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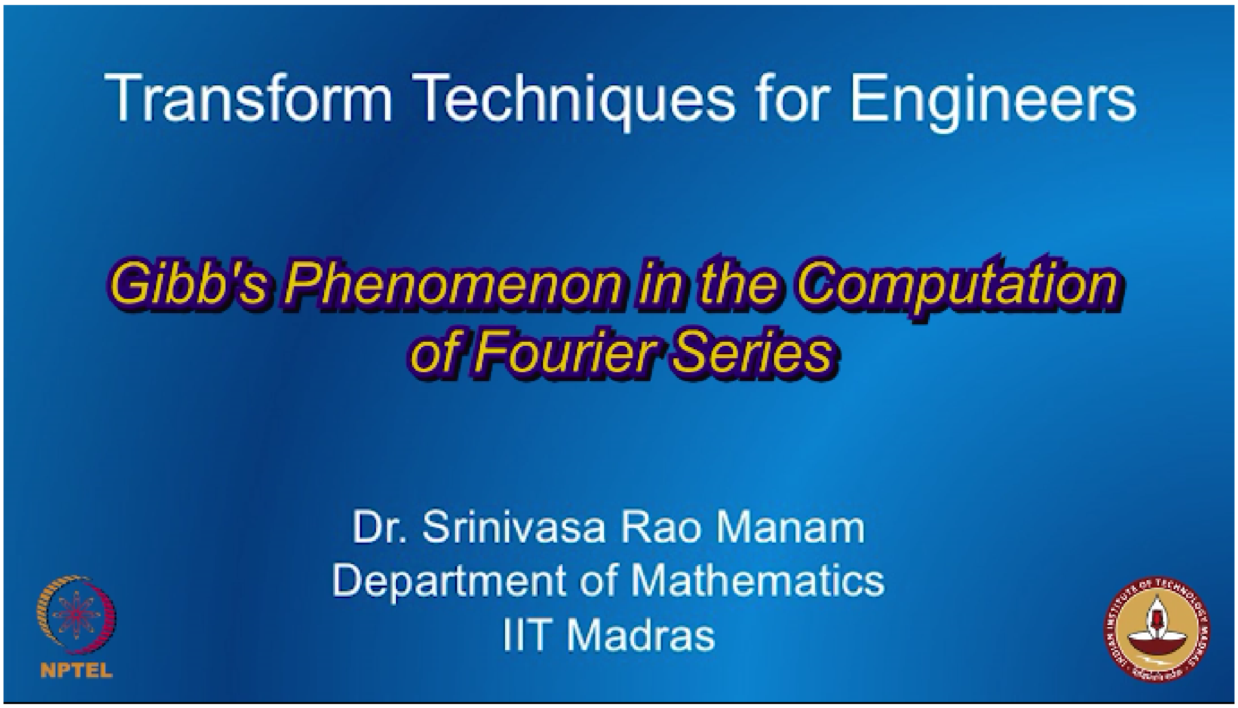
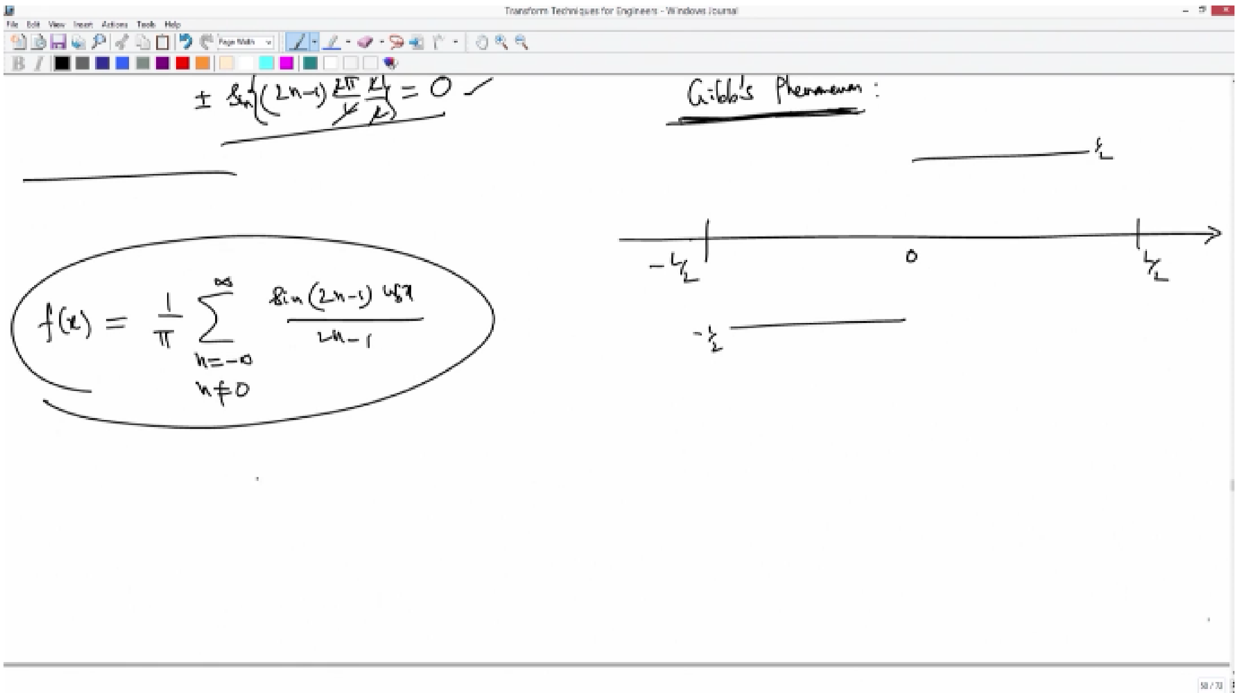
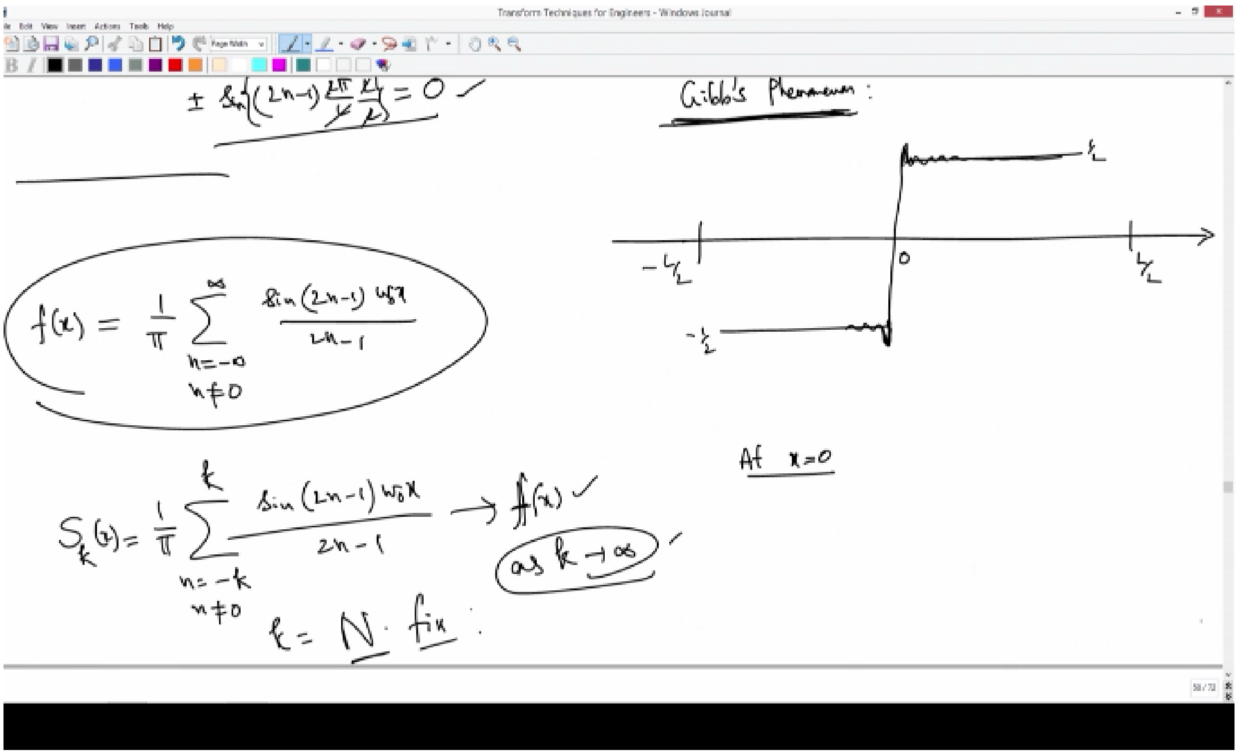
Transform Techniques for Engineers
Gibb's Phenomenon in the Computation of Fourier Series
Dr. Srinivasa Rao Manam
Department of Mathematics
IIT Madras



So welcome back, last video we have seen an example of piecewise continuous function, in fact piecewise a smooth function, piecewise differentiable function for which we have derived the Fourier series, we promised to give you what is called Gibb’s phenomena for a Fourier series at discontinuous points, that is actually it's a technical point of view only we have to explain, so there's nothing more insight on Fourier series when you discuss about the Gibb’s phenomenon.



