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Transform Techniques for Engineers
Use of Delta Function in the Fourier
Series convergence
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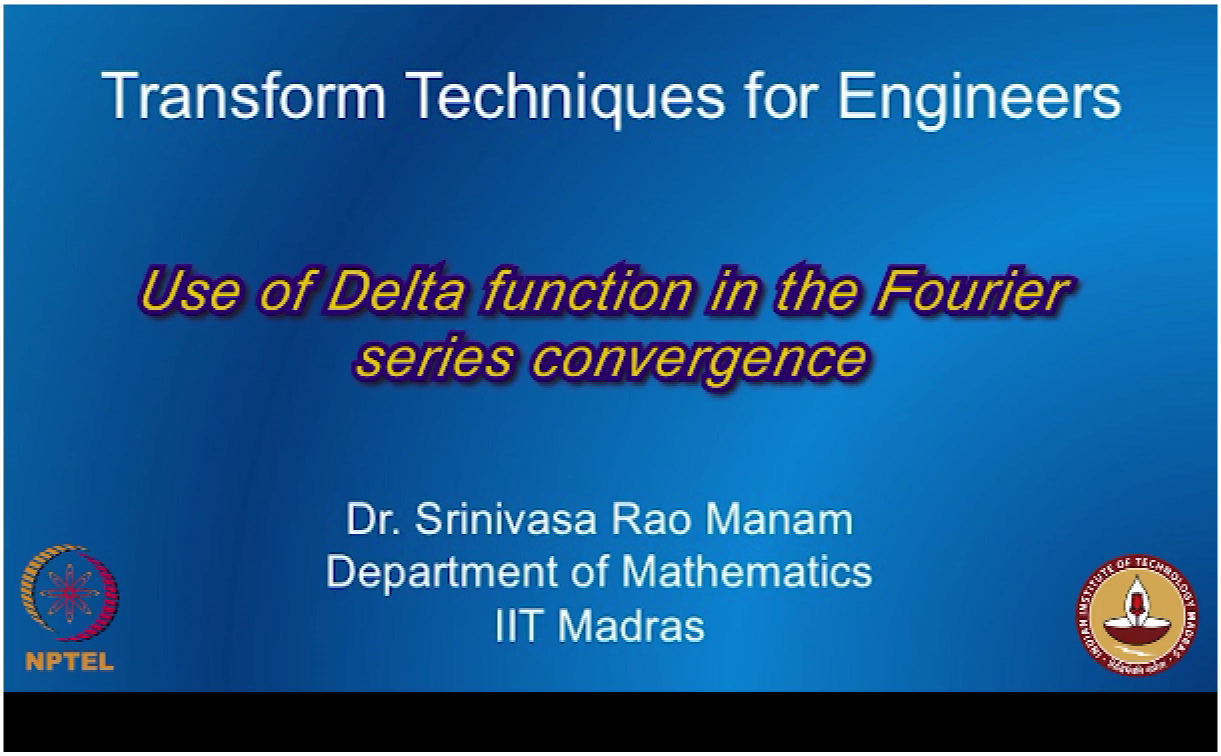
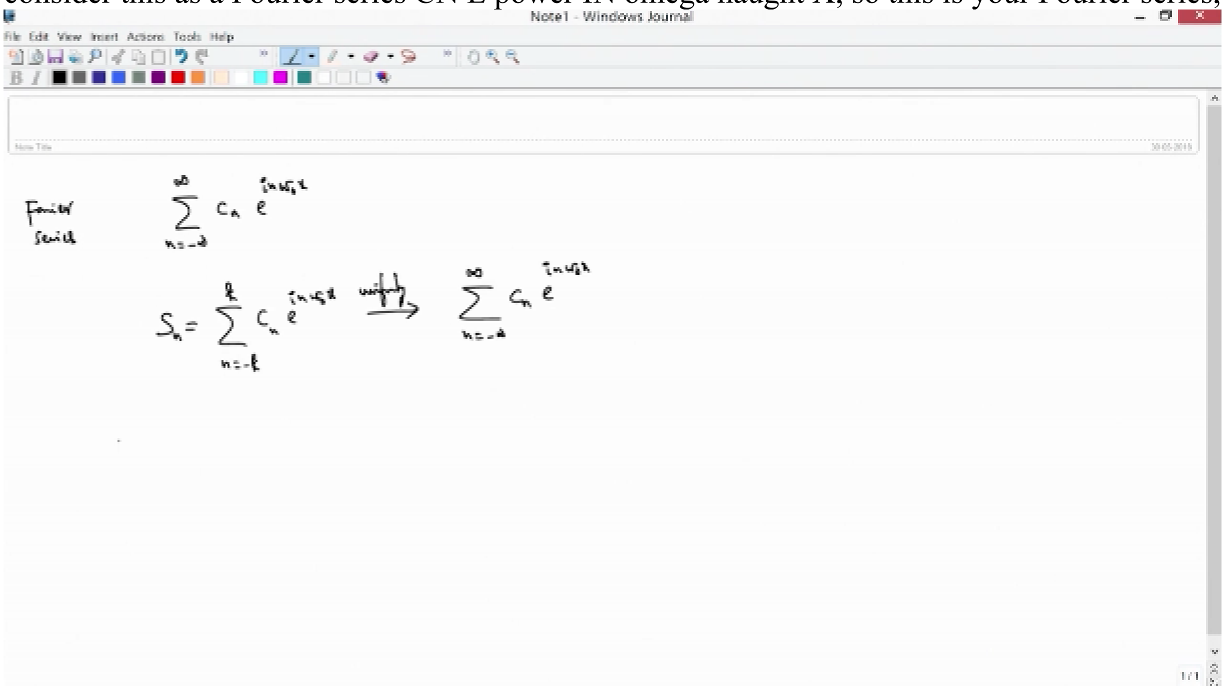


Hello, welcome back, the last video we have seen how to see the convergence of Fourier series, when the signal is piecewise smooth function that means if the function is at finitely many points is discontinuous but solving of jumped discontinuity and other places its continuous function, it's actually differentiable function, so in that case you can have the convergence of the Fourier series to the function at the continuous points, if it is discontinuous points you take the average value of jump, that is half of the jump, average of the value of the jump of the function, that means function at both the ends you consider and take half of it, you sum of it and take the half of the sum, so that is what you have seen in the sufficient conditions for the convergence of the Fourier series.

In this video what we will have is, when the signal is piecewise continuous function and up here you know that if the Fourier series is converging uniformly, okay, we're asking more than piecewise, more than convergence, point wise convergence that means if you assume that it's a uniform convergence of the series, the Fourier series that we have is uniformly converging to a function, two certain function then that function must be, the function F at continuous points, at other points we are at discontinuous, you have half of the jump, okay, that means average value of the jump that's what we will see here, so it's not, result is that not that significant because we are not looking at the convergence of the Fourier series rather if it converges, it has to converges to the function value or the average value of the jump, both sides of the function at the jump, okay.

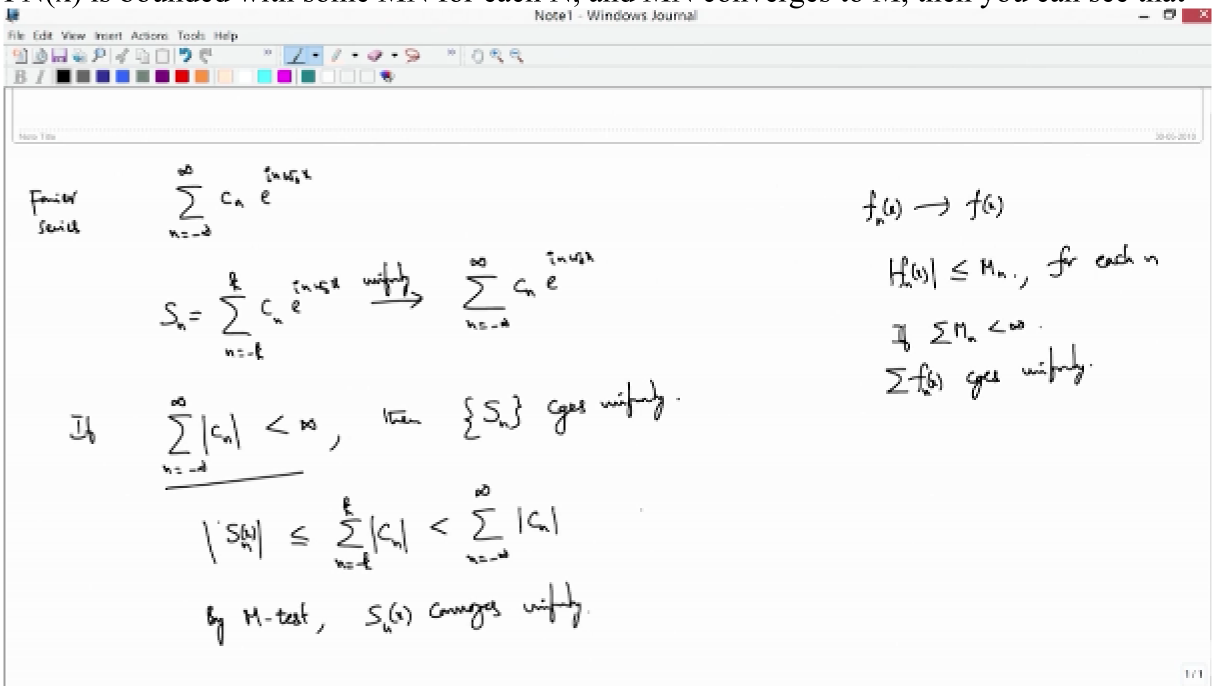
So we are actually assuming that it's a Fourier series itself is converging so why we're still doing is in the process we can make use of a delta function that is useful when you are doing the Fourier transforms and Laplace transform later, so you will understand what is a delta function and it's use in the convergence of this series, Fourier series when apriori you know that it's actually converging uniformly, okay, so we will write it as sufficient conditions before I do this what is the convergence of uniform convergence, on sufficient condition is so if you consider this as a Fourier series CN E power IN omega naught X, so this is your Fourier series,



