

NPTEL
NPTEL ONLINE COURSE
Transform Techniques for Engineers
Conclusions
With
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Conclusions

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So let's conclude the course on transform techniques for engineers, in this course we have learnt how to find some transform techniques, in this course we have learnt a Fourier series, and Fourier transform, Laplace transform, and Z transforms.

So for a periodic signals that is functions of, functions over a finite domains we have a Fourier series as a inverse transform, Fourier coefficients which you can define as a sum infinite, Fourier coefficients as your Fourier transform.

So we have defined Fourier transform, inverse transform as a Fourier series, so that when you have a periodic signal you can represent this as an infinite series, so that is your Fourier series, based on this we have derived Fourier integral theorem formally and also rigorously we have derived this Fourier integral theorem, but provided the signal that $F(x)$ defined that should be smooth domain, that is not just for continuous form, continuous function you cannot have a Fourier series it should be a smooth function that means it should be differentiable wherever it is continuous, just continuous function you have seen that there is no such function for which you have, you have examples in the literature that just continuous function which is not differentiable you cannot have a Fourier series that is finite, that converges, okay.

But you have seen many examples piecewise continuous functions, you might think that this is not continuous, okay, it is continuous but every piece wherever it is continuous you have a piecewise, you have a function that signal is smooth that means it is differentiable within the, wherever it is continuous, and you can allow finitely many discontinuous, jump discontinuities in your function or signal, okay, so based on this we have derived a Fourier transform different

versions of Fourier transforms and we have learned how to find the Fourier transform and inverse Fourier transforms, and then based on its properties we have given a lot of properties based on which we have derived, you know, how to find the inverse transforms, inverse Fourier transform and then we made use of this Fourier transform to find, to apply, to solve some differential equations, to solve some integral equations, and we have seen other applications how to evaluate certain integrals, okay, I have seen this in this course and also based on again Fourier integral theorem we extended the class of functions for which this Fourier integral theorem is valid, we enhance the validity of this Fourier integral theorem so you end up getting Laplace transform from the Fourier integral theorem, so that means based on the Fourier integral theorem we ended up getting Laplace transform and its inverse transform, but the only thing is we paid a price here that is, it's not, you don't have domain anymore over the real line you spilled over to complex plane, so you have to deal with complex integrals, so you have Laplace transform is an integral over 0 to infinity, but your inverse transform is basically in the complex plane.

So that is a price you paid by extending the class of functions for which you have Fourier integral theorem is valid, okay, so they're all interlinked all these transforms are interlinked so far and again we explain how to find this Fourier Laplace transforms for simple functions and then we use the, we derived certain properties based on these properties you can get this inverse Laplace transform for general function you have seen how to evaluate those contour integration techniques you can apply, you can use this contour integration techniques to find the inverse transform, and then use this Laplace transform in the applications, for example you can solve ordinary differential equations, partial differential equations with initial, especially when you have the initial boundary conditions both are provided for your partial differential equations you have, you normally use this Laplace transform technique, that's because for that's why we use this Laplace transform technique to solve hyperbolic equations that is a wave equation and parabolic equations or typical equation is heat equation, so that's where you have time is involved so you have initial value can provide, so but you cannot use this for the steady state heat equation that is elliptic equation or a Laplace equation as an example, so we cannot use this because of this, you have to provide only boundary conditions, if you provide only boundary conditions you will not have, when you apply the Laplace transform you need the initial value set either of the variable we use.

For example if you apply Laplace transform to U_{XX} you need $U(0,y)$ and $U_X(0,y)$, so both these things are required but if you provide any of these two and you may end up a problem, the boundary value problem is not stable, the stable means we'll just give an example of a Hermod example you can see this example in this slide, Hermod example is when you solve this Laplace equation in this upper of plane, this is a upper of plane and you have only boundary is this $-\infty$ to ∞ at which if you provide initial value at $Y = 0$ is this, and U_Y if you provide clearly for this problem and 0 is one solution clear, right, when you have 0 you can easily see