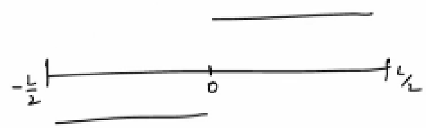
So we'll see what this Gibb's phenomena through this example, so I'll just briefly explain what it is, when you have a Fourier series like this and you try to fix this, you take only finitely many terms for example N is from -K to K 1/pi times, N is not equal to 0 and this is your Fourier series, and you try to compute this Fourier series, you try to plot this thing for K, K let us say 10 terms, 20 terms, and bigger terms, so fix some big N, K = big N, you fixed this and you try to plot, so eventually we know that this converges to this, we know that this converges to F(x), once you fix X as K goes to infinity, so as K bigger and bigger this converges to F(x).
What is actually happening? Once you fix this K which is some big number N, what you see at the discontinuous point 0, 0 is a discontinuous point and what happens here is that, see this is actually converges, 0 actually converges eventually it converges to here at this value, this is at -1/2 this converges, as it approaches the discontinuous point you will see some fluctuations, and you'll see that this is a big fluctuation here, and then it goes to 0 and then it goes to, so it do the same thing, so there is a fluctuation up there and then it goes like that, this is how it converges. So what happens at the point of discontinuity 0, and what you see is that there is a jump here, there is a value actually as at X = 0, at X = 0 you see that there is a overshoot value of this finite sum, this finite sum if you call it SN(x), SK(x) so overshooting value as X goes to 0+, 0+ side if you see that there is a overshoot here, that is also overshoot here, okay, if you observe, if you just plot it you will see that, so maybe I will just show you eventually sometime later in Mathematica, I'll show you how the plotting stuff this function, and you take finitely many sum of a Fourier series, at discontinuous points how it will be looking like this, so you will see that there is a overshoot that means the extra value here so this value is more here, the value at this point, at the end point the value is more.



What happens actually? As K goes to infinity outside this are, right side of 0 whatever fluctuations you see they fizzle out, there won't be any fluctuation, the eventually converge to the constant 1/2, and what you end up but this constant, this constant will remain be there, so this fluctuation at the 0 at extreme point is always be there, so that extra error is always observed to be, is shown to be observed, whatever may be the N value you fixed, eventually for every X positive value it converges to 1/2 here, as converges to 0 negative side will, a negative side this goes to -1/2, here also the negative side here it just converges to the value -1/2, but here and eventually here, here there is a overshoot always, here also you will see the overshoot value, there's a overshoot value here also, so that error happens to be always some percentage let us say some 9%, 9% of some percentage, some percentage of error, some percentage of error, some percentage of error around X = 0 always there, so that is called Gibb's phenomena, will try to explain this in the next video. Thank you very much.

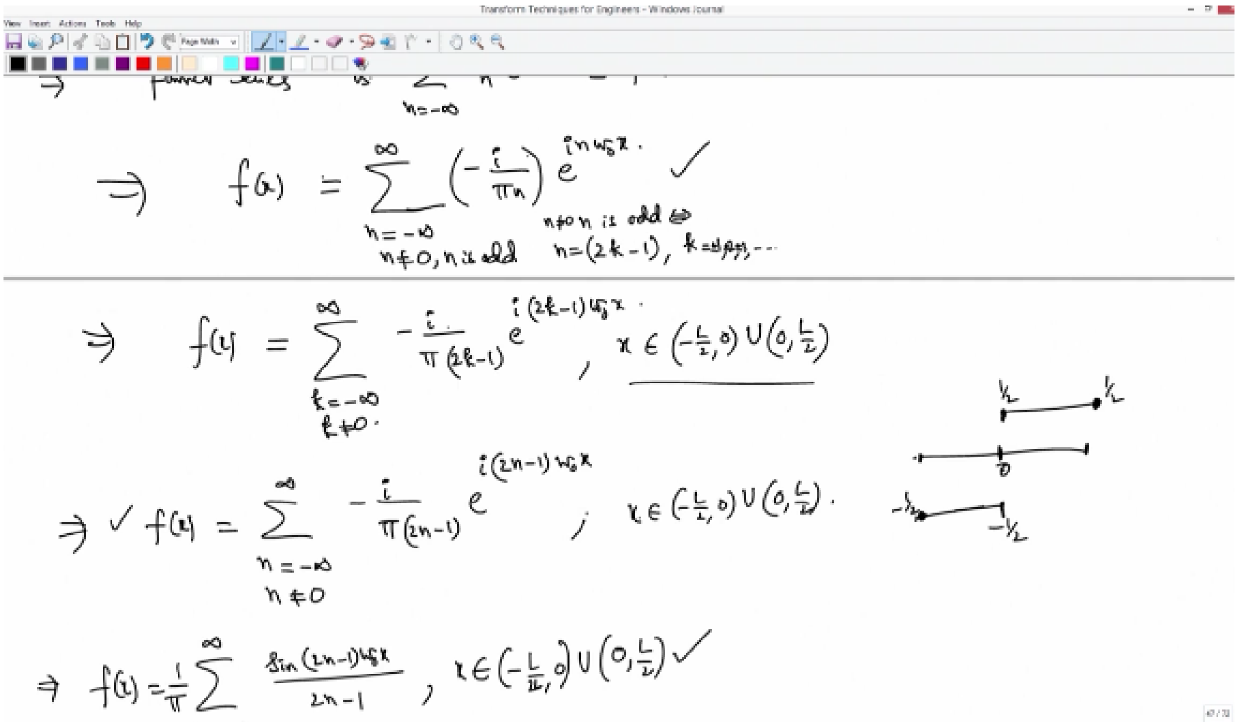
Welcome back we'll try to explain a Gibb's phenomena through an example that I've worked out in the last video, and we start with the example, so that example what we had is F(x) is piecewise continuous function, in fact piecewise smooth function 0 less than X, less than L/2, and -1/2 when you have –L/2 X less than 0, so it looks like you have a positive side and negative side, negative side -1/2, positive side is 1/2, 0 you have a discontinuous point this is from –L/2, this is from L/2, such a function as we have already explained, so normal signal what with elementary functions it's always piecewise a differentiable function, so this is piecewise differentiable function that means it has a derivative which is continuous I mean

$$f(x) = \begin{cases} \frac{1}{2}, & 0 < x < \frac{L}{2} \\ -\frac{1}{2}, & -\frac{L}{2} < x < 0 \end{cases}$$



which is differentiable except at finitely many points, here also discontinuous at finitely many points, it's differentiable so it's derivative 0 is here, 0 here at 0 it is not differentiable, so it is piecewise a differentiable function, even if you do one more derivative F double dash that is also 0, so that is still at 0 it is not defined, if F dash is not defined, F double dash is also not defined so on and so on, so you can easily see that all derivatives F is, F is many times differentiable but it is piecewise a differentiable function, so you can say that F is piecewise differentiable that means F dash exists as a piecewise continuous function, F double dash is also similar thing, F double dash exists except at finitely many points, as those points it has a jump, finite jump okay.

So we have considered the simple example and we constructed the Fourier series, so what we had is this Fourier series that this is what we get the Fourier series, so I'll start, I take, not the final form, and what I get is I go from minus infinity to infinity N is odd, N is odd and this is what you have, and this is CN's, so I'll just try to pick up from what exactly that form, so this is the form we are interested, so this is the form we are interested –I/pi N power IN, so you write



like this –I/pi N, this is my CN and E power IN omega naught X, so this is a Fourier series that is valid between X belongs to –L/ 2 to 0, union 0 to L/2 I have seen this, we will try to manipulate this and eventually we'll show that this phenomena, this phenomena of Gibb's phenomena around the discontinuous point we will just show for this example.

So what we do is I will try to write this from N is from 1 to infinity, and N is odd so what you have is – I/pi times N is odd, N is running from 1 to infinity so you have N is odd, so it starts with 1, 3, 5, and so on, so it should be 2N-1, so when you put N = 1, it is 1, 3, 5 and so on, so this is what you have and here also I 2N-1 W naught X, so I have written N is from 1 to infinity positive side all N odds I considered and this is what is the result, so what is the next part? Other part is N is from rather minus, again one to infinity, and here - I divided by pi times N, N is odd and negative odd, that is negative what, you can consider –(2N-1) when N is from 1 to infinity, and you see that it's going to be -1, -3, -5, and so on, that is exactly what you want, so N is from 1 to infinity, if I replace pi -2N-1, minus minus plus, 2N-1 so these are, this is what happens to when I replace N/-2N-1, so minus minus plus, I divided by pi times this, E power -I 2N-1 W naught X so these are same.

$$f(x) = \sum_{\substack{n=-\infty \\ n=\text{odd}}}^{\infty} \left(-\frac{i}{\pi n}\right) e^{i n \omega_0 x} \, ; \qquad x \in \left(-\frac{L}{2}, 0\right) \cup \left(0, \frac{L}{2}\right).$$

$-(2n-1)$

$-1, -3, -5, \ldots$

$$= \sum_{n=1}^{\infty} -\frac{i}{\pi(2n-1)} e^{i(2n-1)\omega_0 x} + \sum_{n=1}^{\infty} \left(\frac{i}{\pi(2n-1)}\right) e^{-i(2n-1)\omega_0 x}$$

So these are the two sums I can combine it and what you get is N is from 1 to infinity, so you can combine this and you write - I divided by pi times 2N-1, and this sum is 2 times, this is difference, so this is basically because I have taken negative outside, this difference is 2I times sine 2N-1 W naught X, when I add this one and this one together and this is equal to I square minus, so minus minus plus so what you get is N is from 1 to infinity, 2 divided by pi times 2N-1 sine 2N-1 W naught X, okay, so this is exactly your F(x) your function, given function X belongs to –L/2 to 0, union 0 to L/2 this is always true function value.