okay, N is from minus infinity infinity, so this is your Fourier series, so if this converges uniformly means this sequence of partial sums, that is N is from -K to K CN E power IN omega naught X this has your SN, this sequence converges to the function which is finite, so it's E power IN omega naught X so uniformly.

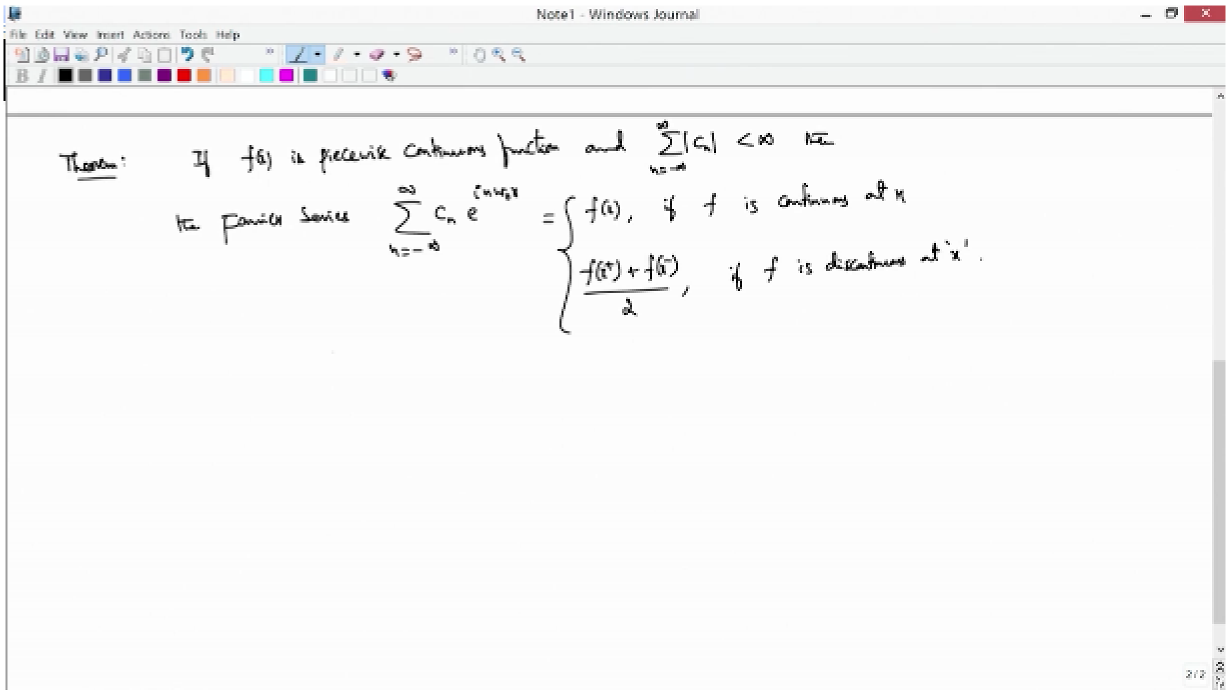
Uniformly means irrespective of the value of X so you have the convergence is uniform, so a pace of convergence is you have a minimum pace of convergence at all values that converges, so you have a minimum pace of convergence that works for all values of X, that's crudely saying uniform convergence, point-wise convergence means for different X values you have a different pace, and so as when you consider some X values, so if you consider the limit of these X values it may go to the minimum pace may not be positive, so that may go to 0, okay, so that means it may not be convergence going with the same pace, okay, there's no uniform pace at which this converges for all values of X, so what are the sufficient conditions for this uniform convergence is if I assume that these coefficients, Fourier coefficients, N is from minus infinity infinity, if this is finite, if this is finite I can say that this is uniformly convergent, then this SN converges uniformly.

So this is easy to see because if you look at the SN, modulus of SN this is less than or equal to sigma CN, N is from -K to K, okay, mode CN so you can have this mode of CN because of this so SN is a function of X, so I can write it as a function of X, this is significant to this, so this converges by M test, so each of this is bounded with this, okay, so and this is bounded with this because this series is a finite as a number series and this is from minus infinity infinity, so this is finite, so once you have this so you can easily see that this is by M-test of convergence SN(x) converges uniformly, irrespective of X value this is still less, always converging, so that's what you can easily see, okay. So FN(x) converges to F(x) uniformly, so this M-test is, if you have FN(x) is bounded with some MN for each N, and MN converges to M, then you can see that



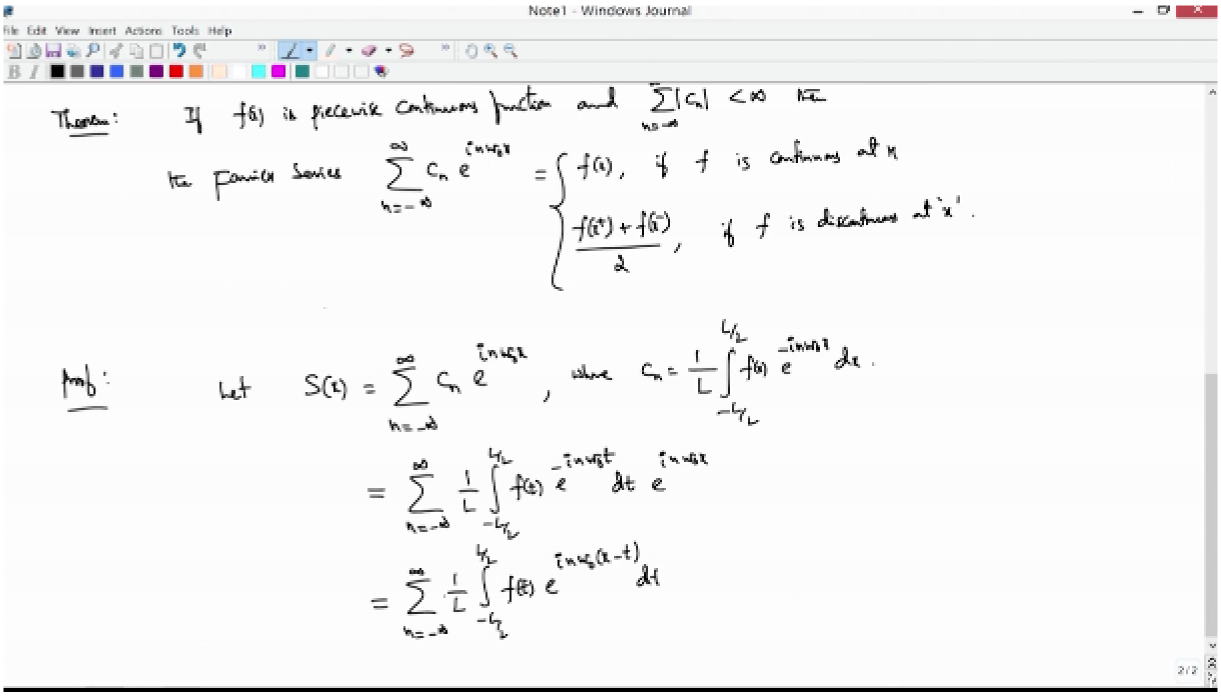
FN(x) converges to or rather sigma, sigma MN is finite, if sigma MN is finite then FN(x) sigma FN converges uniformly, so that is M-test, so we using from calculus, so if you use this so you can see that if this is the sufficient conditions to ensure that this Fourier series is converging uniformly, so I make use of this and to state this result, so let me write as small theorem, so if the signal F(x) is piecewise smooth, not smooth, so piecewise only continuous function and sigma CN, N is from minus infinity infinity, if this is finite then the series the Fourier series converges, N is from minus infinity to infinity CN E power IN omega naught X = F(x), if F is continuous at X, otherwise you will have to take the jump value, okay, those average of the jump, so that is sum of the limiting values, take the half of it, if F is having jumped this discontinuous, if at all is discontinuous it should be jump because it's a piecewise continuous function, okay.