Conclusions

(Hadamard's example)

$$u_{xx} + u_{yy} = 0, \quad x \in \mathbb{R}, y > 0$$

I.C's:

$$u(x, 0) = 0$$

$$u_y(x, 0) = \frac{\sin nx}{n}$$

Solu:

$$u(x, y) = \frac{1}{n^2} \sin(nx) \sinh(ny)$$

that 0 is 1 solution, okay, for example if you provide let us say $U(x,0)$ is 0 okay, and $UY(x,0) = 0$, you take 0 condition here 0 0 here if you take let's take $U_{XX} + U_{YY} = 0$, X belongs to \mathbb{R} , Y positive, so this problem has a 0 solution, so solution here is 0, $U(x,y)$ is completely 0, identically 0, okay, this is one, now if we consider, if I change this initial data for example the second initial data that I give sine NX/N , if you take N very big value then sine NX/N is very

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$$\left\{ \begin{array}{l} u_{xx} + u_{yy} = 0, \quad x \in \mathbb{R}, y > 0 \\ u(x, 0) = 0 \\ u_y(x, 0) = 0 \end{array} \right.$$

Solu: $u(x, y) \equiv 0$

small quantity, so that means I give little perturbation in this initial data, and if you give such initial that sufficiently big N , what happens? This problem as a solution here $1/N^2$ sine NX into sine hyperbolic NY , so if N is sufficiently big N^2 is growth is smaller than sine

hyperbolic \cosh , because it is exponential growth $e^{NY} + e^{-NY}$ divided by 2, that is your sine hyperbolic.

So as N goes to infinity, N is sufficiently big this solution is away from 0, so as you see, you give the initial condition as sufficiently big N this quantity is very close to the initial condition of this problem, so these are very close but the solutions are not very close, okay, but as N goes to infinity these two deviate from each other, so that means the solution is not depending on initial data continuously, continuously the solution is not continuously depending on the initial data, so that is why we don't solve this initial value, initial values we don't provide to this Laplace equation.

So assume that one of this data is you can think of as unknown, as an unknown you keep it and then you provide, you are using the boundary data whatever you have you apply the Laplace transform and you may end up at the end you try to apply, you try to whatever you get the solution that using that solution you can get your, you can apply this other initial condition to see it may work so you can apply such technique that means you have, if you apply, if you have a boundary value problem for the Laplace equation try to apply Laplace transform to one of the variables and you need another initial data that is for example U_x or $U_y(x,0)$, $U_y(x,0)$ or $U_x(0,y)$ okay that you don't know because you have to provide only boundary data for the Laplace equation, because it's elliptic type.

So you assume that whatever is required, requirement after applying the Laplace transform you call this some general function as an unknown function, okay, let's call this some $C(x)$ and as though you know everything so you apply everything the only thing is that you may not be using all the boundary data when you if you apply the Laplace transform, so in that case whatever you have not used you try to apply at the end, and try to get this unknown function $C(x)$ which is an initial data, initial data $U_y(x) = 0$ if you call this as $C(x)$ which you assume you can get this unknown function arbitrary function $C(x)$ from one of the boundary data that you have not used, so you can apply that that technique may work or may not work, okay, so we don't normally that is the reason we don't normally use Laplace transform for the boundary value problems alone, so you need if you have a initial boundary value problems this technique is very useful to solve some partial differential equations, okay.

So that's what we have seen in the Laplace transform applications and we have in the communication theory you have signals that are discrete signals we normally report we have a plenty of data that is discrete type so you can apply the signal processing technique to get this discrete transform that is similar to the Laplace transform, so it's called Z transform, again we derived based on only Z , definitions of Laplace transform and it's inverse transform you get, you get Z transform and it's inverse transforms and again we proved for all different signals sequences discrete signal is nothing but sequence data that is for different sequences we have derived Z transforms and also how to find out and we give some properties to find the inverse Z transform.

And then we applied Z transforms to solve some differential equation, discretize version of differential equations those are difference equations, so you can solve this with these difference equations with initial data by Z transform technique. Also as an another example we have evaluated certain infinite sums involving a parameter using these Z transforms with additional properties that we have provided in the last video of this course, okay. So with this we have learned different types of continuous and discrete transform techniques and their applications that are useful for most of engineering and physical sciences. Thank you for watching.

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