is like if you want it function something like, this is your F(x) and F of, G is another function and what you do is you just translate G(x-2) for example, this one this is F(x), the convolution is the product at like this, the product of this that will give, and you integrate, once your product, you take the, consider the product and you integrate and what you end up is the convolution F x G at the value 2, okay, that's what it means. 2-X you should consider like this, then that is called this is convolution value at 2.

So like that we consider for every X, so for every T so you consider always T- X2 functions



you consider so, such is the definition of a convolution, so convolution of two functions, so because the F and G are, F and G product is, this product there are periodic functions and the product is integrable, square integrable, they are absolutely integrable functions because of that this is always finite so it is well-defined, because this integral this modulus of this is less than or equal to modulus of less than or equal to integral modulus of this, that means and which is finite so that means this is well-defined function, okay, so this is well-defined, this is a convolution of product of two functions, convolution product of two functions F and G, so if I consider them as piecewise smooth periodic functions, their product is well-defined because this integral is finite in that case.

Then what is this one? What I have is then F convolution G, the convolving function this is the convolution function is F convolving G(x) together or is also periodic function, it is a periodic function, is also periodic. If you find G are periodic this is also periodic, so how do we prove this? You consider the same period so you consider X+L and you simply write or the definition –L/2 to L/2 F(t) G(x+l-t) DT, so just by change of variable you can see that because it's a periodic, F and G are periodic functions the integral will not change, integrand will not, this limits will not change though they actually change and there is actually same as again that is equivalent to, that is equal to the same –L/2 to L/2, F(t) so what you do is, X+L you take it as X+L you take it as X1, X dash, then rather what you do, no it's not a straightforward, you don't have to do anything, so don't do anything so simply G, since G is periodic with period L this is exactly G(x-t) DT, so this is nothing but your F convolution of G(x), that is your, so that means it's periodic function.



Seventh, again on convolutions, and if you consider two such periodic, smooth periodic functions, piecewise smooth periodic functions you convolve these two functions and you consider the Fourier transform of that you get that as F cap(n) into G cap(n), so the way we defined as this convolution this constant term, so this is in your hands, so if you change you may end up this relation will also change, if you try to change your definition here and this result will change, okay, so we will see exactly whether by defining 1/L that factor will get this equal or not, okay, or you may get some other constant here.

So let us see what it is? So LHS is 1/L of –L/2 to L/2, F convolution of G(x) into E power -IN omega naught X DX, that is my LHS, and now 1/L square –L/2 to L/2 F composition of G, you know, 1/L that comes out that is L square, again –L/2 to L/2 this is now convolution F(t) G(x-t) DT into E power – IN omega naught X DX, so this is equal to 1/L square –L/2 to L/2 one more integral L/2 to L/2 F(t), what you do is I try to replace X-T as X1, so you have G(x1) so I try to change so, so what happens? If I replace this with the outer integral, so that is DX integral, so you have DX = DX1, so I have DX1 and this limits will become when you put X =–L/2 - T + L/2 –T, so outer will be –T –T, and this because they are periodic they are same as –L/2 to L/2 so they don't change, so and you have this and DT is as it is, and this is E power -IN W naught, X is now X1+T so DX, so now these are separated.

Now these are the two integrals 1/L integral –L/2 to L/2 F(t) so T integral you can write separately, now they are separated this integrant is 2 separable functions F(t) E power - IN omega naught T DT into 1/L –L/2 to L/2, now G(x1) is a dummy variable so you can write again DT you can write if you want as G(x1) E power - IN omega naught X1 DX1, so you can

(7) $\widehat{(f*g)}(n) = \hat{f}(n)\cdot\hat{g}(n)$

$$L.H.S = \frac{1}{L}\int_{-L/2}^{L/2} f*g(x)\cdot e^{-in\omega_0 x}\,dx.$$

$$= \frac{1}{L^2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} f(t)\, g(x-t)\,dt\; e^{-in\omega_0 x}\,dx \checkmark$$

$\underline{x-t=x_1}$ $\quad dx = dx_1$

$$= \frac{1}{L^2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} f(t)\, g(x_1)\,dt\; e^{-in\omega_0(x_1+t)}\,dx_1$$

$$= \frac{1}{L}\int_{-L/2}^{L/2} f(t)\, e^{-in\omega_0 t}\,dt \cdot \frac{1}{L}\int_{-L/2}^{L/2} g(x_1)\, e^{-in\omega_0 x_1}\,dx_1$$

see that these are same, so these are your Fourier transform of F(n) and G Fourier transform of G(n), so this is exactly, so if you define it like for this function composition, that is a convolution of two functions so in fact you don't need really piecewise smooth functions you simply consider as long as they are well-defined the inner rather, the convolution product is well-defined between two function, the Fourier transform will always give you of course you take the periodic functions, periodic functions if you consider you always see that the convolution, Fourier transform of convolution will be product of the Fourier transforms, so this is the property, important properties.

$$= \frac{1}{L^2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} f(t)\, g(x-t)\,dt\; e^{-in\omega_0 x}\,dx \checkmark$$

$\underline{x-t=x_1}$ $\quad dx = dx_1$

$$= \frac{1}{L^2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} f(t)\, g(x_1)\,dt\; e^{-in\omega_0(x_1+t)}\,dx_1$$

$$= \frac{1}{L}\int_{-L/2}^{L/2} f(t)\, e^{-in\omega_0 t}\,dt \cdot \frac{1}{L}\int_{-L/2}^{L/2} g(x_1)\, e^{-in\omega_0 x_1}\,dx_1$$

$$\underline{\widehat{f*g}(n) = \hat{f}(n)\cdot\hat{g}(n)} \checkmark$$

Now you have one important last property which you have, and we will see this in the next video, what we see is so far, we're just giving you some kind of properties of these transforms that will be, so similar properties will be doing for other transforms in the later videos, so these are the very important properties, sometimes may be useful so it's just they're simple application of known results in calculus, and we have one more simple property along with that and I will give the recap of what we have done and we also give some result on delta functions, delta functions which are, that are useful, one simple result that will be useful, so we have seen that delta function, we have defined what is delta function, we have seen that by definition an integral of the delta function over the full real line minus infinity to infinity is 1. And if you actually consider only between 0 to infinity of delta function, delta X DX and it should be 1/2, why is it so, we will see in the next video along with a recap of what we have done for the Fourier transform of a periodic signal, and its inverse transform or Fourier series of the periodic function or signal okay.

So we will see this, we will have a recap of everything what we have done, final example we will see, will recap and then we will do some more examples, we will try to do some more example in the next video. Thank you very much.

[Music]

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