So the if at all this, if you assume that this is true then you have this Fourier series converging uniformly, so if it is a piecewise continuous function or rather just continuous function you cannot say that Fourier series need not converge, okay, you cannot say that Fourier is converge, so the Fourier series need not converge for a continuous function that is in the literature, we



will not look into those details, but why when you choose certain piecewise continuous functions which we normally assume, if you just write it as elementary functions and certain piece how you represent as an elementary function, another certain piece, another certain piece you have a elementary function that is represented as a function, then these are, because these are elementary functions they are actually not just continuous, they are also differentiable, that is why it is actually piecewise a differentiable function, or piecewise smooth function that is why the Fourier series converging, that is what you are seeing all these examples so far, okay, and you will also see many examples because by considering, when you consider function with elementary functions it's actually piecewise a smooth function so that your Fourier series converging actually to the function F or its average value of the jump.

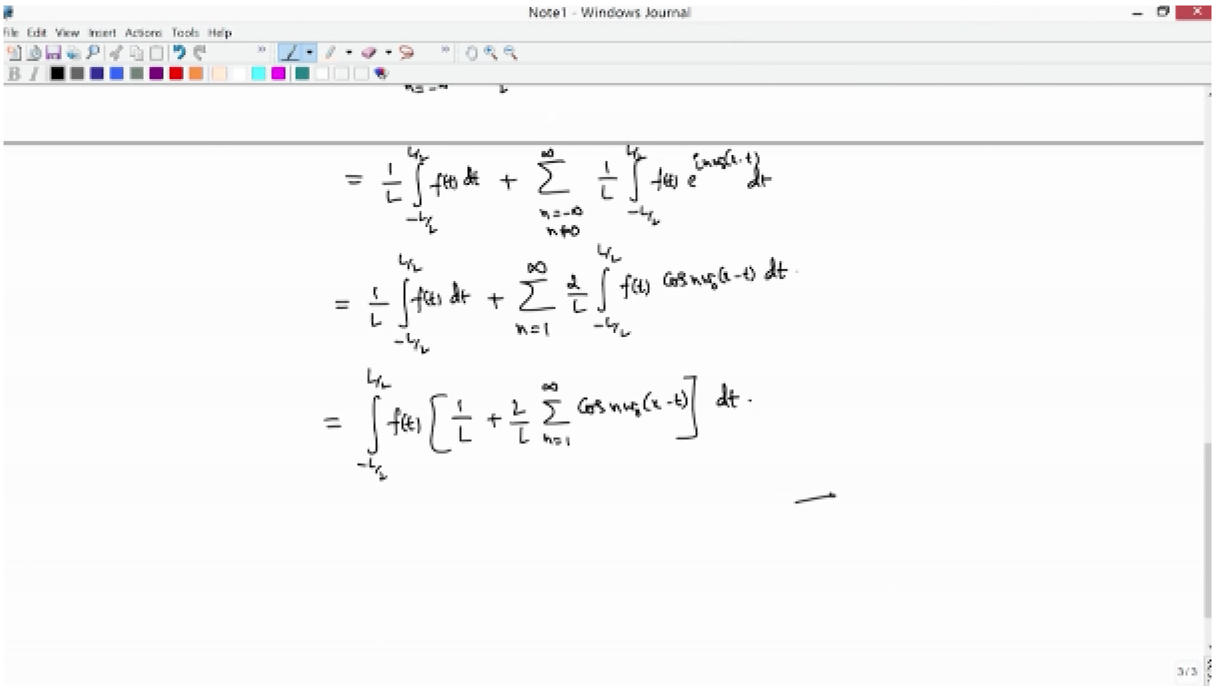
So if it is a piecewise continuous function or a continuous function, the Fourier series need not converge, okay, we have examples in the literature but here if the Fourier series converging and it has to converge to the value, it cannot converge to some other thing except but it has to converge to the function F(x), if F is continuous at discontinuous point that's the jump value, average jump value of the function at the discontinuous point, okay, so to show this result is, as I said earlier result is not that significant, the proof we make use of delta function so that will be useful later, okay, so how do I see this, this is actually this series is converging to F(x) so let S(x), let me call this full series as S(x), N is from minus infinity to infinity CN E power IN omega naught X, okay, so this you can represent as, now we know because it's a Fourier series and we know what is the CN, CN's are where CN's are 1/L –L/2 to L/2 F(x), these are the Fourier coefficients - IN omega naught X DX, so if you substitute here and you see that is going to be minus infinity to infinity, CN I substitute here these Fourier coefficients F(x) E power –IN omega naught X DX times, so let me put it as, because I already have X let me use this dummy variable T, so this is T, E power IN omega naught X, so if you clubbed together so



what you get is N is from minus infinity to infinity, 1/L – L/2 to L/2 F(t) E power -N omega naught or + omega naught X-T DT, so here I split this I take N = 0, I write separately, N = 0 if you take this is 1, so you have 1/L – L/2 to L/2, average value of F(t) plus, now if you write this as N is from minus infinity to infinity, but N is not equal to 0 what is left here, so that is – L/2 to L/2 F(t) E power IN Omega naught X-T DT, so this is equal to, if you actually combine negative and positive, okay, so negative part if so this is where I'm using the uniform convergence, okay.

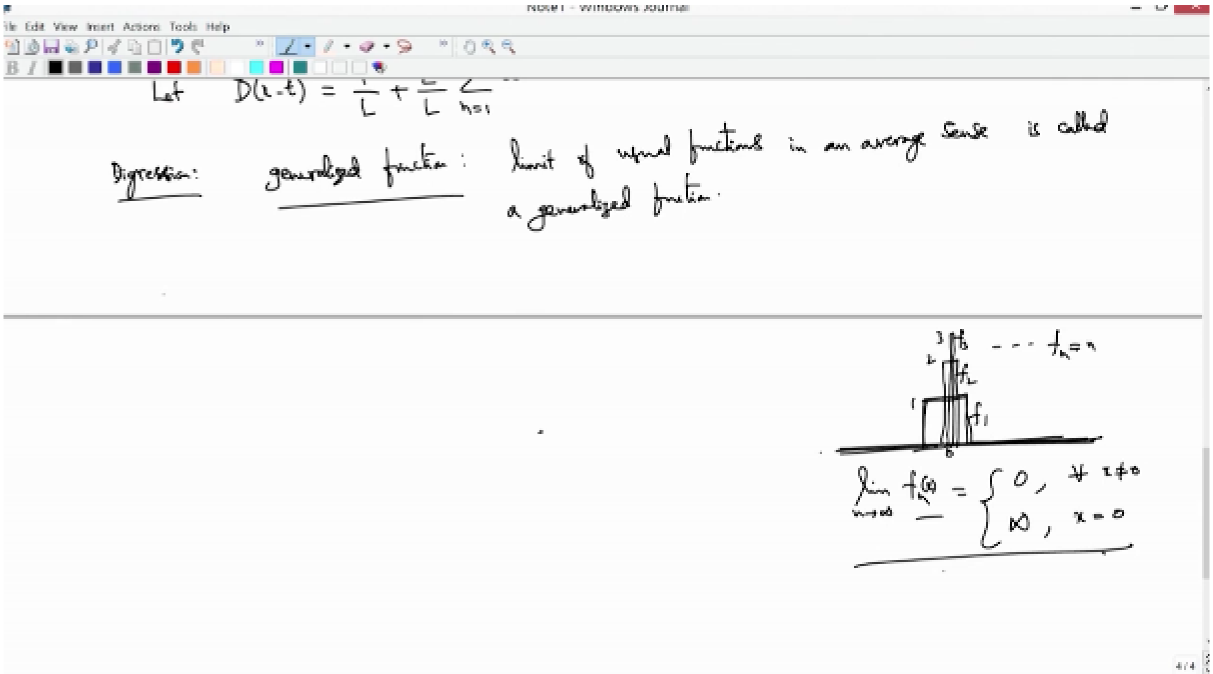
So uniform convergence so this is what I'm using, so if this is true this is a kind of series is uniform convergent, if it is uniform convergent it's actually uniformly and absolutely convergent, so that means I can do the, I can shuffle all the terms and I can add N term to, for example here minus infinity term, plus infinity term, - K term, + K term and so are all kinds of sum, you reshuffle it, rearrange the terms and we can add it you still converge to the same value, so if I do that I have 1/L – L/2 to L/2 F(t) DT + here if - infinity to + infinity if we add, so it's going to be from 1 and -1 if I add, so what I have is –L/2 to L/2, so you have F(t) E power, N is from, N is 1 and N is -1, so we have a 2 times, you have a cos instead of this thing you have cos N omega naught X-T DT, so this is what you have, because of rearrangement and then addition of those 2 each, addition of -K and +K terms if you do, this is what you get, okay, so that we have this nice form, this is possible first of all this step is possible, because of uniform convergence.