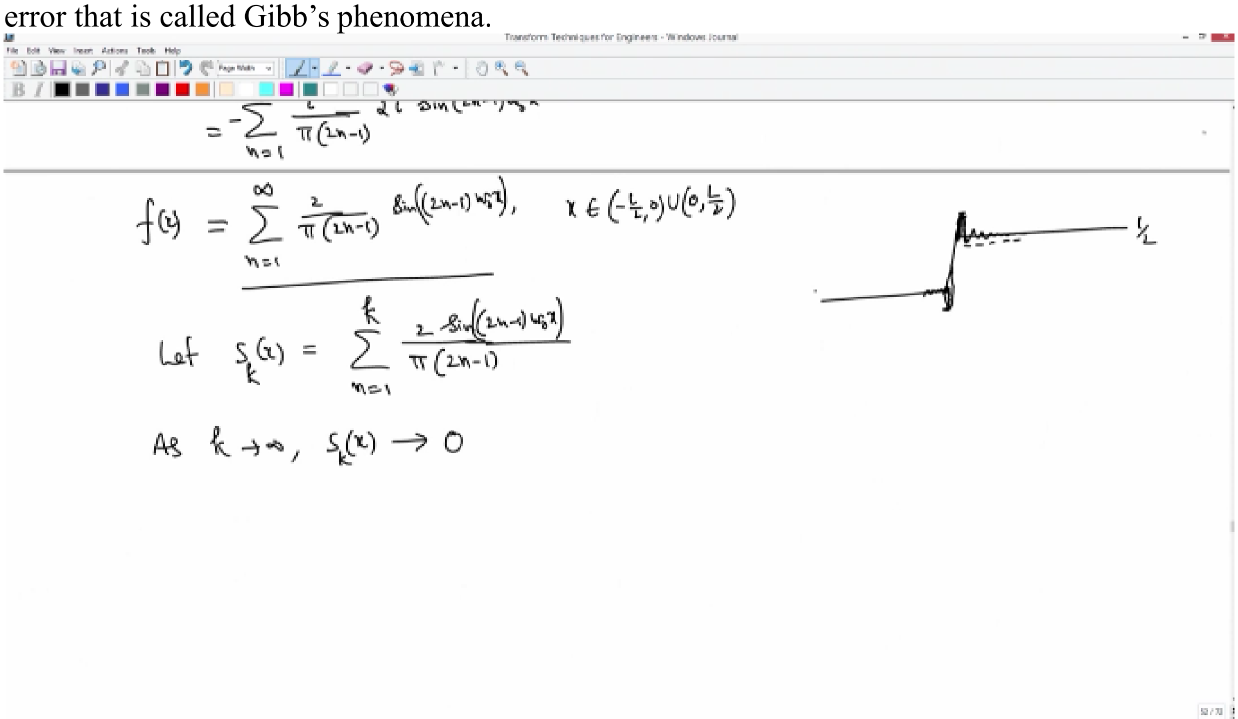
$$= -\sum_{n=1}^{\infty} \frac{i}{\pi(2n-1)} \, 2i \sin(2n-1)\omega_0 x$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{\pi(2n-1)} \sin(2n-1)\omega_0 x, \qquad x \in \left(-\frac{L}{2}, 0\right) \cup \left(0, \frac{L}{2}\right)$$

At discontinuous point we have already seen that if F is a piecewise differentiable function, and we have seen that the Fourier series actually, this Fourier series converges to the average value of the function at a discontinuous point, so what we try to do from here is that we'll just see if you actually see if you consider in this series, if I consider only finitely many terms, okay, so if I consider finitely many terms of this series let SN be, SN(x) be simply let us take SK, let us take K, N is from 1 to K, if I take K terms in this series what you get is 2 divided by pi times 2N-1 sine 2N-1 times W naught X, this is for full sine, sine function of this thing, this is the whole thing, sine of the whole thing.

So if I consider this, if I plot this you will have because these are composition of sine functions you will see that it will have oscillated, if you plot it, it looks like oscillatory function, so if you do so, so when you do this around the 0 the plotting is like this, eventually as K goes to infinity this SK(x) converges to half of the value so this is -1/2 and this is +1/2 by the convergence of the Fourier series this plus this divided by 2 that is what we have, right, so that is 0, so we know that is going to be 0, okay, at 0 okay.

At 0 is going to 0, if I choose what happens and then the Gibb's phenomena what we have is, what happens is that around this point you will have, it will go like this and there is a phenomena like this, there is a, eventually it fizzled, so as K goes to infinity these small oscillations will vanish, but this bigger one whatever may be the small quantity that will remain, similarly here that is what is the Gibb's phenomena, so this exactly, around 0, negative side of 0 and positive side of 0, this peak at which this value so this peak is always will be there, so that is exactly the overshoot value of the function that is what we will see that is the error that is called Gibb's phenomena.

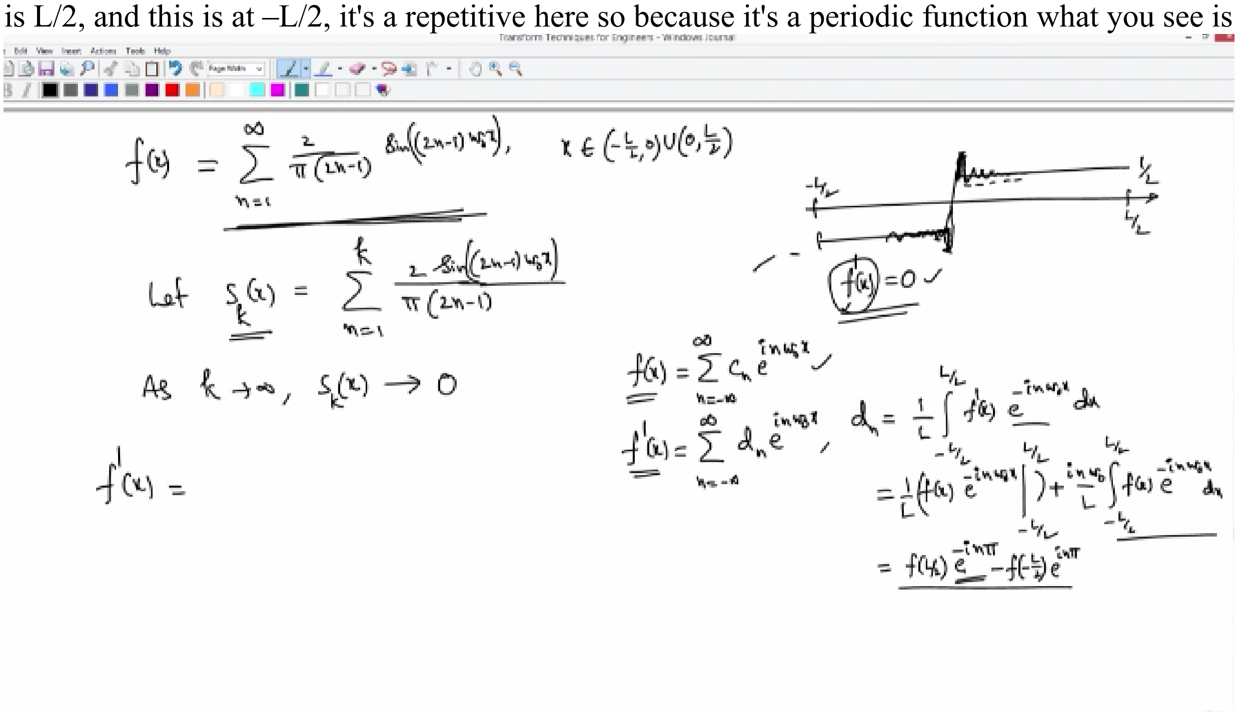


So at what value this peak will happen? So first peak that eventually will be there, okay, as K goes to infinity otherwise there are many peaks this is another peak, this is small peak, these are local maximums, similarly these are local minimums when you have oscillations this is a local minimum and this is local minimum this and so on, so among the first local minimums right side and left side, first local minimum at which you have this peak happens, so we will try to see that, to do that as we know if you want to find the local minimum or local maximum of a function F(x) you need to find the derivative make it equal to 0, so the roots of this, those are roots of this function F dash(x) = 0 will give the critical values, at those values you will have local maximum or local minimas okay, so we will consider exactly what those values around 0, first value will be, will just take, will find out what exactly those, for that we need a function derivative, so SK is this, so we need SK is basically as K goes to infinity this is actually F(x) this is the Fourier series, so for this Fourier series we need derivative.

So we'll just try to find what is F dash(x), because F dash(x) exists, you see that they are given that F(x) is a piecewise differentiable function and it has a Fourier series CN E power IN Omega naught X this is what we have minus infinity infinity, you have a Fourier series because F is piecewise differentiable function, and we have considered an example that it is a piecewise differentiable function, one is 1/2 and -1/2, -1/2 and +1/2 half and because these are elementary function, constant function it is also, it's derivative F dash(x) is also piecewise smooth.