We're not proving the convergence of the Fourier series here, if it converges it has to go to only a function not anything else, so if I pull this together, so again I am trying to pull this integral out of this series, that is also possible because of uniform convergence, if I do this you see there it's going to be –L/2 to L/2 F(t), if I write it 1/L +2/L sigma, N is from 1 to infinity cos N omega naught X-T DT, so this is the series to DT, again this step is also because of uniform convergence you have this.

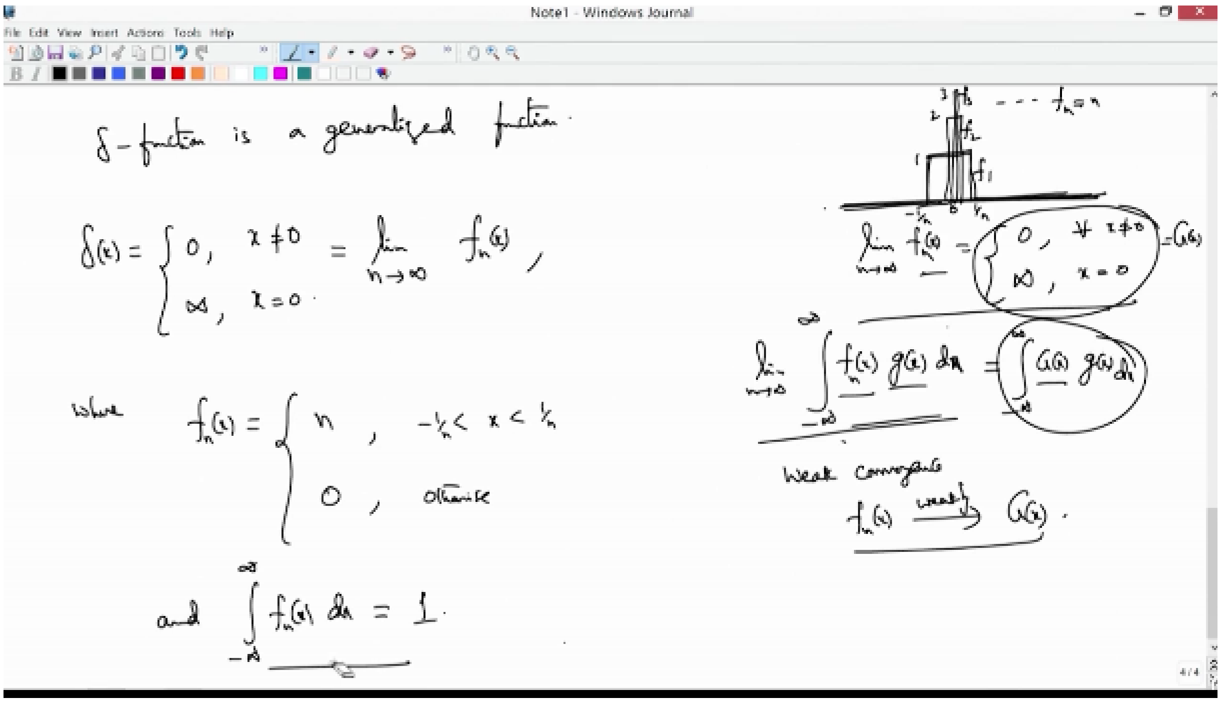


So let me call this as some new function so let D capital D(x-t) as 1/L + 2/L sigma, N is from 1 to infinity cos N omega naught X-T, let me represent like this, now in this here I'd take the digression, digress little bit to define what is a delta function here, a delta function is not a function, so delta function so first of all unusual functions which you have here continuous functions, differentiable function which you, all these usual functions which you see there are functions that are defined, it doesn't take any infinite value, okay, so any the generalized functions are a limit of usual functions, okay.

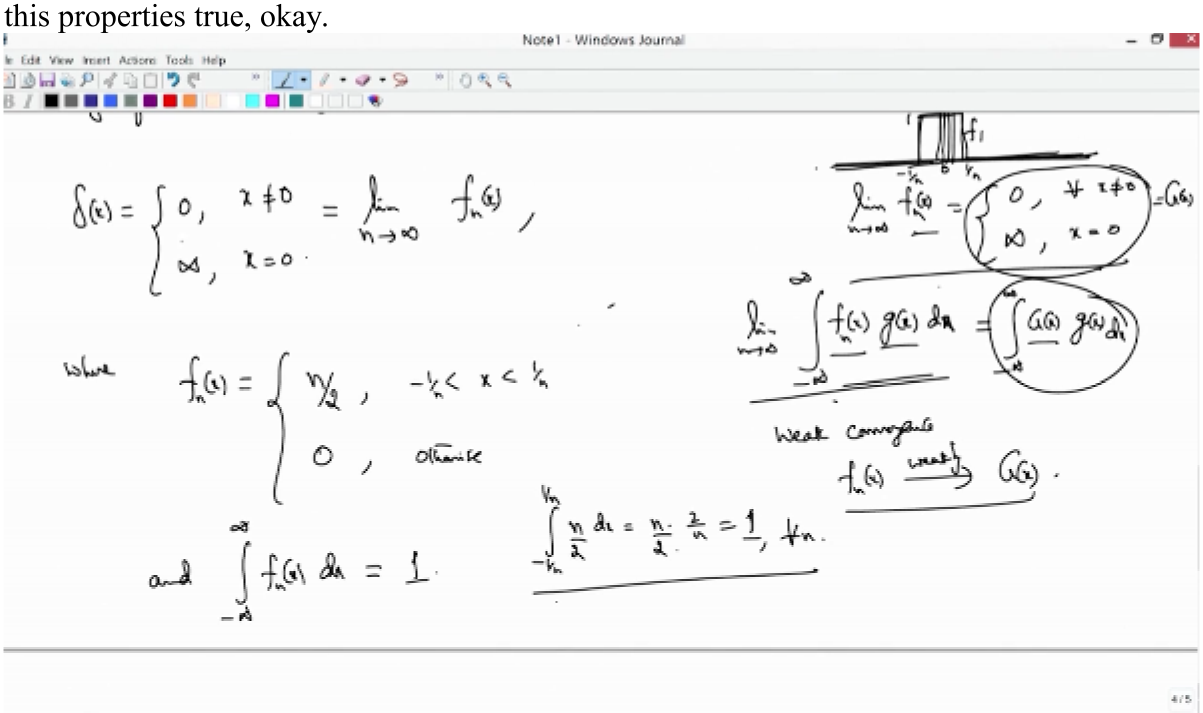
So first of all if I define what is generalized function? Generalized function if I define as a limit of usual functions, limit of usual functions in an average sense is called a generalized function, limit of usual functions in an average sense is called a generalized function. For example I'll just give you, so I choose a delta function itself, let me choose this as my first function 0 here, suddenly there is a jump, there's a 0 again here from here to here, okay, so this is 1, again next this is your F1, F2 is 0 up to here, go up and you go up and again 0 and this is your F2, like this you go on getting it like this, okay, 0 if it's 1 here, 2 here, 3 here and so on, okay, and if you go on as N goes to infinity as like that you go on, you have FN, FN if you consider for any each N this is a usual function because 0 at some place it's a finite number and other places it is 0, okay, this is a usual function but you consider the limit as N goes to infinity what you get is 0 for every value X is not equal to 0, okay, because eventually this is 0, around 0 both your eventual is 0, at 0 you are getting as you see there is a 3, 3, N is value is N at 0, that's some small neighborhood of 0 it's N.



So at X = 0 that is infinity, so such a function, so this is not a function what you get the result of this usual function, but this is called the generalized function and it's a limit to, what do you mean by this limit, this converges to this function? This is a some average sense you have to look at it that is F(x) you consider some G(x) take the average wherever this integral makes sense, now you look at this N goes to infinity this average minus infinity to infinity, so that this integral makes sense, you consider all G(x) such that this integral makes sense, this is equal to, if I call this as some new function let us say generalized function capital G(x), so what you have is G(x) into your any test function if I call this G, any G(x) that makes sense, okay, so this converges to the finite quantity like this, so this limit converges on an average FN converges to G(x) that is the meaning of this, so that is a weak convergence, okay, on an average means a weak convergence, so that means FN(x) converges weakly on an average to G(x), that is the meaning, okay, so that is what happens for certain limit of functions such as this.
So delta function is one such thing, so delta function is generalized function, so how do I define this delta function as a delta(x), which is 0, at X = 0 at X equal to, X naught = 0, infinity at X = 0, so this I can represent this as a limit of some usual functions FN(x) as N goes to infinity, where FN(x) I define it as, I can define this as, how do I define? Yourself construct, if you take this -1/N to 1/N for example, okay, so if X is between -1/N to 1/N this is value is, its value is N for example, so what is the difference? This integral value is, and so I define like this N outside otherwise 0, for example let me put it, it's not, so what happens? For each N I maintain, and integral value for minus infinity to infinity FN(x) DX if you take the integral value that I maintain as a constant, okay, so such a limit of functions is called delta functions.



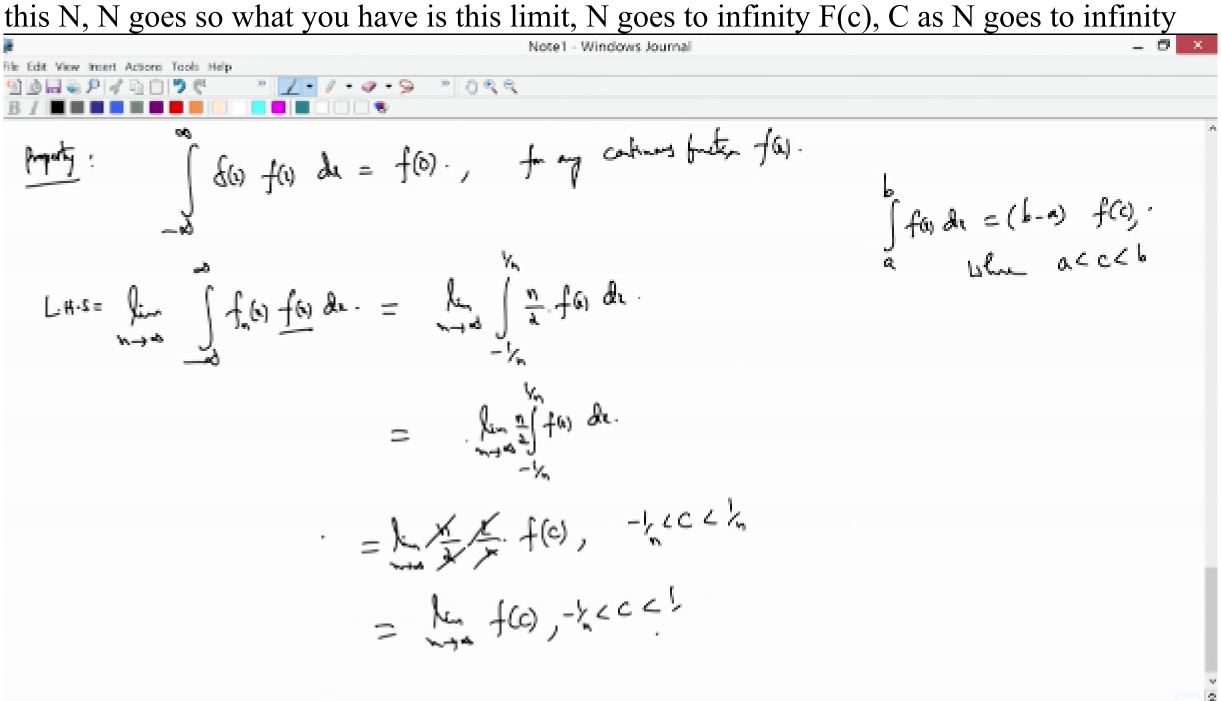
So delta function not only, this is a limit of usual functions but you also maintain this area for example this integral value has to be 1, so if I choose this here FN(x) N times -1/N to 1/N DX, so this is N times, what is the value of this? One, so that is 1/N, 1/N - +1/N, so you have 2/N, so you have 2/N, so in order to get as 1 so this should be N/2, so I define this as N/2, if I define N/2, so this will be simply 1, value is maintained for every N, okay, so because of this you have this properties true, okay.



So you have many representations for such delta function, so you can have a different FN values, different sequences converges to this property, converges to this type of function and also keeping the average value is 1, integral value is equal to 1, such a function is delta function, that place an important role in the physical and engineering sciences, for example when you consider instant instantaneous loads in a civil engineering so you consider, you use these delta functions, okay, so many other places you can use this delta function, so we also use, make use of this, we also use this delta function, you can also apply Fourier transform, you can also do Fourier transforms on these delta functions in our course so these are also useful, you can also apply Laplace transform on this delta function.

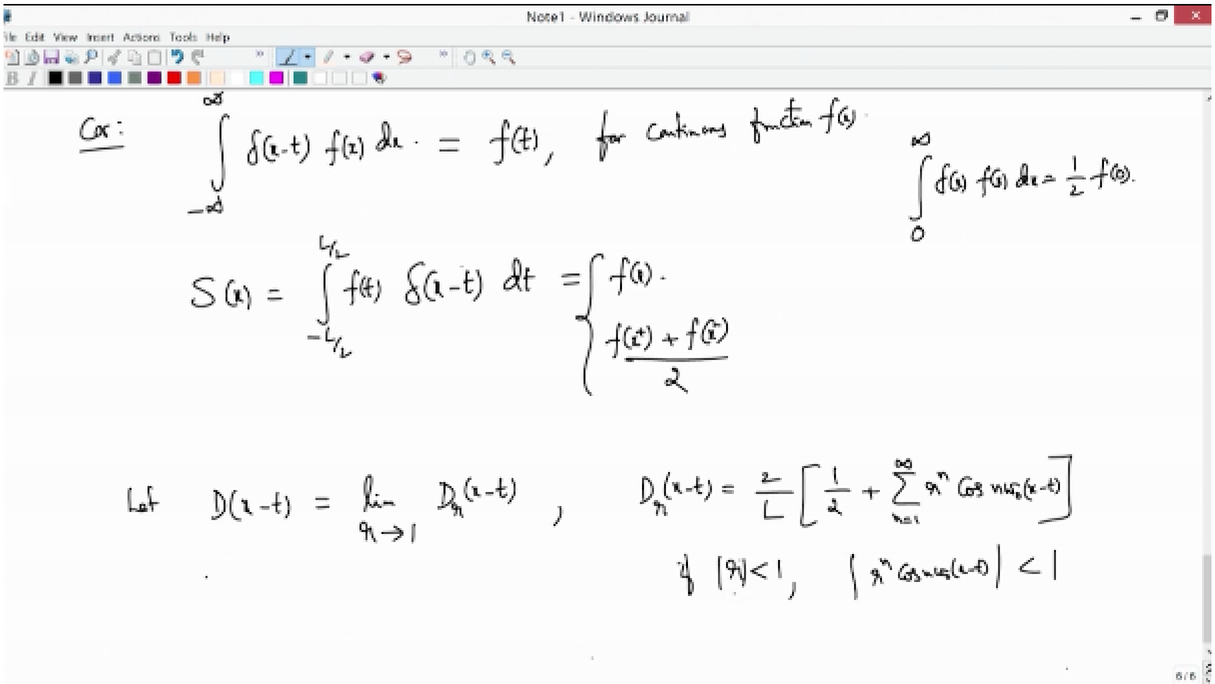
So once you have this delta function so I will try to prove here that one of the property of the delta function is, once you have such a delta function what you have is average value of this FN(x) if you multiply any F(x) or rather a delta(x) you multiply with the F(x), this value equal to DX = F(0), so how do I prove this? This is the property, property of delta function, so this proof you can easily see as minus infinity to infinity, delta you have seen that is usual limiting value of, this is actually you can write it as limiting as N goes to infinity, FN(x) F(x) D(x) okay, so LHS is this, so this you can write it as a limit N goes to infinity, so this is from – 1/N to 1/N and you have FN(x) is N/2 and F(x) D(x) okay, so this is N/2 which of your constant comes out, limit N goes to infinity, this is from -1/N to 1/N F(x) D(x), so by calculus you can have this average value you can consider N/2 as 2/N times F at some value, so you have if F is the usual function so F is a, this here multiplication function that we have for any usual function F(x) okay, for any continuous function, let us say, continuous function or a differentiable function, okay, F(x), so if I choose such a F(x) you have this theorem in your calculus F(x) D(x) this value actually B - A times F(c), as C is, F is continuous, C is the value between A and B, okay for some C like that, so you have you can write like here, so you can have some C where C is between -1/N to 1/N.