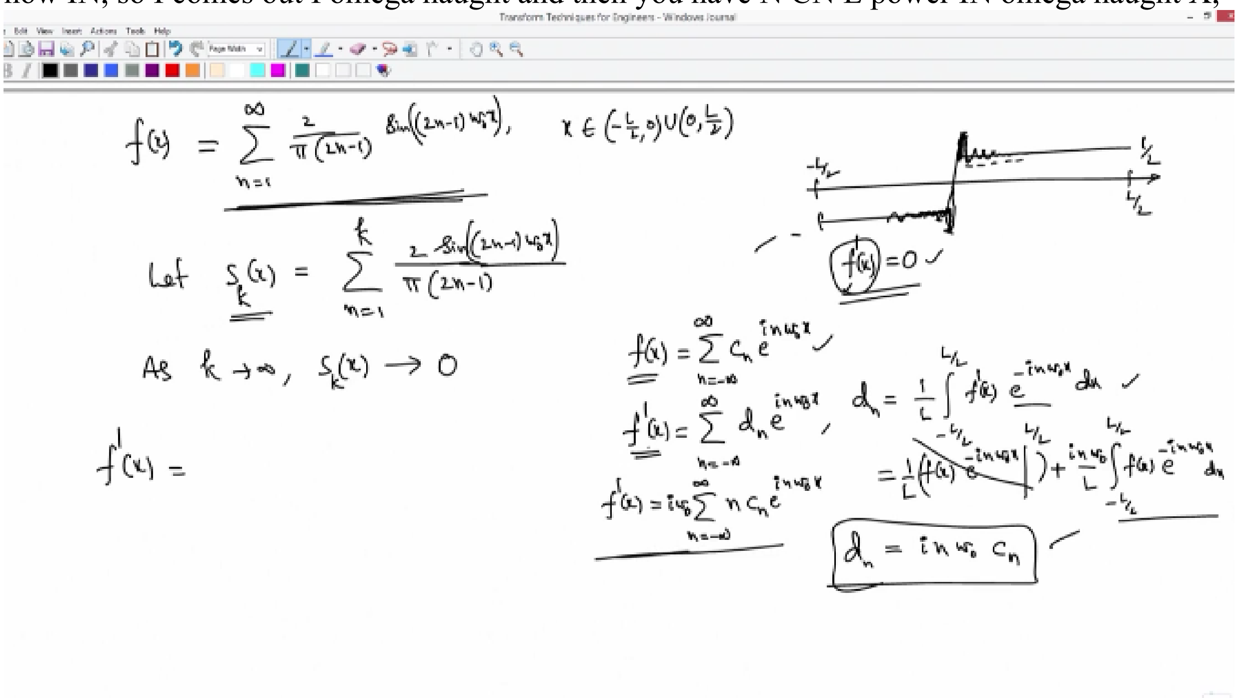
If you consider F double dash that is also piecewise continuous function, so for that reason I will have F dash(x) also will have a Fourier series that let me call it some DN E power IN omega naught X, N is from minus infinity infinity, okay, so what are this DN’s? DN’s are basically Fourier coefficients, that is from 1/L –L/2 to L/2 only for this function, derivative function, that is F dash(x) E power -IN omega naught X DX.

Now if you simply do this, if you do this integration by parts here what you get is F(x) E power -IN omega naught X for this 1/L, substitute the limits and you have a -1/L integral –L/2 to L/2 F(x), now you differentiate this function so you get IN so that comes out -IN comes out that becomes +IN omega naught F(x) E power -IN omega naught X DX, so this is exactly CN, so what happens to this one? This becomes at F(L/2) and F(-L/2) and this becomes, simply if you calculate this contribution will be 0,we'll see what it is F(L/2) then E power –IN, W naught is 2 pi/L into L/2, so L/2 goes, so IN pi, E power IN pi, okay, so you have simply E power IN pi – F(-L/2) E power + IN pi, so this is what you have, okay so at L/2, L/2 and –L/2 there is actually repeating, so this is the value at –L/2, okay, this is –L/2, this is at here, so this is the X axis, this is L/2, and this is at –L/2, it's a repetitive here so because it's a periodic function what you see is

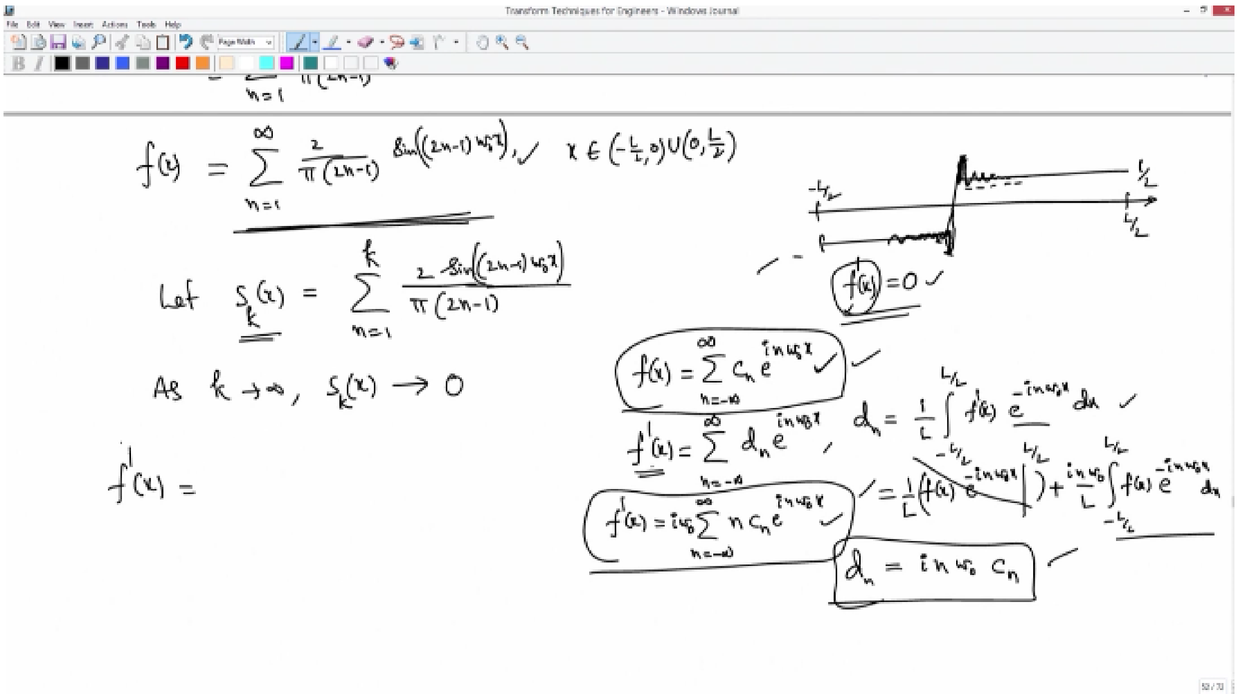


L/2 – L/2 is same as +L/2, this is a periodic function, so whatever repeats here it will go, so that way, and this is simply cos N pi cos N pi so this together so it's 0, so this contribution is 0, so I

will remove now, and then what is the other, what remains is IN omega naught 1/L this integral is nothing but your CN, so DN is nothing but, that is this one.
So you know the difference because F dash is a piecewise smooth function, piecewise differentiable function implies it has a Fourier series like this, if you calculate its Fourier series and that is exactly in terms of CN that is nothing but N is from minus infinity infinity, DN is now IN, so I comes out I omega naught and then you have N CN E power IN omega naught X,

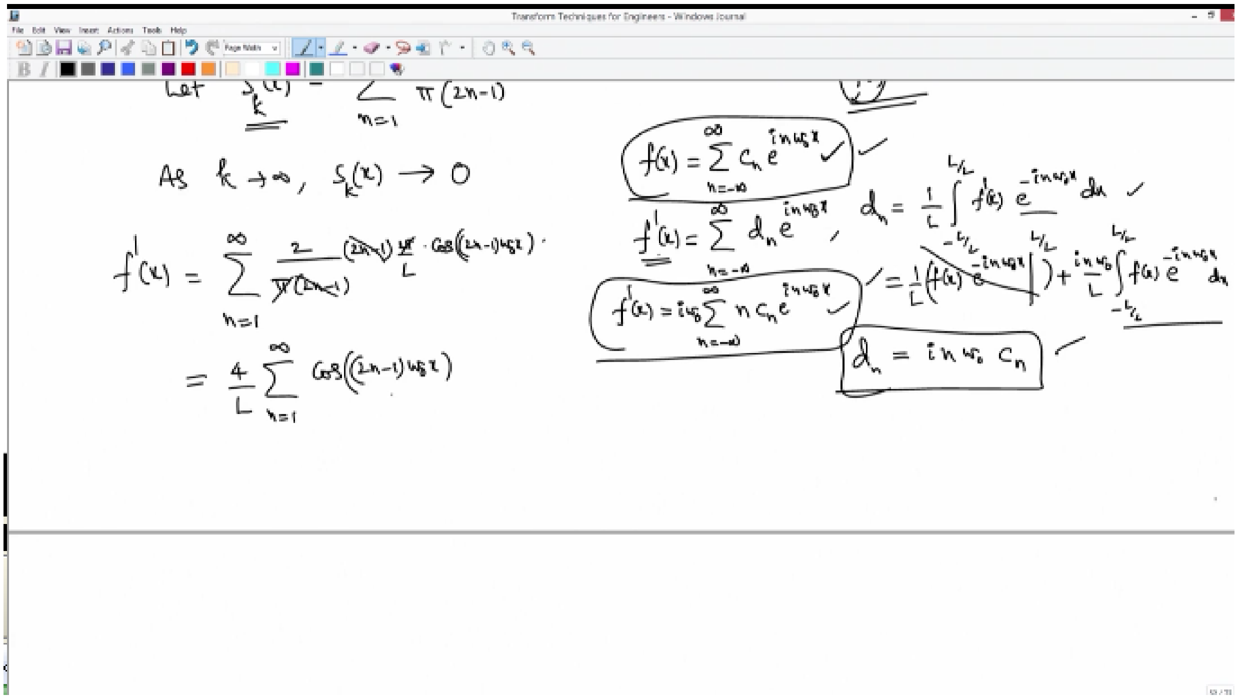


so this is your Fourier series and that converges to F dash(x) wherever F dash is continuous, otherwise it converges to the average value of the F dash at those two values, okay, so this is well-known, this is how we show, so we can actually calculate, so because we have considered such a simple example with elementary functions whatever may be the elementary functions you consider, whatever may be the function F piecewise continuous function involving elementary functions is always piecewise smooth, that means it is piecewise differentiable function and it is also piecewise twice differentiable, piecewise tries differentiable and so on,

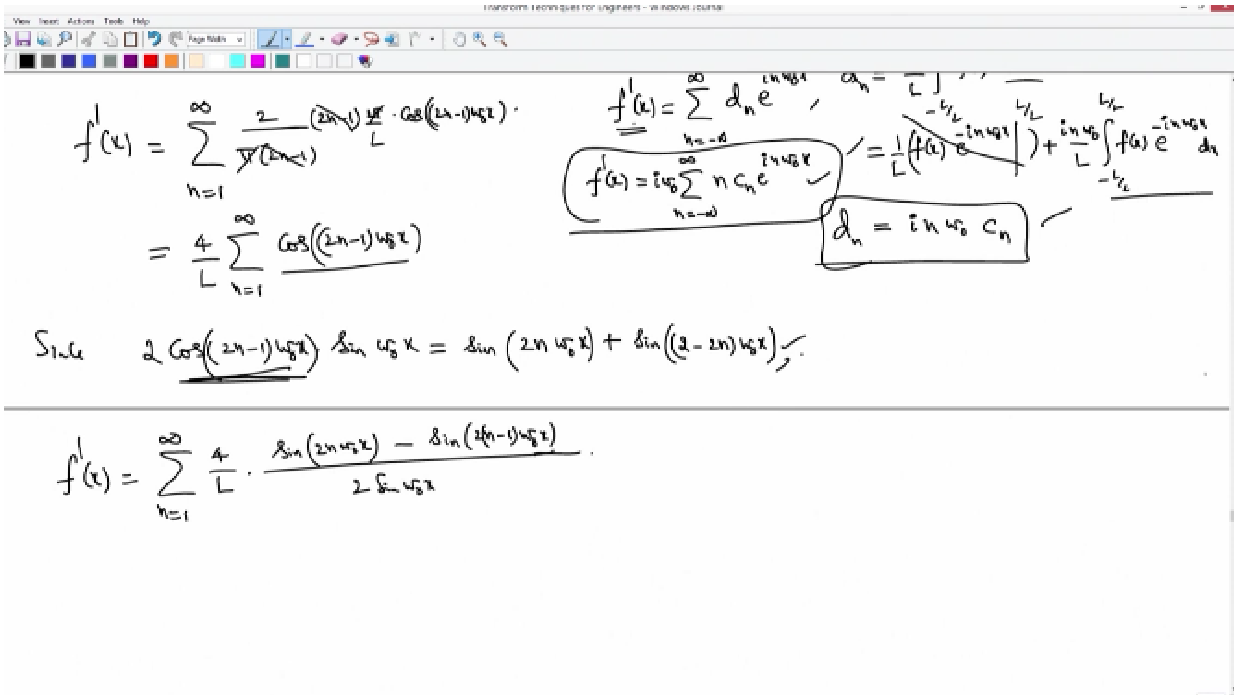


okay, for that reason you can get this Fourier series for the derivative, so this F dash(x) is now, so now your F dash(x) is this given function, and you have already this Fourier series, and then if you actually do this F dash and what you see, you can see that this is exactly, this is actually if you observe this is simply differentiate this term by term, if you differentiate term by term In omega naught, so actually given function F this Fourier series if you differentiate both sides that is exactly what you got, okay, so that way I'll just derive it here, so in that sense this function given function this given function which is at given piecewise function in our example, we can simply differentiate both sides, okay, I consider F dash(x) that is now the Fourier series I can simply a differentiate term by term, so n is from 1 to infinity 2 divided by pi times 2N-1 and this becomes cos, okay.

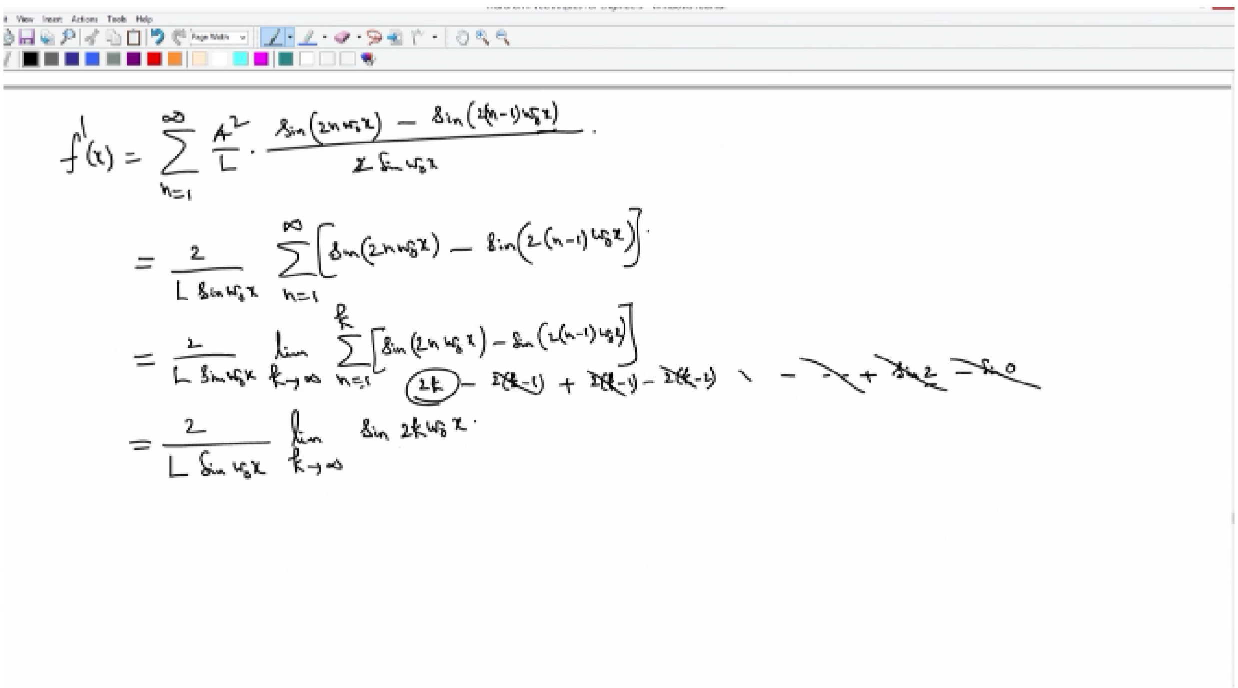
So cos and we have a 2N-1 omega naught into W naught into cos, this is cos 2N-1 W naught X, this is what is the Fourier series now, it's derivative, Fourier series is this, derivative of the function for which the Fourier series has become this, this gets cancelled omega naught is 2 pi/L and you have 4, if pi pi goes 4/L and you have N is from 1 to infinity, simply have cosine, cos 2N-1 W naught(x).



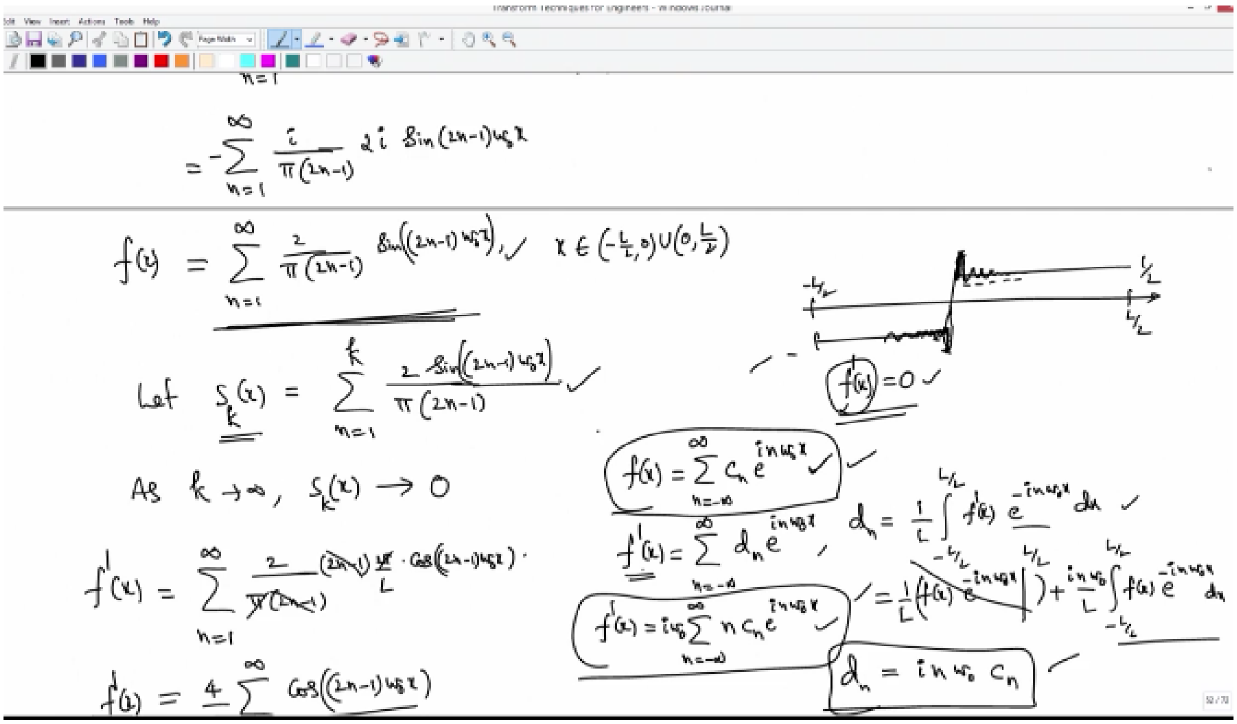
Now you can rewrite this cosine cos, so I use cos 2N-1 W naught X into sine W naught X, two times of this, this is equal to sine sum plus sine difference is 2 sine sum +2, 2 sine A cos B is sine sum, sine A+B + sine A-B, that's what I'm going to use here, so if I use this, if I use this what is the sine sum? Sine sum is sine 2N, if I add this 2N W naught, so 2N W naught X -, because this is starting with cos and that makes sine sum + sine difference, sine sum, sine difference will be -2, so this is sine sum, so let us say this is sine sum, so sine A cos B + cos A sine B, so that right, so that is this 1+, plus or minus when you add this goes so what you have left with is sine A is this, this is your B, okay, so A-B so sine sum is this, sine difference is, A-B is W naught X, so 1- +1 so that is 2 - 2N into W naught X, that is what it is, okay.
So I'll just remove this, so this is what you get. So if I use this, what you end up is F dash(x) is equal to sigma N is from 1 to infinity, 4/L let me write inside you know this cos, I'll write I will use this one here, so this right hand side divided by 2 sine omega naught X, 2 sine omega naught X and this will be sine 2N omega naught X and this I'll write, I’ll take minus, you have a sine 2N-1, okay, 2N-1 W naught X, so this is what it is, so I replaced cos function which is here is by using this identity, okay.