So if you have this now as you, now you have the limit, so you still have the limit, so you can try at outside so this is the limit okay, so this limit once you have this limit N goes to infinity. so this N, N goes so what you have is this limit, N goes to infinity F(c), C as N goes to infinity



this becomes -1/N to 1/N you end up getting, as N goes to infinity this is nothing but F(0), so this is the property that we have, this is your right-hand side, so with this preamble this delta function we can have, we can make use of this delta function here. so what we have is this

Fourier series that we have S(x) is actually F(t) times this if I show that this D(X of T) is delta function behaves like, if this is actually kind of a delta function then I can show that this is value is equal to F(x), okay, so that's what we will see.
So one immediate thing which we have is, if I have instead of 0 if my instantaneous load is, my concentration is only at 0, okay this is how you are putting your concentration, limit of sequences, so what is the delta function of X-T is a concentration, as a function says the limit of FN(x-t), the concentration it's some value T, okay, so you can have such a thing.
Then you have some immediate corollary of these properties, integral delta(x-t) into F(x) DX as minus infinity to infinity is actually equal to F(t) okay for continuous F(x) continuous function for any continuous function F(x) so this is what exactly we use there your Fourier series, what we have is one by, what we get is –L/2 to L/2, F(t) so what we have is a D(x-t), if I show that this is actually kind of delta function, so we have DT this becomes F(x) if F is continuous, okay, so that is exactly what we are going to show.
You can also use make use of the delta functions, if it is a discontinuous points, if it's discontinuous, function is discontinuous you can also write this as, you can have one more property that if you use 0 to infinity, delta F(x) DX this will give you because of your concentration only, you're only looking at the positive side so you have only this part if you look at it, it contribution will be half so you have half of F(0), so if I do like this, if F is discontinuous point you can use this, if F is continuous this one, otherwise F(x+) + F(x-) divided by 2, half of F(x+) for F(x+2) infinity, other one is, other part is minus infinity to F(x-) part up to here will have half contribution here, half contribution here, because of this property, so only thing I have to show is D(x-t) is actually delta(x-t), so how do I show this? So to do this let D(x-t) as, I put it as a limit, X goes to, sorry, so I introduce some new parameter here as DR(x-t), so where DR(x-t) is, you have this series L/R 2/L times, 1/2 + this is actually 1/L I am writing like 2/L times 1/L so that I can take this series, 2/L out this series, N is from 1 to infinity, so instead of simply cosines that we have for D(XT) you write introduce R power N times cos N omega naught X-T DT, X-T, so this is what you have.
So by introducing this as R is less than 1 what you have is this modulus of this inner product in this sum is always less than 1, if mode or less than 1 then what you have is mode R power N

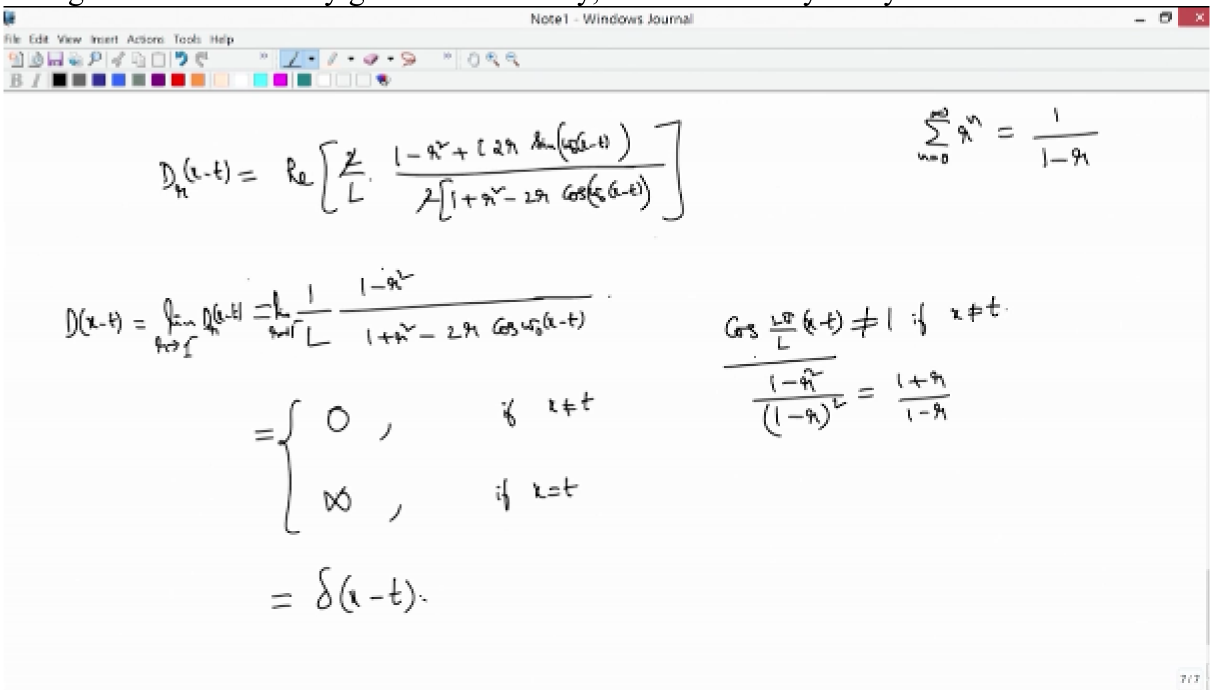


cos N omega naught X-T is also less than 1, so because of this I can sum because this becomes a geometric sum so I can sum it up easily, so for that reason only for R less than 1, more or less than 1 this makes sense, is finite, so as R goes to 1- because less than one side you approach the limit that you define it as D(X of T), so if you allow R goes to, limit of R goes to 1- this is actually D(x-t) that you can easily see, put R = 1 that is exactly what you have, that is your D(xt).

So by defining like this your D and then you can write, you can sum it up this you can add it and you can write this as limit R goes to 1- and you have a real part of 2/L times 1/2 + sigma N is from 1 to infinity, R power N so I write simply instead of cosine I write exponential function so that I write real part of it, E power IN omega naught X-T okay, so remember omega naught is 2 pi/L here, so now if you can add this 1/2, so this becomes, so if you add this series, this series I can add and you can easily see that 2/L that is our DR okay, DR(x-t) if you add this series you'll see that 2/L, so if you add this what you see is real part of 2/L times, this becomes 1 - R square + I 2R sine X-T omega naught divided by 2 times 1+R square -2R cos omega naught X-T, you can write here as sine omega naught X-T as one term for sine.