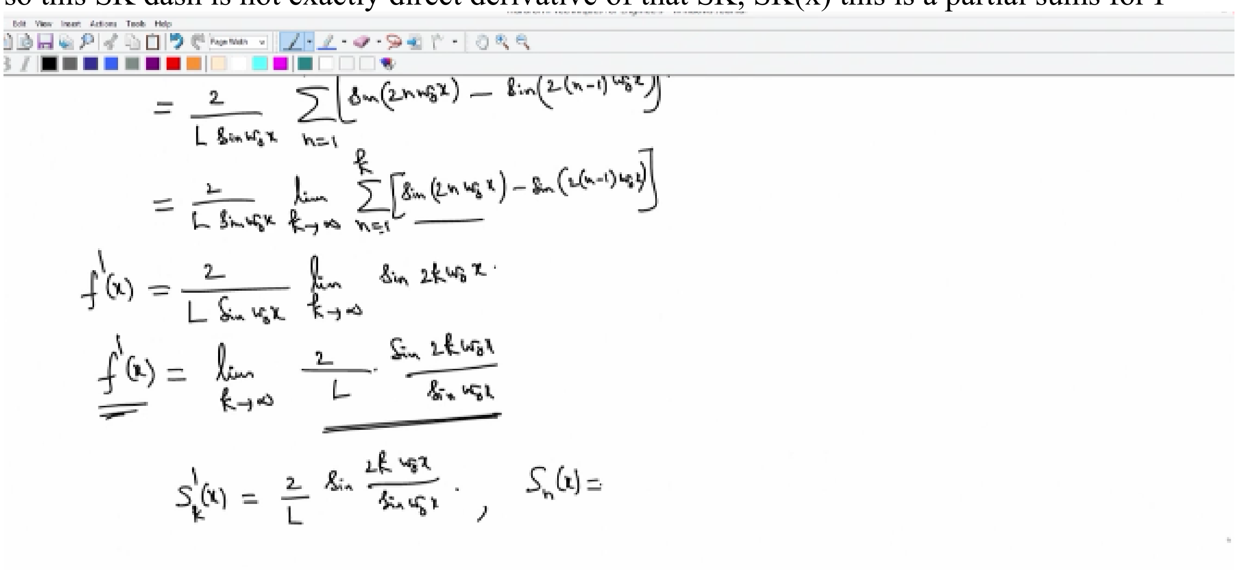
So once you have this, this is equal to I'll just rewrite this as a limit of, some limit which is 2 divided by, so of course you have 2 2 goes, 2 divide by L sine omega naught X, and you have this sum, N is from 1 to infinity, sine 2N omega naught X - sine 2N-1 omega naught X, so what is this one now, you look at this sum you take partial sums here, if you takes partial sums and limit K goes to infinity, this partial sum N is from 1 to K and whatever here, so this is sine 2N omega naught X - sine 2N-1 omega naught X, okay.
So now the, because this is a partial sum if you expand it, starts with 2 omega naught X - sine 0 and then plus, so and then what is remain? So 2N-1, so that is sine N = 2, sine 4 W naught that is going to be sine, this gets cancelled, so eventually what you end up is sine omega naught X limit K goes to infinity, this is simply sine 2N W naught X, so 2K, 2K W naught X, you write 2K, so basically 2K - 2K-1 again you add when you put N= K-1, so you have 2K - 2K-1 - 2K-2, so this gets cancelled like this and end up finally an N = 1 this is simply sine 2 and then - sine 0, 2 into 1, here 2 into 0 is 0, sine 0, that is 0 and this gets anyway cancel, so you end up only 2K



term that is here, okay, so this is how I explain, so you simply expand this finite sum everything gets canceled except the, when N = K the first term this term only contribution will be there.
So this is exactly what you have as of now, so this is my F dash(x) which is, this is simply a limit K goes to infinity 2 divided by L times sine 2K omega naught X divided by sine omega naught X, this is my F dash(x).
Now I consider F dash(x) is a Fourier series, okay, F dash(x) is actually a Fourier series, so this is your F dash(x) your Fourier series, so as partial sums this is a limit of a partial sums is actually your F dash(x), so that is exact, so that means this is your partial sums, so SN dash of, SK dash(x) is nothing but 2/L, sine 2K omega naught X divided by sine omega naught X, okay, what is your SN? SN(x) is a partial sum of your Fourier series, actual Fourier series that



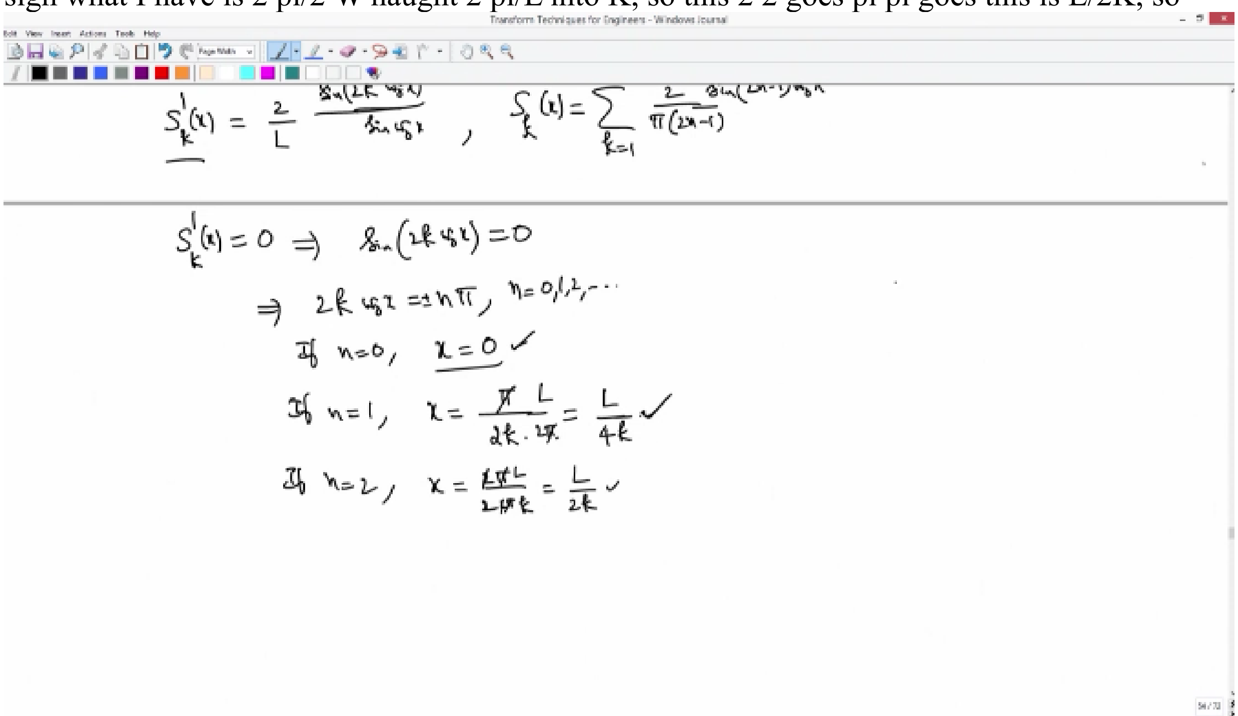
is this one, okay, so this is your partial sum and its derivative is simply what you end up is this, so this SK dash is not exactly direct derivative of that SK, SK(x) this is a partial sums for F



dash(x), okay, so this partial sums you have this one, so if you differentiate I think you will get the same, SK dash(x) is eventually end up as saying you may end up this one only, okay, that will not bother.

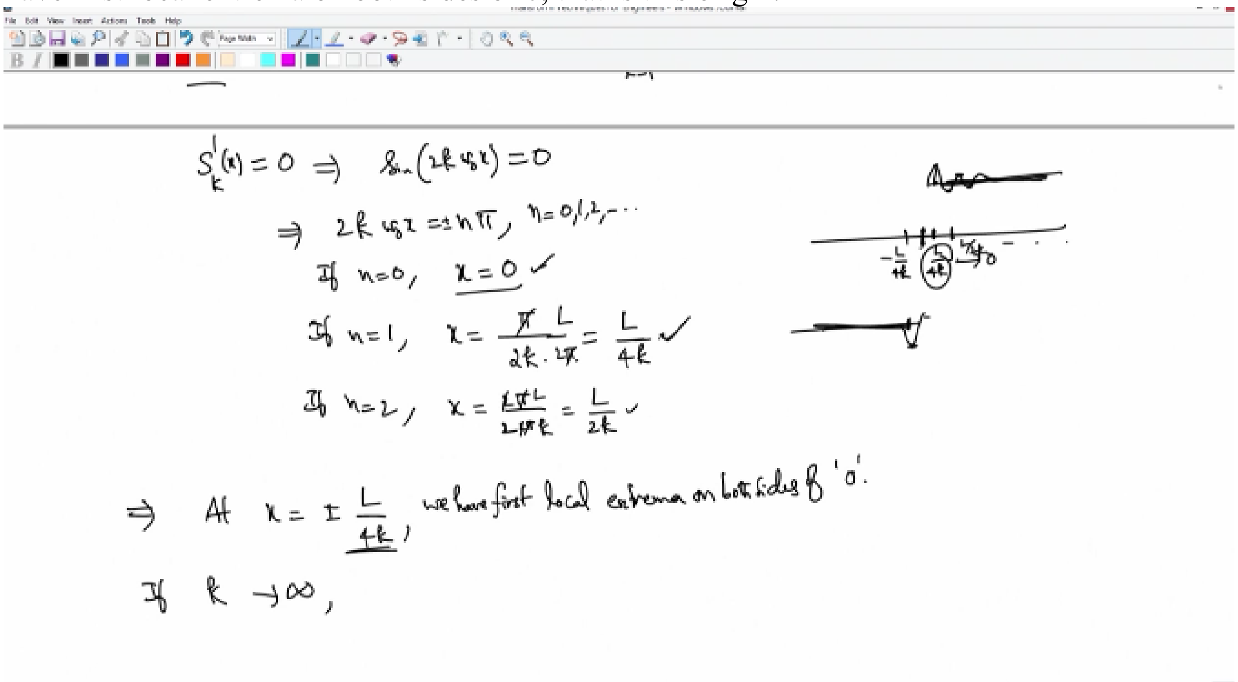
So let me write what is that SK, SK(x) this is from SK(x) let me write, SK(x), K is from 1 to N 2 divided by K times, what is that SNK? 2 divided by PI 2N-1 sine 2N-1 W naught X, that is exactly your SK(x), so this is our derivative, so if I make this SK dash(x) is 0, SK(x) = 0 what you end up is sine, sorry, I think I have written wrongly, so this is sine of this divided by sine of that, so you have sine 2K W naught X divided by, so it has to be 0, if this has to be 0 implies this has to be 0, so this means 2K W naught X is N pi, n is running from 0, 1, 2, 3, any 0, 1, 2, 3, okay. At 0 you have seen if we plot it a finite sum Fourier series, you see that is eventually you have a jump here, so this will go from this to this, and at 0 it crosses like this, if you plot like this so 0 is an inflection point that you can see by just differentiating one more time and see that it is still 0, okay, right, at N = 0, okay, so 0, if I choose N = 0, K = 0, if N = 0, if N is from 1, 2, 3 onwards, if N = 1 what is K? K is pi by, so what are the points you need? 2K, I'm sorry so if N = 0 if I choose then X equal to obviously 0. K is actually running from 1 to infinity, okay, if N = 1 and so on, so if N = 1, X = N pi by, or rather N = 1, so you have pi divided by 2K divided by W naught, W naught is 2 pi/L, so you have pi pi goes you have L/4K, so this is the first value.

And what happens N = 2, if N = 2, X will be equal to, of course this can be even plus or minus right, so when I choose the plus side this is the case, the positive side if I look at it at X = 0, you have critical value, at X = L/4K you have another critical value, when I take N = 2 with positive sign what I have is 2 pi/2 W naught 2 pi/L into K, so this 2 2 goes pi pi goes this is L/2K, so

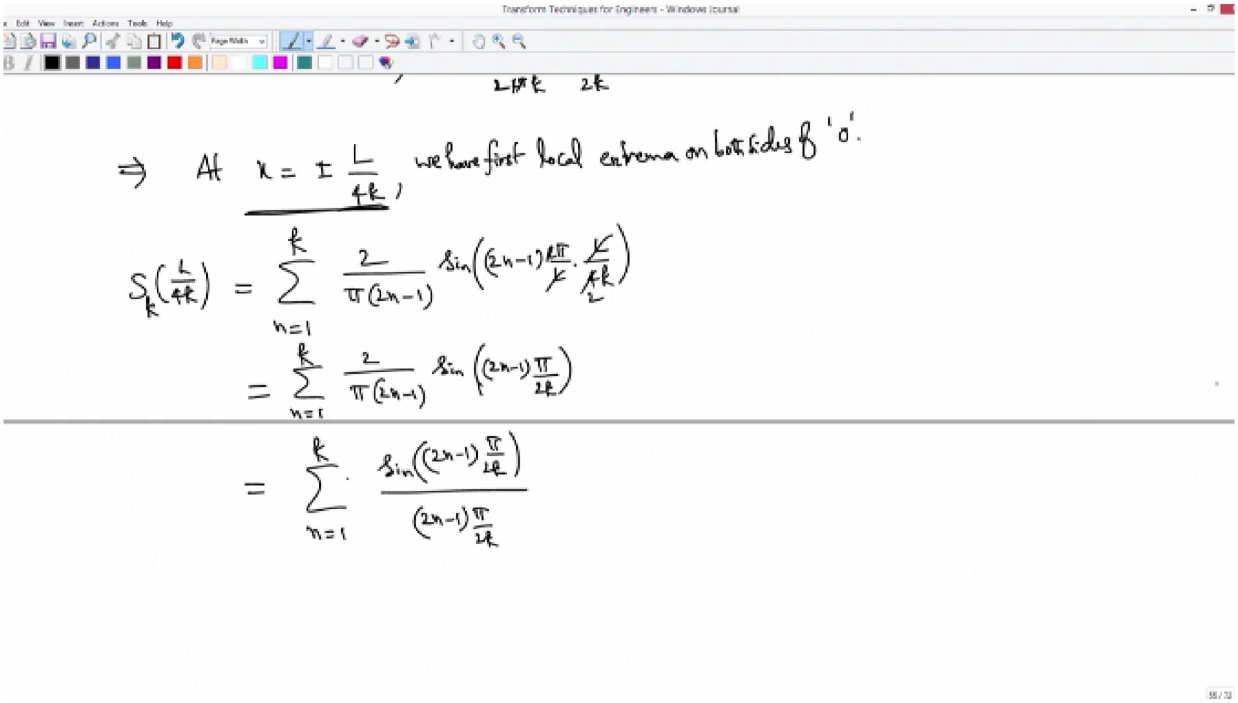


this is bigger than this one, this is bigger than ,this quantity is bigger than L/4K, so the first value on your right hand side when you have this one, so the first peak, oscillatory peak value here will be around L/2K, okay, if you consider K values K is running from, what is SK(x), and K is from 1 to, right, so SK, what is SK? So you have a partial sum, SK is partial sum you have N is running from 1 to K, K terms if you choose, if you choose K terms if K becomes bigger this becomes close to 0, this goes to 0, so it will be closer to 0, otherwise the first peak will be L/2K, next one will be L sorry, L/4K, L/4K is the first one, L/2K is the second one and so on, so the first peak is here.

Similarly the negative side when you choose N = -1, what you have is –L/2K this peak, this lower peak local minima the value of X that is -L by, actually yeah that is negative side, so I have –L/4K, so both plus or minus L/4K that implies at X = + or – L/4K we have local extreme points, extrema, okay local extrema, if I increase, if K goes to infinity, so first local maxima will be always be there okay, as you increase K there are other extremas, this is a local minima, local maxima all those things will be there, they're all vanishing slowly because it has to converge to 1/2, here also it has to converge to -1/2, so but this first peak will always be at this one as your K, you fix your K at these points you have local extrema, first local extrema, we have first local extrema on both sides of 0, that is this origin.



So we will just see at this point that these values, what is the value of the function, okay, so we'll just try to calculate what is your SK(L/4K), we'll just look at one side, other side is same, similar, so this is running from N is from 1 to K, 2 divided by pi times 2N- 1, sine 2N-1 into omega naught X, that is omega naught is 2 pi/L, X is now L/4K, right, so 2 2 goes two times, L L goes, and what you end up is simply N is from 1 to K, 2 divided by pi times 2N-1 sine off, this is the whole of sine, so sine(2N-1) pi/2K, this I'll try to rewrite like this, I'll try to rewrite as a sum, finite sum, N is from 1 to K, sine function of sine 2N-1 pi/2K divided by 2N-1 pi/2K, so

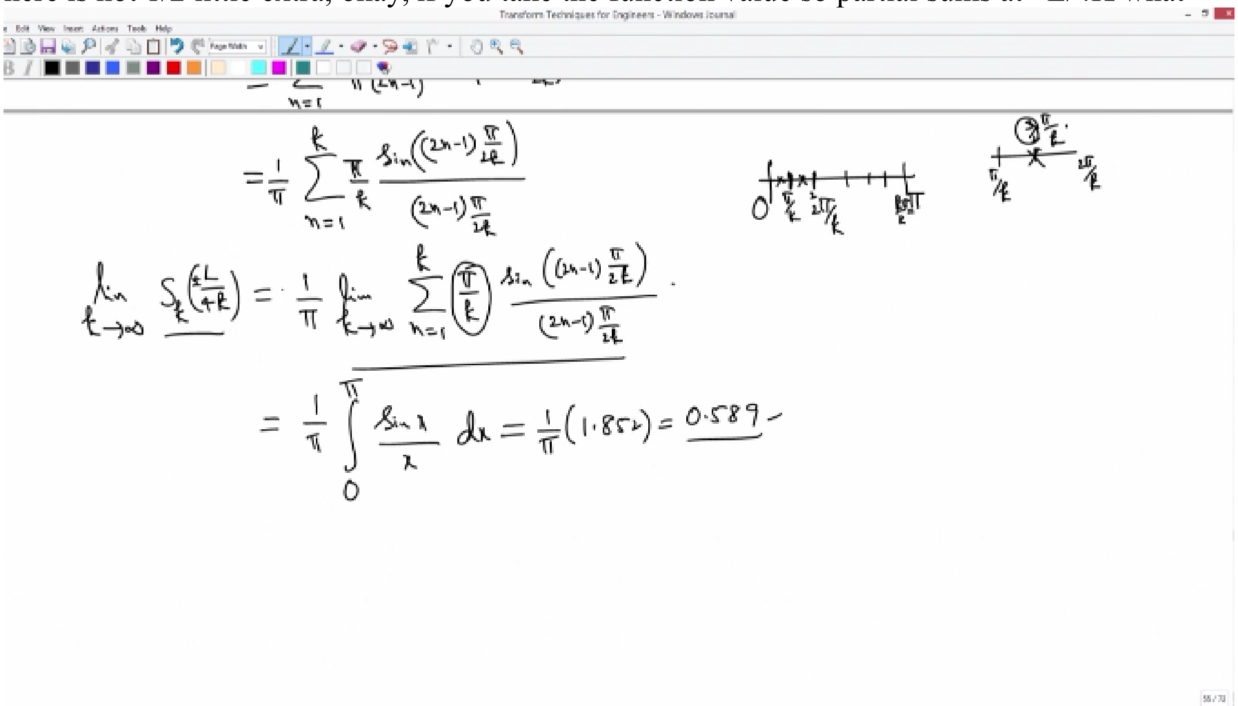


I have added what, so this divided by, this is already there 2K, so 2K is 4K, I have to add 4K or rather pi/2 is already there, K is also there, yeah, this K is extra, and so K is extra, so if K goes up, so if I have to do this I have to simply divide with K, right. So if this K K goes, 2 goes up and pi into this, this 2 divided by pi times 2N-1 is same as this quantity, this is what I exactly rewrote.