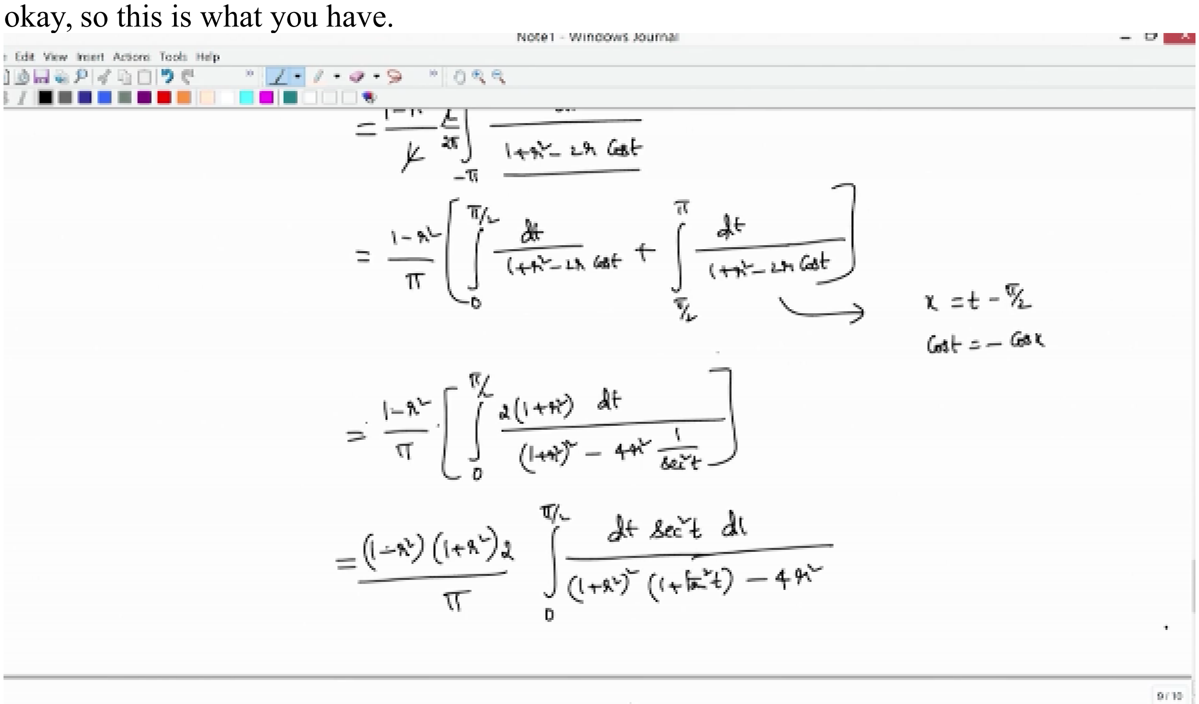
Similarly for cos omega naught as put it as one term, so this is what you can easily see by adding this as geometric series that is like sigma R power N, N is from 1 to infinity is actually R power 1 is R, so R divided by 1 - this geometric thing is that is R, okay, something like this or you have N is from 0 to infinity, R power N is simply the first term is 1, 1-R, okay, this is if I use the sum you can easily see that this is the result, okay.

So this you have to take the real part, if you nicely see so 2 2 goes so you end up getting 1/L, 1-R square/1+R square -2R cos omega naught X-T, okay, now what you do is you know that D(x-t) is a limit of our DR(x-t), as R goes to 1 -, so you take this limit R goes to 1 -, so if R=1 you can easily say if X equal to, so you can easily see if X is not equal to T this cosine function never be 1, if X is not equal to T cos omega naught, cos 2 pi/L times X-T as long as this is not equal to 0 this never be 1, if X is not equal to T, so for that reason this denominator never be 0 and what you get is numerator is, but numerator is going to be 0 so you have 0 here, otherwise what you have is, if it is 1, if X = T you have this becomes 1 and you have this becomes 1-R whole square, numerator is 1-R square, so you have this is nothing but 1+R divided by 1-R, so as R goes to 1 - is actually given to be infinity, so this is exactly like your delta function.

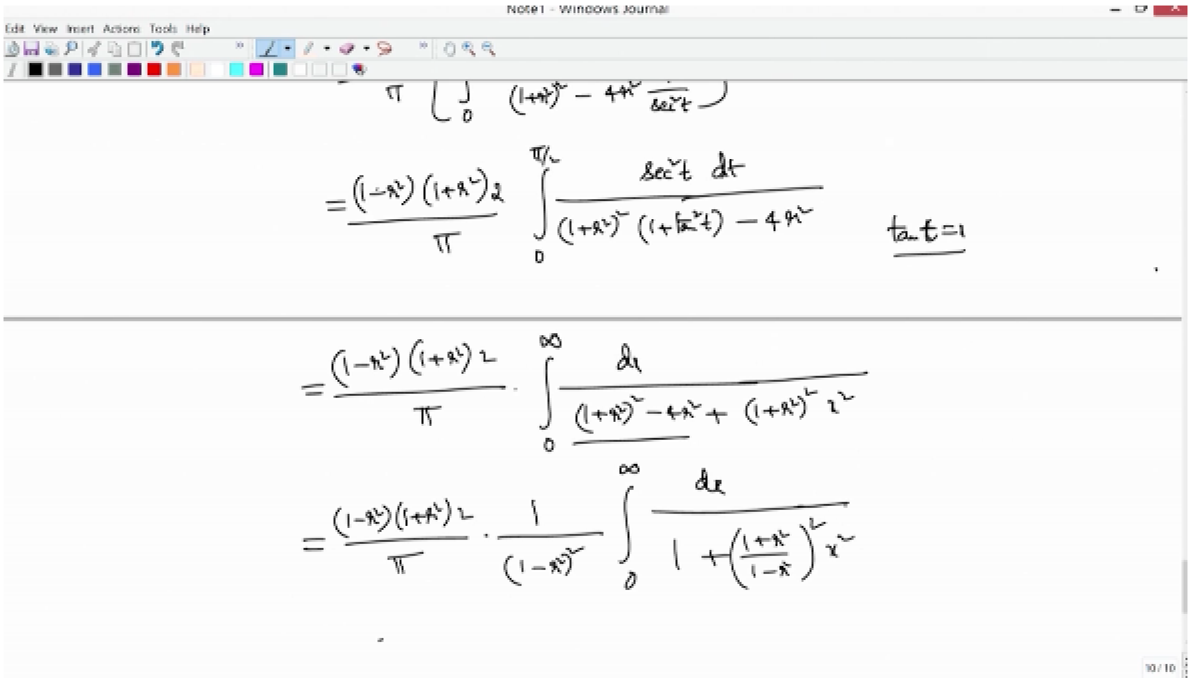


You cannot still say unless I show that integral other property of the delta function, integral value of delta, and integral value of D(x-t) DX minus infinity infinity, so you have to show that this value is equal to 1, if I just multiply with any function 1, and if any continuous function if you multiply it has to become F that function value at T, so here I choose this as 1, so if I choose this this is sufficient, so if I choose this, if equal to 1 then then D(x-t) is actually delta function, actual delta function, okay, here are still here I cannot say though it's taken 0 at X =T X is not equal to T, at X = T it's taking infinity, it looks like delta function, but if I provide only this then we clearly say that D is actually delta function.
So how do I say this? Again because we know that this is a D(x-t) DX is actually a limit of R goes to 1 -, okay, so this is minus infinity infinity, and this is your DR(x-t), because of the uniform convergence of this DR(x-t) which is a series, and I am just putting R, I can bring this integral inside and you end up getting this limit, limit you can take it as inside okay, so I can exchange these limits and you have this DX, so if I show that this value is equal to fixed, if it is equal to 1 then I am done, so to show this, this is equal to let me look at only this one, okay, to show this, this is equal to 2, to show that this is equal to 1 let me choose this DR(x-t) DX, this if you calculate minus infinity infinity, as you see DR(x-t) outside because it's coming from the Fourier series, so outside you can assume that it is 0, okay, DR(x-t) or D(x-t), between X-T is, if X-T is between –L/2 to L/2 only this is true, this is outside you can safely assume that is 0, okay, just like you have a periodic signal which is only periodic between –L/2 to L/2, outside you can take it as 0, so if I do this –L/2 to L/2 this will become –L/2 to L/2 and the DR(x-2) is 1-R square/L times 1 over 1+R square -2R cos omega naught, that is 2 pi/L X-T DX, okay, so this is equal to, so you can easily show that this quantity is equal to 1.
How do I show this? This is equal to, so you can take this out 1-R square/L –L/2 to L/2 and here before I do this, so I'll write X-T as X dash so that you have DX is DX dash divided by 1+R square -2R cos 2 pi/L times, X-T as X dash and this becomes when you put X = -L/2 to –T, this -T because of this periodicity and I can simply, this is again same as –L/2 to L/2, okay,

because DR(x-t) is 0 outside, so it's anyway periodic so it doesn't matter, so you can, this is cosine function is periodic so you can have this, this is same as this, okay.
So this is actually -T and -T because of this cosine function which is periodic so that is actually same as this one, you have 2 pi/L so you have, that's why, that is the reason, okay, so X is a dummy variable I can remove these dashes, so this is what you have.
And the next step is let's choose 2 pi/L X as T, so if I do this you have 1 - R square/L this becomes when I put this, this becomes -pi to pi, DX is L/2 pi times DT 1+R square -2R cos T, so L L goes and you have this 1 - R square/2 pi, so again this is even function, integrand is even function so you have 2 times 0 2 pi DT/1+R square -2R cos T, so 2 2 goes here so you end up simply pi.
So again this one I can split this into 1/pi part pi/2, and pi/2 to pi DT/1+R square -2R cos T, so again if I choose in this integral, if I choose T as X-pi/2 and we get X = T – pi/2, at T = pi/2 so it becomes 0, and T = pi, it is pi/2, and cos T becomes - cos X, okay, so if I use this, this change of variable in the second integral, so you end up getting 1-R square/pi times, so you see that this is going to be 0 to pi/2, so if you add this you get 1+R square whole square -4R square cos square T, DT what you have is 2(1+R square) times DT, so you have 1-R square, 1+R square into 2 /pi times integral 0 to pi/2, DT by and here one more step I do is I write cos square T as 1/secant squared T, so if I put this as 1+R square whole square, 1 secant square that I write it as 1 + tan square X, and tan square T -4R square so that you can take this secant square T DT okay, so this is what you have.