So if I multiply 1/pi, I multiply pi here, so right, so what is this one? This quantity is, this is now if I take the limit K goes to infinity SK(L/4K) this value that means the value when you plot, what happens to the value if you take K goes to infinity, full Fourier series as take more and more terms of Fourier series and you sum it up and plot it, what happens is this fizzle, this oscillatory parts here, here and this side will be fizzled out, we will have only this part, this what happens here, so we'll have the Fourier series, the value of the function, we are now trying to calculate value of function here, we will see exactly the value, whether this is going to be ½ or it's going to be something always, will see here what it is, so this is going to be 1/pi this limit K goes to infinity this sum N is from 1 to K, pi/K, okay, times sine 2N-1 pi/2K divided by 2N-1 pi/2K, okay, so this is exactly, this is our integral sum between 0 to pi, and you divide with K equal intervals with pi/N size, pi/K size so that I have pi/K times sine function, so between any two intervals I simply have, this is 1, 2, 3 and so on, right, so this is pi/K, first one is pi/K, the second one is 2 pi /K and so on, finally K pi/K is pi, okay.

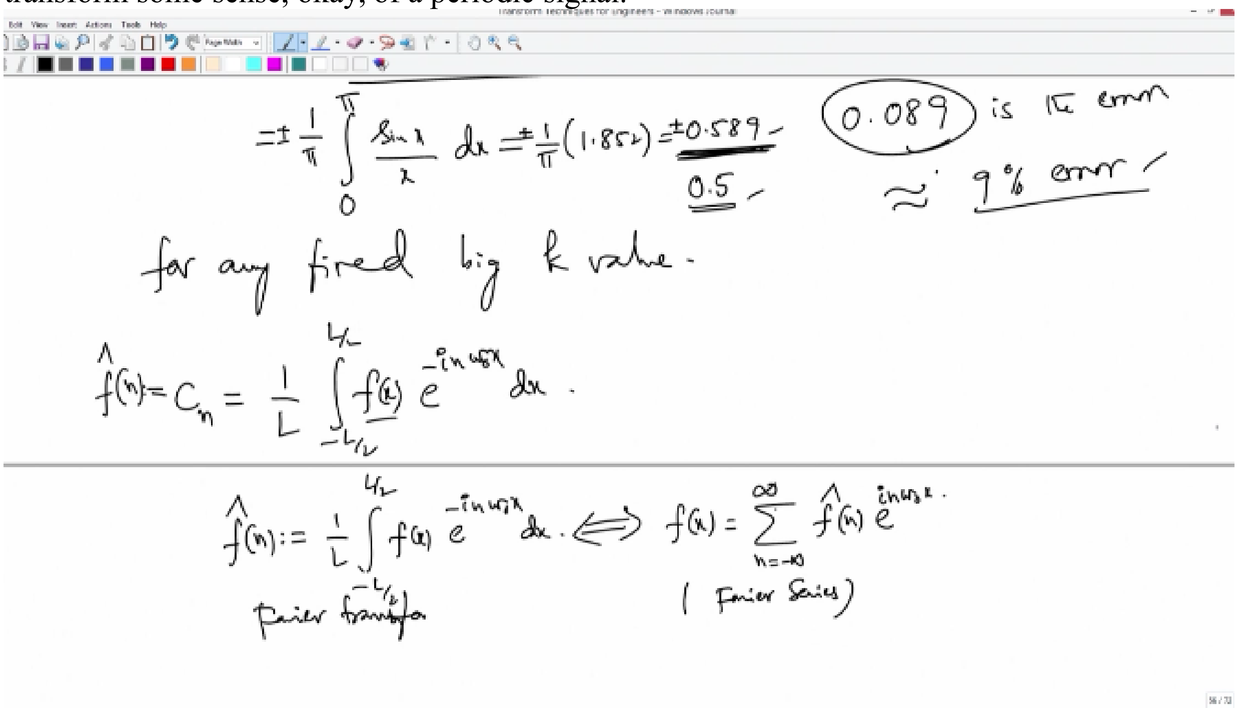
So you have 2N-1/2 will always be in between, okay, when N = 1 this is 1/2, so you're going to be here, if N = 2, 3/2 so it's going to be here, so exactly middle of that, function value middle, in the middle of this interval okay, middle of the sub interval, so this is exactly equal to 1/pi times, this limit, this is the, this pi/K is the interval length and the function value at this middle point, middle point is 2N-1/2 pi, okay, by 2 pi/K, so what exactly the middle point? If this point is pi/K and this point is 2 pi/K, middle point is 3/2 pi/K, okay, 2N-1/2 is 3/2 pi/K, so exactly the middle value, function value, what is a function? Sine X/X, so you have this integral, so this is an integral sum that means it is going to be 1/pi sine X/X DX, this integral sum is that, and if you calculate from the table of integrals this value is actually 1/pi times 1.852 which is, the value is 0.589.

So as limit K goes to infinity, so if it K goes to infinity what happens at this value L/4K? It's eventually, so if you take bigger large values of K this Fourier series the more terms, so partial sums of the Fourier series you take more and more terms you choose, the value of the function here is not 1/2 little extra, okay, if you take the function value so partial sums at –L/4K what



you have is minus of that, that is exactly so you will get + or – 1, so if we choose this one and what you have is, you will have only + or – 1/2, so that will give me + or -, so you will have more at one side, one side is more here 0.589 something, here - 0.589, so what you are supposed to get always as K goes to infinity you are ending of this one, instead of 0.5, at this value L/4K as you increase bigger and bigger values K, SK at those peaks you should get, you should actually, you expect it to converge to 0.5, instead you are actually converging to 0.589 always, whatever may be the fixed K value.

For any fixed big K value you're ending up this result that means 0.089 is the error, okay, so this is approximately 9% error, okay, so this is called the Gibb's phenomena, whatever may be so this is simply technical point of view, when you plot this Fourier series whatever maybe the number of terms you choose in the Fourier series you always, at discontinuous points you always overshoot when you plot it, okay, and when you try to compute you always overshoot the value of, actual value of the function, so this is called Gibb's phenomena, okay, so this is the Gibb's phenomena, so normally so in the next video we'll try to give, the next video we will try to give you a properties of the Fourier transforms, so basically what we have is you have a Fourier coefficients N CN's are 1/L – L/2 to L/2 F(x) as a given signal E power –IN omega naught X DX, so these are your Fourier coefficients if you call them, so it depends on N, so if you call this it depends on F and N, so you try to call this as F cap(N) this is now function of N, so if you call this, so you can say that this is the definition of your Fourier transform, Fourier transform of a periodic signal is defined as 1/L –L/2 to L/2 F(x) E power – IN omega naught X DX, then the inverse transform is FX which you have is, what you have is F cap(N) that is your CN's, N is from minus infinity to infinity E power IN omega naught X, this is your Fourier series, this is the inverse transform which is the Fourier series, okay, and this is your Fourier transform some sense, okay, of a periodic signal.



So we'll see what are some properties of, will see some properties of this Fourier transform, so we'll look into this Fourier transform properties and some of, when you say that this Fourier series converges uniformly all these small, small results will just prove in the next video. And these results will be useful when you do eventually Fourier transform on the full real line, okay. So continuous Fourier transform that is what we will see eventually these properties, and some of the results on delta function that integral, integral 0 to infinity, delta function delta X DX, what is that value? That value is actually half of that, we will prove this small result so these remaining, these small, small results properties and some of the results that will be useful later on delta function, we will see them all of them in the next video. Thank you very much.
[Music]

Online Editing and Post Production

Karthik

Ravichandran

Mohanarangan

Sribalaji

Komathi

Vignesh

Mahesh Kumar

Web Studio Team

Anitha

Bharathi

Catherine

Clifford

Deepthi

Dhivya

Divya

Gayathri

Gokulsekar

Halid

Hemavathy

Jagadeeshwaran

Jayanthi

Kamala

Lakshmipriya

Libin

Madhu

Maria Neeta

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Muralikrishnan

Nivetha

Parkavi

Poonkuzhale

Poornika

Premkumar

Ragavi

Raja

Renuka

Saravanan

Sathya

Shirley

Sorna

Subash

Suriyaprakash

Vinothini

Executive Producer

Kannan Krishnamurthy

NPTEL CO-ordinators

Prof. Andrew Thangaraj

Prof. Prathap Haridoss

IIT Madras Production

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