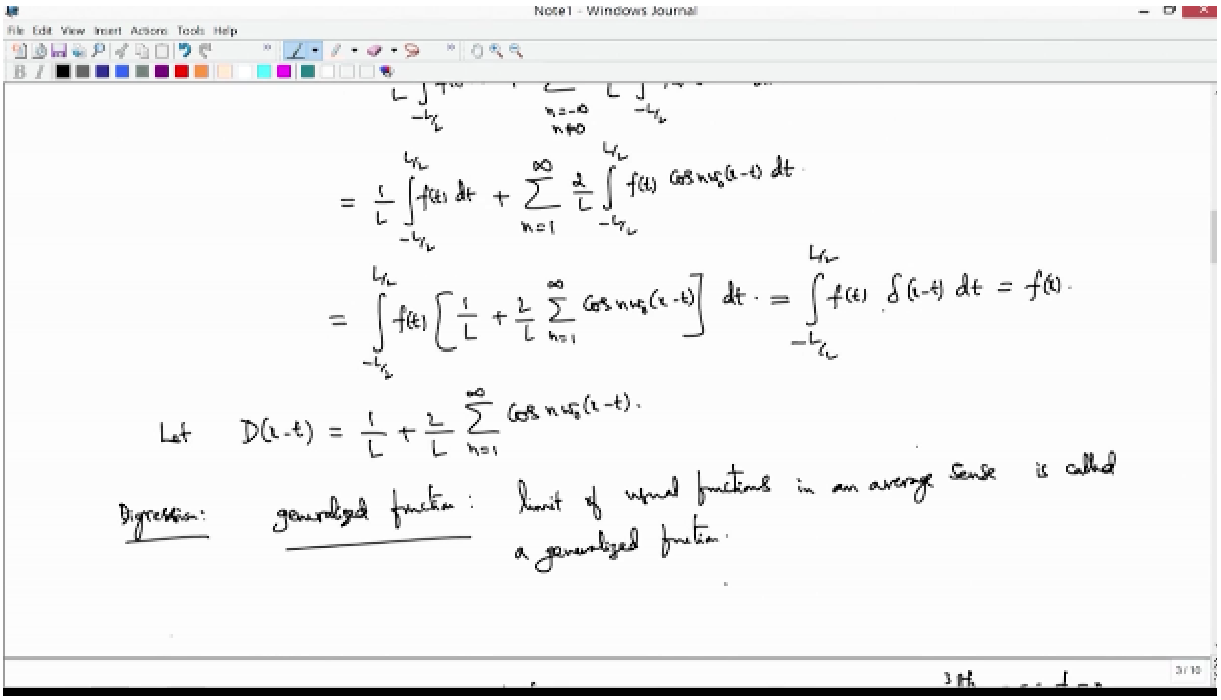


Now put tan T as X in this, so you end up getting 1-R square, 1+R square 2/pi times here, 0 to pi/2, so if I do this T = 0, tan 0 is 0, T = pi/2, tan T there is X is, X becomes infinity so you have DX 1+R square whole square -4R square + 1+R square whole square times tan square T that is X square, this is what you have so this if you pull it (1-R square) (1+R square) 2/pi, if I pull this part outside what you get is this is actually equal to 1-R square whole square, okay, so this if I pull it apart and you have 0 to infinity, DX divided by 1 + 1+R square/1-R square whole square

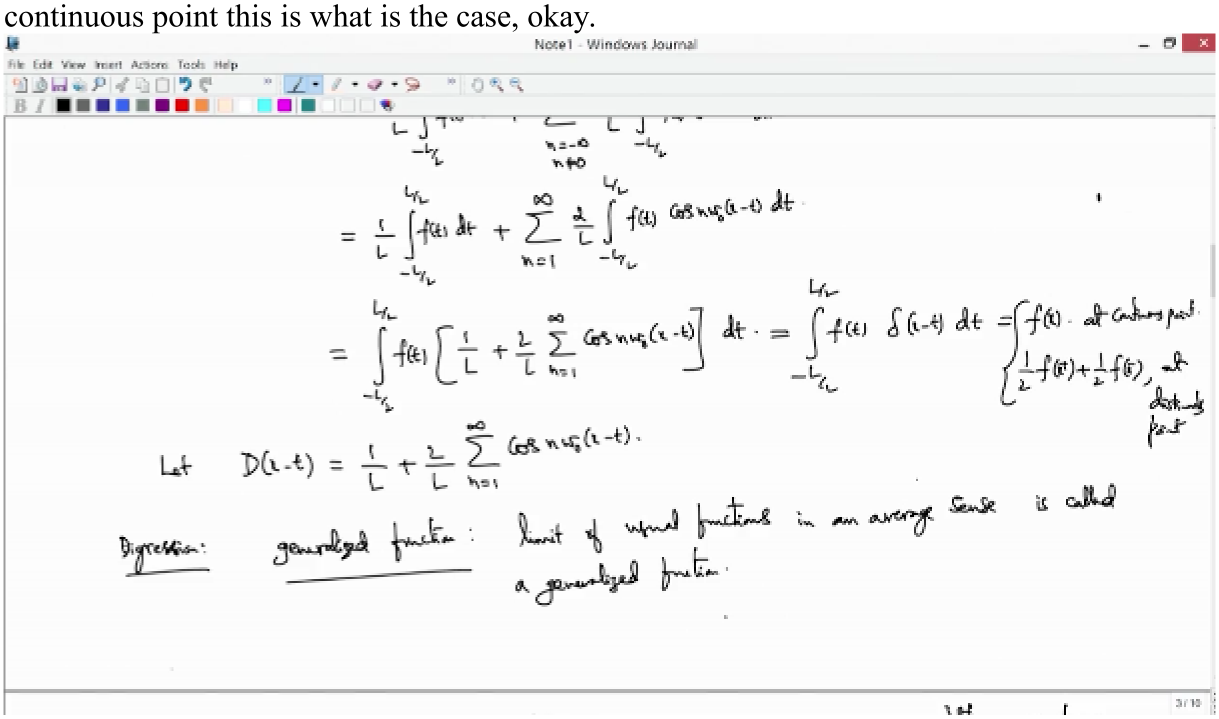


X square, so this gets cancelled here, so you end up getting 2/pi times 1+R square/1-R square integral 0 to infinity DX /1 + 1+R square/1 +1-R square X whole square, so this is actually equal to 2/pi times tan inverse 1+R square/1-R square X, this you apply 0 to infinity limits, this is at infinity this is pi/2 at 0 this is 0, so you have 2/pi times pi/2 – 0, so this is actually equal to 1.

So this is what we have shown, so this implies the DR, so this means when you come back here and write it here so this integral value have shown that for any irrespective of value of R this limit R goes to 1- is actually constant ,so it is actually equal to 1, so delta function is this and delta function integral value is 1, so this implies D(x-t) is, there's actually a delta function, so that means what we have is this Fourier series if you come back and use here this is actually if you use the property of the delta function is –L/2 to L/2 F(t), now you have this, in the place of this you have a delta function delta(x-t) DT, so this is nothing but F(x), if it's a continuous, okay, otherwise because again using the delta function property if it is a discontinuous you can



have, if F is discontinuous at X, X+ and X- both the places you have a jump, such a jump you have if it is a discontinuous jump here to here, okay, so it's minus infinity to here 1, and here to infinity 1 if you consider so each contribution by the delta is 1/2 F(x+) + 1/2 F(x-), so this is exactly what we have if at discontinuous point, discontinuous point, continuous point, at continuous point this is what is the case, okay.



So that's what we have seen that if it is continuous function this Fourier series need not converge, and if it converges actually we strongly assume that it is uniformly convergent, a Fourier series actually converges to F(x) in the process we have defined what is a generalized

function and such an example is the delta function, we made use of this delta function to show that the series actually converges to F(x) and average jump value, okay.
So with this we'll do more examples in the next video for the convergence of the Fourier series when you consider such examples usually they are in terms of elementary functions, they are piecewise smooth so that a Fourier series convergence is guaranteed, okay. Thank you very much.

[Music]

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Funded by

Department of Higher Education

Ministry of Human Resources Development

Government of India

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